THE VISCOUS STRUCTURE OF BAROCLINIC CRITICAL LAYERS IN STRATIFIED SHEAR FLOWS WITH BACKGROUND ROTATION

Patrick HUERRE, Meng WANG & Philip MARCUS

Department of Mechanical Engineering, UC Berkeley
Lanoratoire d'Hydrodynamique (LadHyX) Ecole Polytechnique

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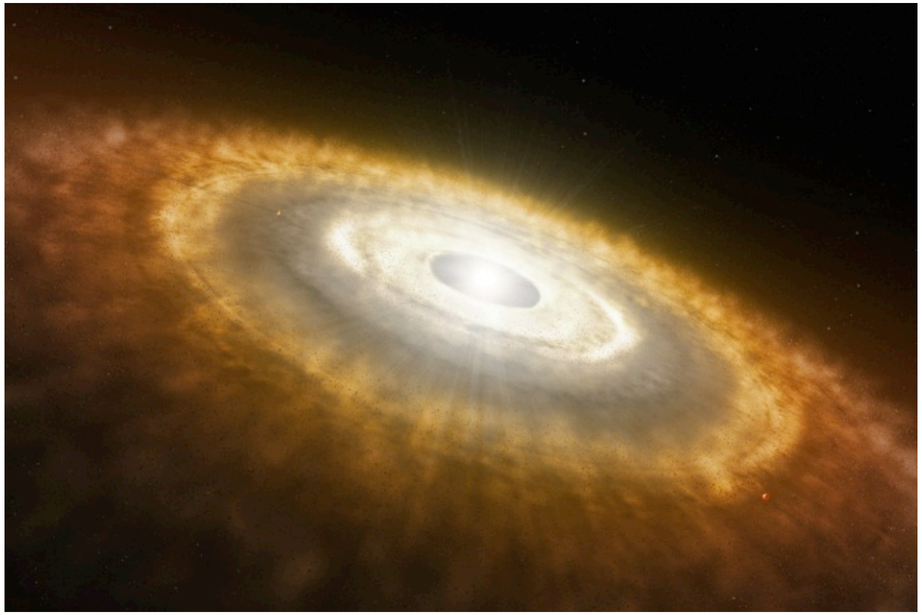
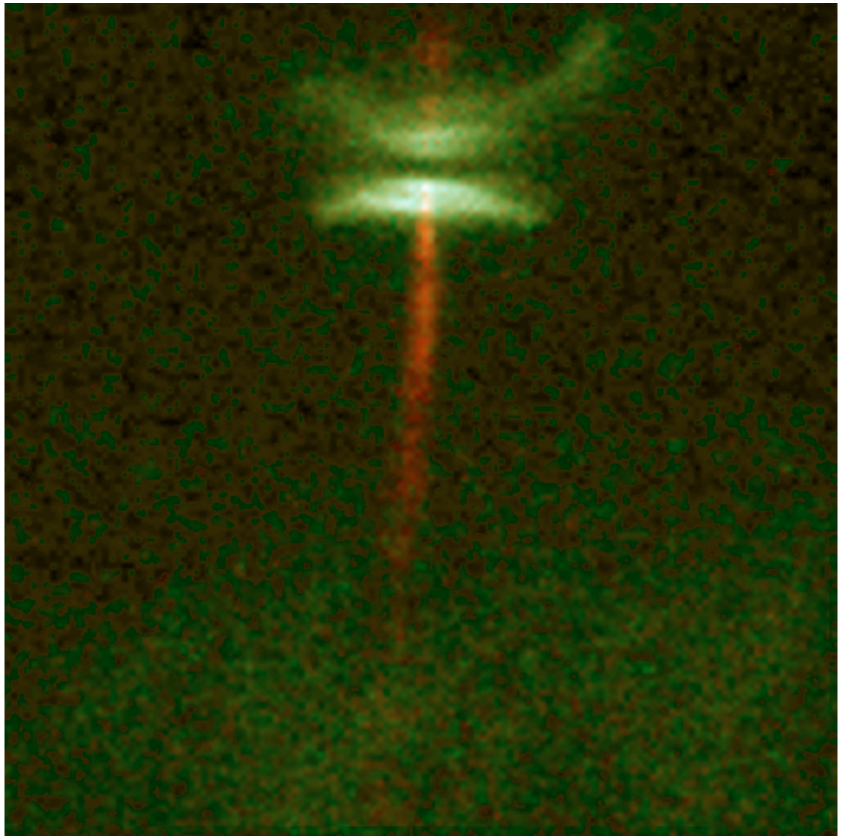
PROTO-PLANETARY DISKS (PPD): HOW DO STARS AND PLANETS FORM FROM PPD'S ?



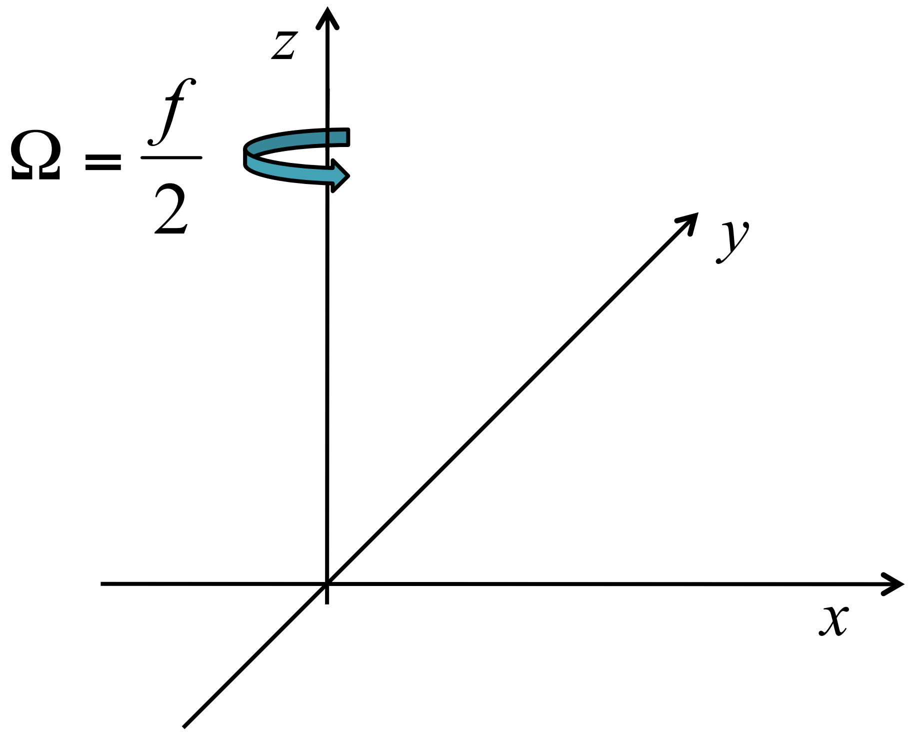
Artist's impression of " baby star " surrounded by a proto-planetary disk



Proto-planetary disk of proto-star HH-30 in Taurus constellation (ESA-NASA-Hubble)

ZOMBIE VORTICES

Idealized uniform shear flow schematic and baroclinic critical points



Marcus, Pei, Jiang & Hassanzadeh (2013)

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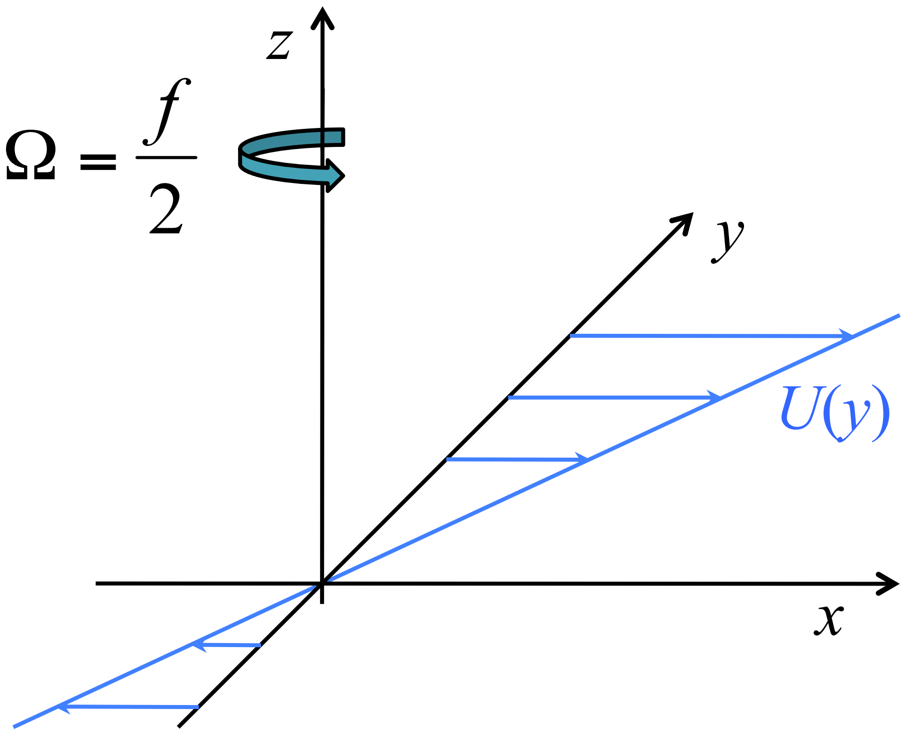

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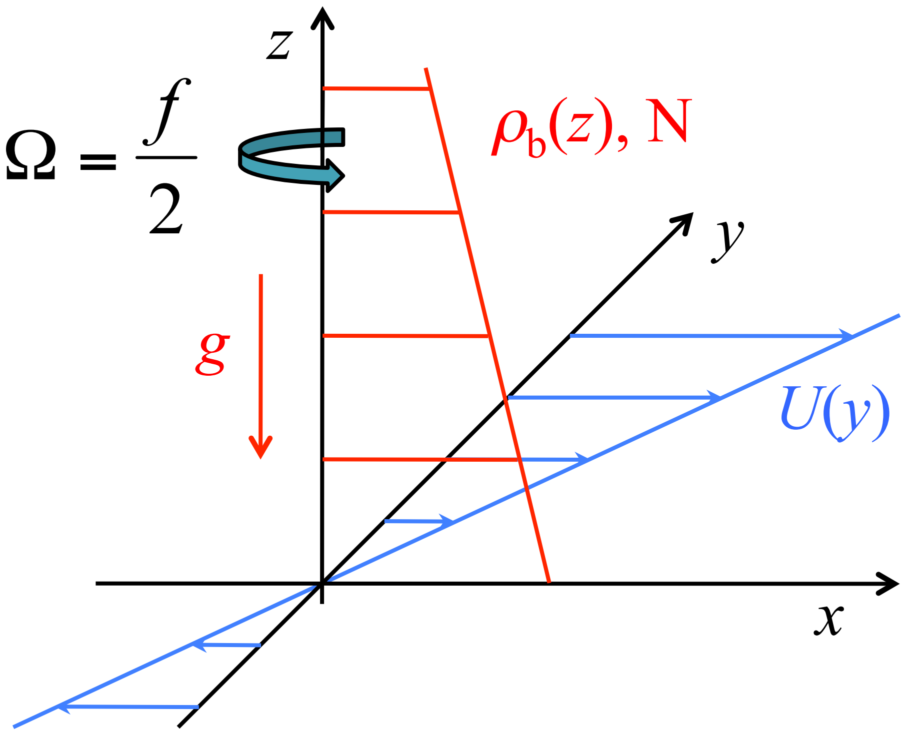
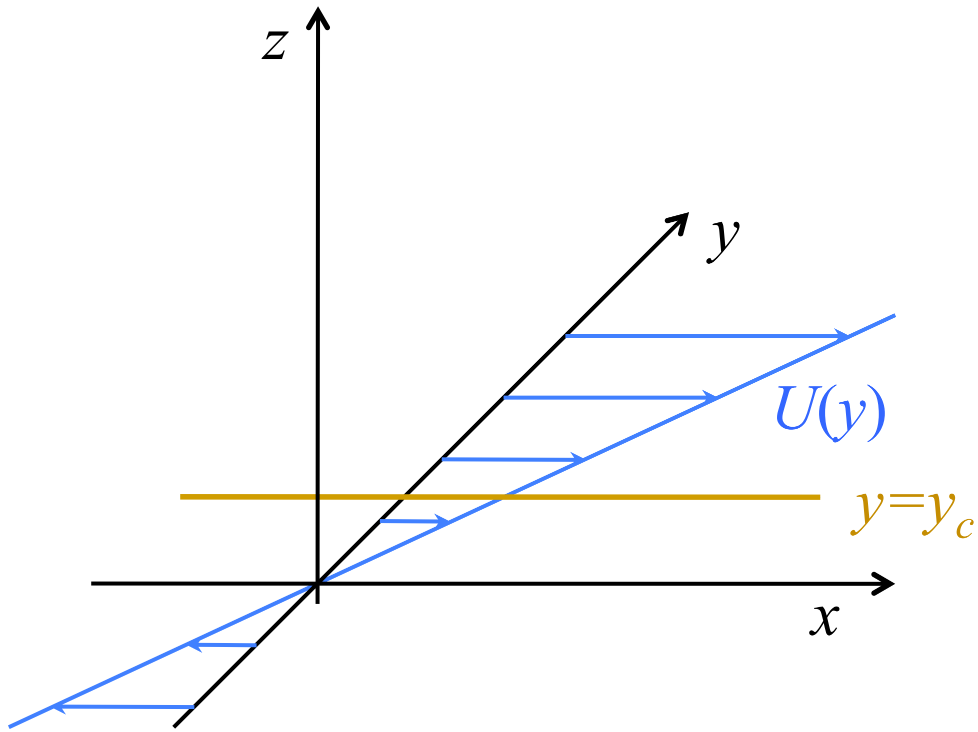


Marcus, Pei, Jiang & Hassanzadeh (2013)

ZOMBIE VORTICES

Idealized uniform shear flow schematic and baroclinic critical points

$$\omega - k_x U(y_c) = \pm N$$

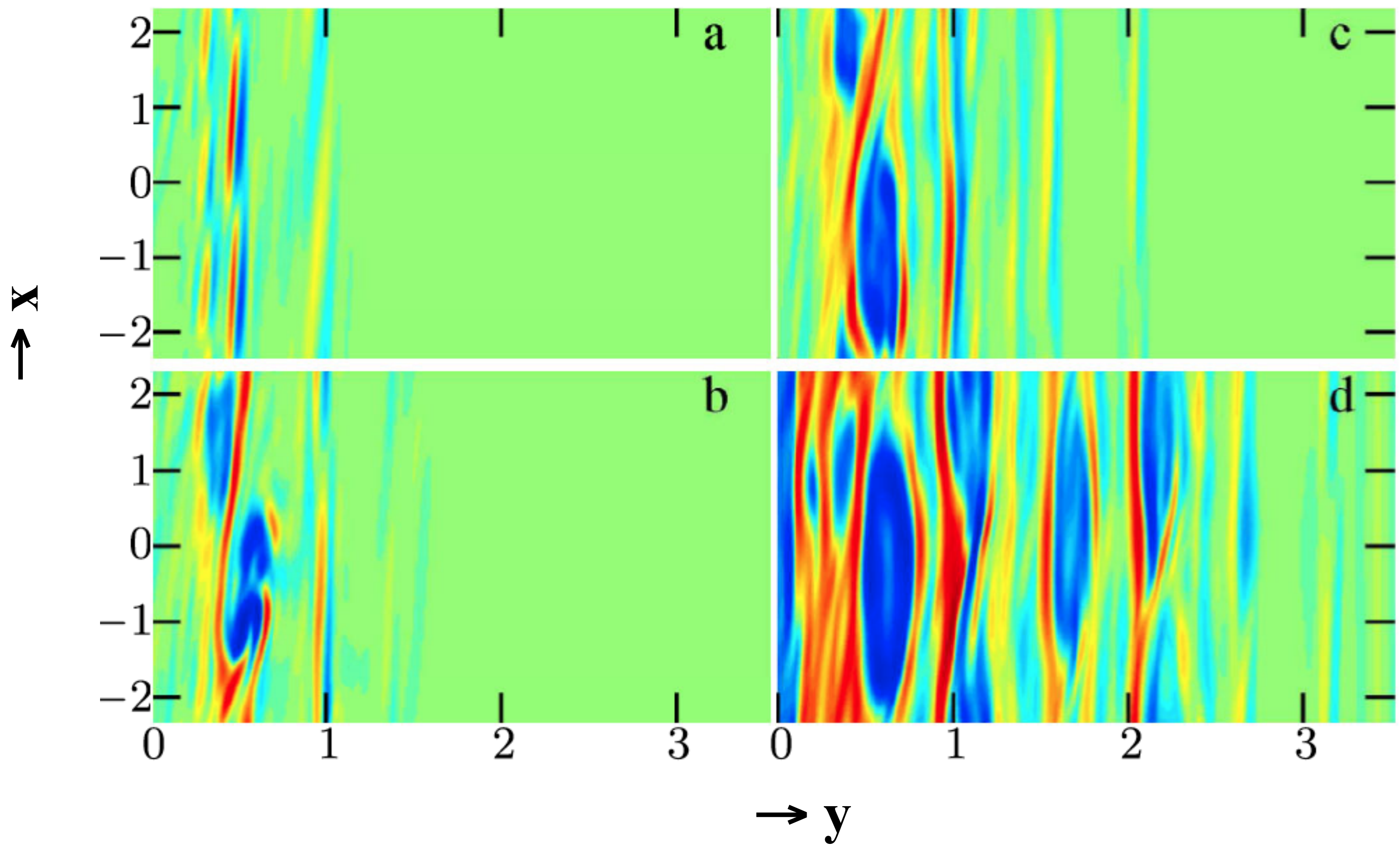


Marcus, Pei, Jiang & Hassanzadeh (2013)

ZOMBIE VORTICES

Inviscid numerical experiment:

vertical perturbation vorticity ω_z in horizontal x-y plane



Marcus, Pei, Jiang & Hassanzadeh (2013)

LARGE REYNOLDS NUMBER CRITICAL LAYER ANALYSIS

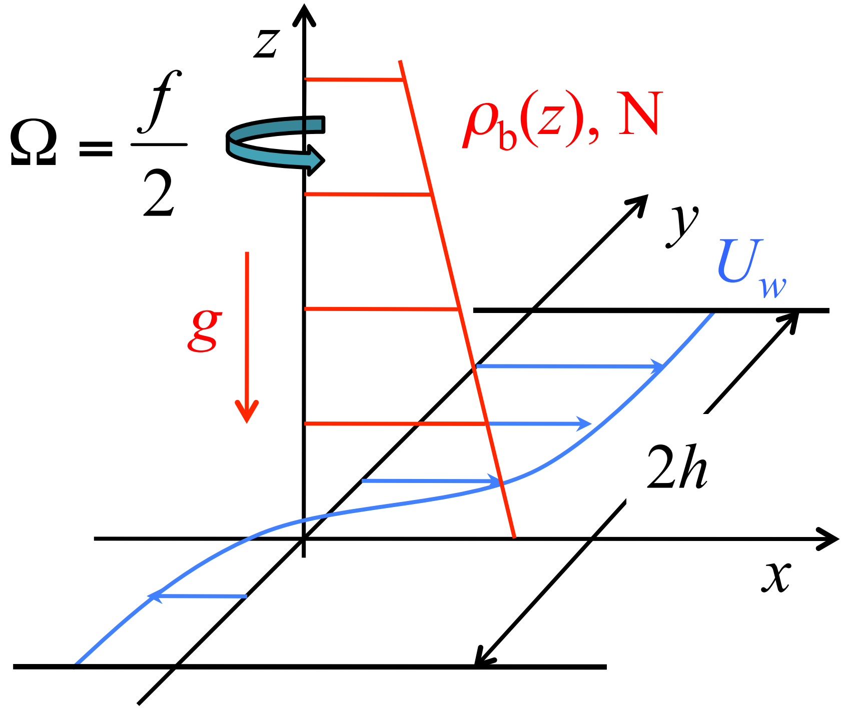
Linear stability analysis of horizontal parallel shear flow with vertical linear stratification and background rotation

Horizontal Froude number: $Fr_h = \dfrac{U_w}{Nh}$

Rossby number: $Ro = \dfrac{U_w}{fh}$

Reynolds number: $Re = \dfrac{U_w h}{\nu} \gg 1$

Schmidt number: $Sc = \dfrac{\nu}{\kappa}$



Normal mode: $q(x,y,z,t) = \Re\left\{\hat{q}(y)\, e^{i(k_x x + k_z z - \omega t)}\right\}$

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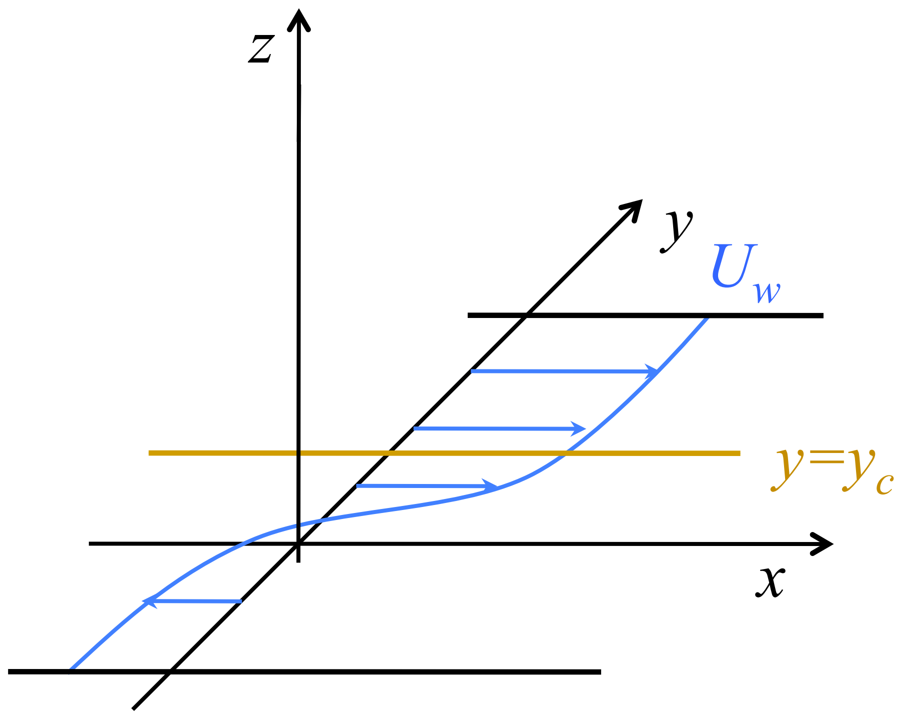

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LARGE REYNOLDS NUMBER CRITICAL LAYER ANALYSIS

Inviscid "Rayleigh-like" outer region
Nature of critical point singularity

Baroclinic critical points: $U(y_c) - c = \pm \frac{1}{Fr_h k_x}$

When $|y - y_c| \ll 1$:

$$\hat{u} = \frac{\left(U'(y_c) - \frac{1}{Ro}\right) a_0 \ln(y - y_c) - \left(U'(y_c) - \frac{1}{Ro}\right)(c_1 - a_0) - k_x \frac{1}{Fr_h} c_0}{\frac{1}{Fr_h^2} + \frac{1}{Ro}\left(U'(y_c) - \frac{1}{Ro}\right)} + \cdots ,$$

$$\hat{v} = \frac{-i \frac{1}{Fr_h} a_0 \ln(y - y_c) + i \frac{1}{Fr_h}(c_1 - a_0) - i k_x \frac{1}{Ro} c_0}{\frac{1}{Fr_h^2} + \frac{1}{Ro}\left(U'(y_c) - \frac{1}{Ro}\right)} + \cdots ,$$

$$\hat{w} = -\frac{k_z}{2k_x U'(y_c)} \frac{c_0}{y - y_c} + \cdots ,$$

$$\hat{\rho} = i Fr_h \frac{k_z}{2k_x U'(y_c)} \frac{c_0}{y - y_c} + \cdots$$

LARGE REYNOLDS NUMBER CRITICAL LAYER ANALYSIS

Inner viscous critical layer structure
"Smoothing" of critical point singularity by viscosity and density diffusion

"Zooming" inner cross-stream variable: $\eta = \dfrac{y - y_c}{\epsilon}, \quad \epsilon = i(k_x Re)^{-\frac{1}{3}}$

Vertical velocity: $$\hat{w}^*(\eta) = \frac{\pi k_z}{k_x}\left(1 + \frac{1}{Sc}\right)^{-\frac{1}{3}} [2U'(y_c)]^{-\frac{2}{3}}\, \hat{p}^* Hi(\xi)$$

with $\xi = \mathrm{sgn}[U'(y_c)]\left(\dfrac{2|U'(y_c)|}{1 + Sc^{-1}}\right)^{\frac{1}{3}} \eta$

Scorer function Hi(ξ): $\dfrac{\mathrm{d}^2 Hi}{\mathrm{d}\xi^2} - \xi Hi = \dfrac{1}{\pi}$

LARGE REYNOLDS NUMBER CRITICAL LAYER ANALYSIS

Inner viscous critical layer structure
"Smoothing" of critical point singularity by viscosity and density diffusion

Density: $$\hat{\rho}^*(\eta) = -i\pi \frac{k_z}{k_x} Fr_h \left(1 + \frac{1}{Sc}\right)^{-\frac{1}{3}} \left[2U'(y_c)\right]^{-\frac{2}{3}} \hat{p}^* Hi(\xi)$$

Cross-stream velocity: $$\hat{v}^*(\eta) = -i\pi \frac{k_z^2/k_x}{2U'(y_c)} \hat{p}^* \int_0^\xi Hi(t)\,\mathrm{d}t + b^*$$

Similarly for $\hat{u}^*(\eta)$.

LARGE REYNOLDS NUMBER CRITICAL LAYER ANALYSIS

Jump conditions across critical point between $\mathbf{y_{c+}}$ and $\mathbf{y_{c-}}$

Streamwise velocity:

$$\hat{u}(y_{c+}) - \hat{u}(y_{c-}) = -i\pi s Fr_h k_z^2 \hat{p}(y_c) \frac{U'(y_c) - 1/Ro}{2k_x U'(y_c)}$$

Cross-stream velocity:

$$\hat{v}(y_{c+}) - \hat{v}(y_{c-}) = -\frac{\pi s k_z^2 \hat{p}(y_c)}{2k_x U'(y_c)}$$

Reynolds stress:

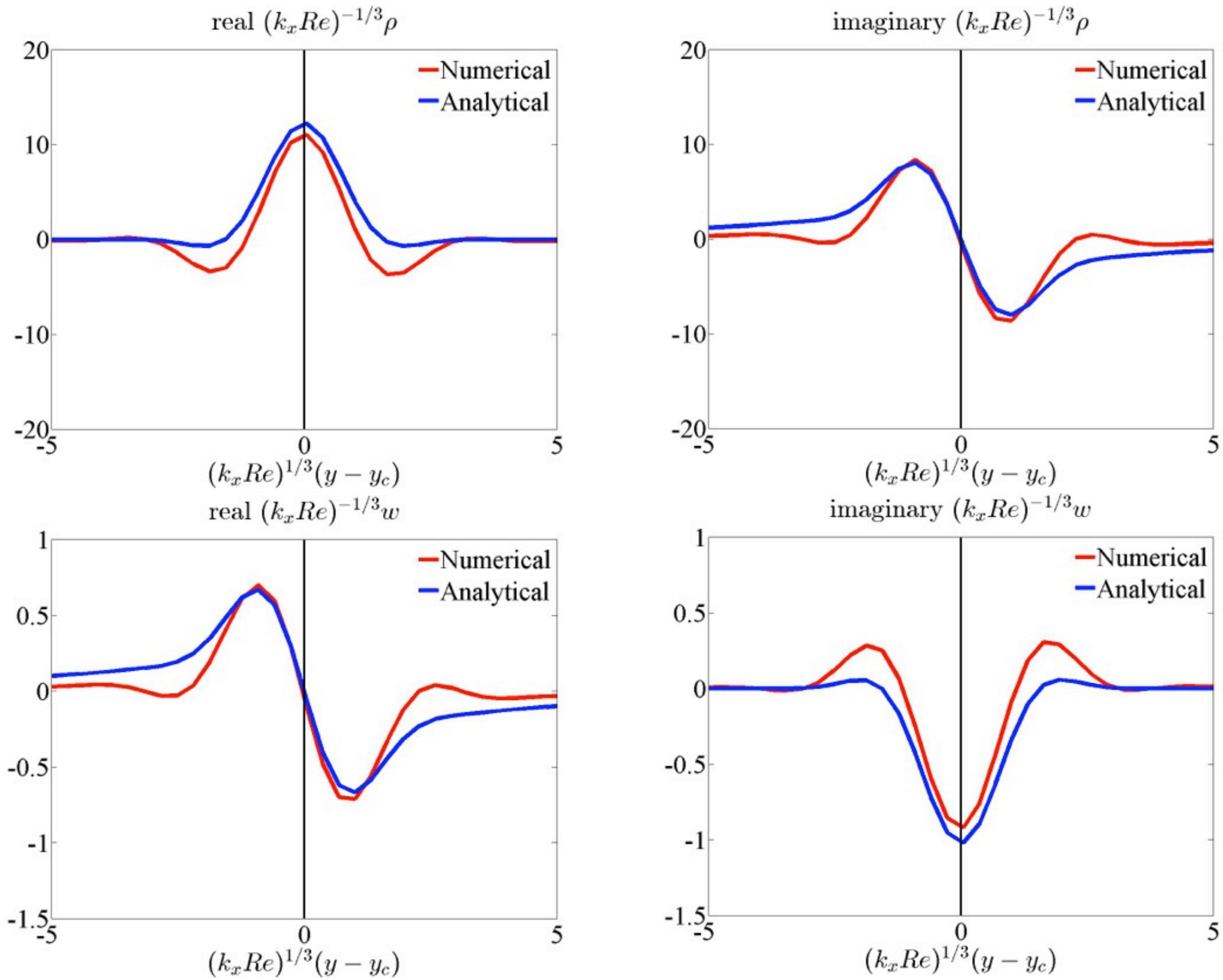
$$\tau_{xy}(y_{c+}) - \tau_{xy}(y_{c-}) = -\pi s Fr_h k_z^2 \frac{|\hat{p}(y_c)|^2}{4U'(y_c)}$$

LARGE REYNOLDS NUMBER CRITICAL LAYER ANALYSIS

Inner viscous critical layer structure

Cross-stream distributions of density ρ^* and vertical velocity w^*

$$\frac{\rho_r^*}{\sqrt[3]{k_x Re}} \qquad \frac{\rho_i^*}{\sqrt[3]{k_x Re}}$$



$$\frac{w_r^*}{\sqrt[3]{k_x Re}} \qquad \frac{w_i^*}{\sqrt[3]{k_x Re}}$$

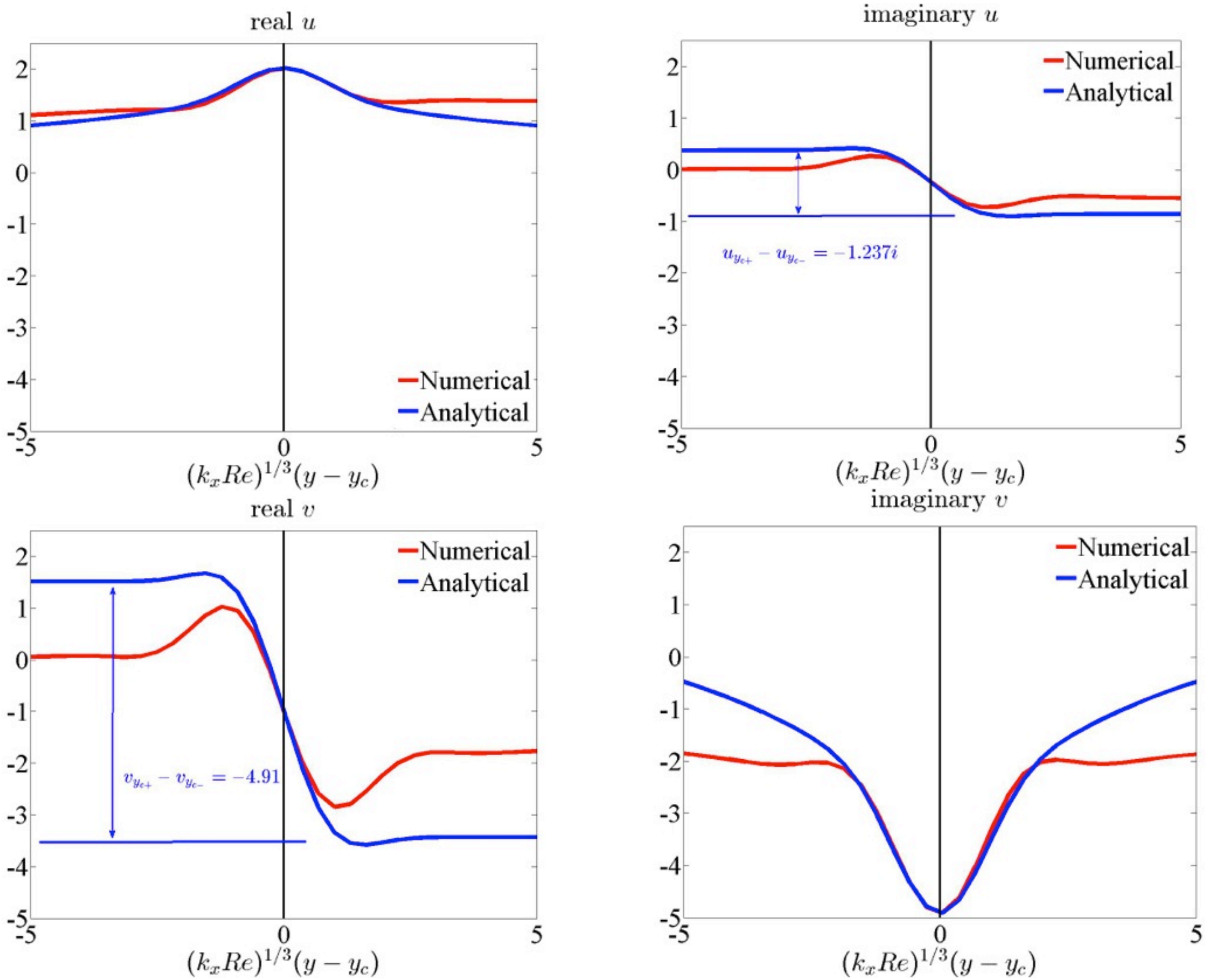
LARGE REYNOLDS NUMBER CRITICAL LAYER ANALYSIS

Inner viscous critical layer structure

Cross-stream distributions of streamwise velocity u* and cross-stream velocity v*

u_r u_i

v_r v_i



CONCLUSIONS AND ONGOING WORK

Horizontal shear flows in vertically stratified and rotating flows generate *baroclinic* critical points that are more singular than classical 2D *barotropic* critical points

The structure of *baroclinic critical layers* has been determined in the *linear and viscous régime.*

Strong jet-like and shear-layer-like vertical velocity profiles are produced which may lead to large-scale vortices (Boulanger et al. 2007; 2008) A Floquet instability analysis will tell whether these periodic flows give rise to such structures.

Ongoing investigation of the *nonlinear and viscous critical layer régime*