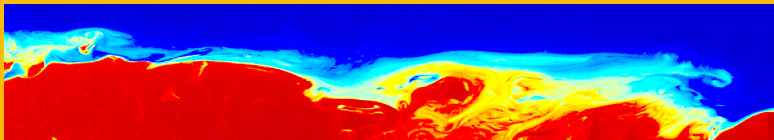


## Length scales of stratified turbulence : new insight on Thorpe displacements statistics



**L. Gostiaux**, A. Delache, E. Horne, *LMFA, CNRS, EC Lyon, France*

H. van Haren, *NIOZ, Texel, The Netherlands*

A. Cimattoribus, *EPFL, Lausanne, Switzerland*

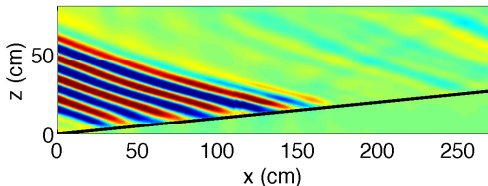
A. Venaille, *ENS de Lyon - Joel Sommeria, LEGI*

Emil Hopfinger Colloquium

11-13th May 2016, LEGI, Grenoble, France



# From linear internal waves to stratified turbulence

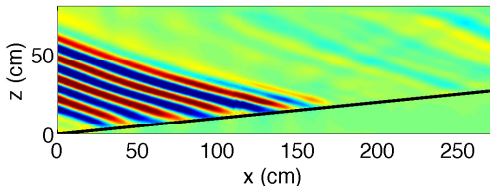


Plane waves emitted by a sinusoidal wave maker at the Coriolis Platform LEGI

T. Dauxois, J. Sommeria, C. Saquet,  
H. Didelle, S. Viboud

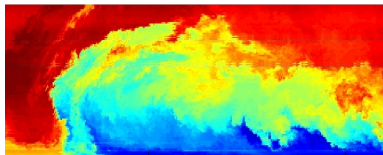


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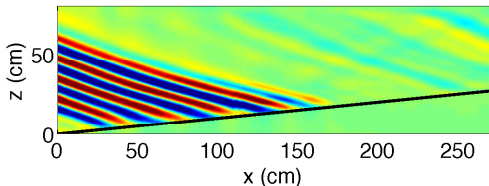


Internal wave breaking over a deep seamount, NIOZ

H. van Haren, A. Cimattoribus

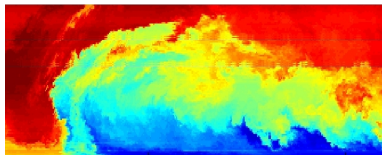


# From linear internal waves to stratified turbulence



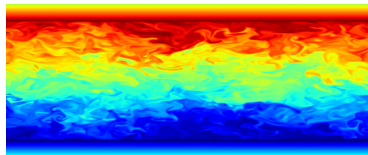
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Numerical stratified turbulence at LMFA

C. Cambon, F. Godeferd, A. Delache



# From linear internal waves to stratified turbulence

2009 version of  
Hopfinger and Toly experiment.

A. Venaille, J. Sommeria LEGI





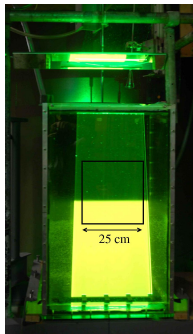
# From linear internal waves to stratified turbulence

2009 version of  
Hopfinger and Toly experiment.

A. Venaille, J. Sommeria LEGI



- Dimension :  $40 \times 40 \times 90\text{cm}$
- Grid :  $L = 10\text{cm}$ , 5Hz, 8cm amplitude
- Lower fluid : water + salt + rhodamine
- Upper fluid : water + ethanol
- $\Delta\rho = 0.1, 0.3, 0.8$  &  $1.1\%$
- $1024 \times 1024$ , 10Hz,  $25 \times 25\text{cm}$



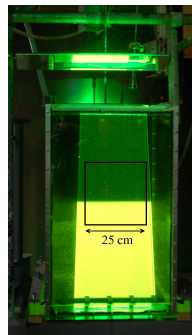
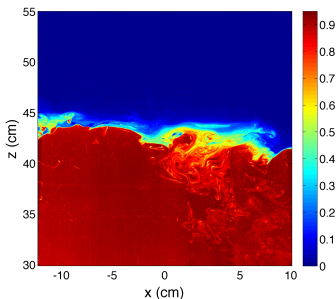
Hopfinger, E. J. & Toly, J.-A. Spatially decaying turbulence and its relation to mixing across density interfaces  
Journal of Fluid Mechanics, 1976, 78, 155-175



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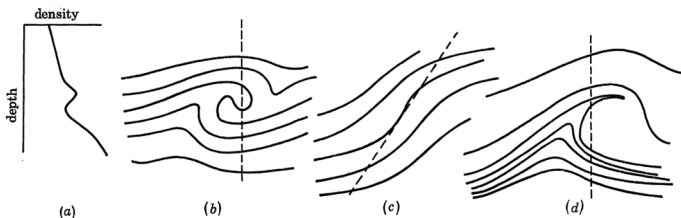
Hopfinger, E. J. & Toly, J.-A. Spatially decaying turbulence and its relation to mixing across density interfaces  
Journal of Fluid Mechanics, 1976, 78, 155-175



# Thorpe scale mixing estimate

Thorpe 1977:

*The presence of inversions may be used as a gauge of the vertical displacements which occur in the Loch.*



Thorpe, S. A. Turbulence and Mixing in a Scottish Loch, Royal Society of London Philosophical Transactions Series A, 1977, 286, 125-181





# Thorpe scale mixing estimate

Re-ordering density profiles  $\rightarrow$  Thorpe displacement  $d$ .

Thorpe scale  $L_T = \langle d^2 \rangle^{1/2}$ .

$\langle \rangle$  : vertical + successive profiles

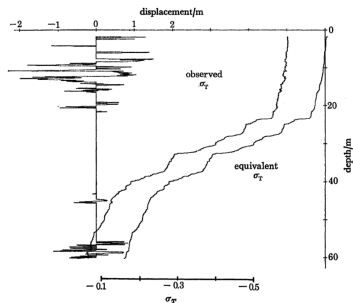
In stably stratified regions,  $d = 0$  :

$\chi$  is the proportion of unstable fluid  
(also named intermittency index)

Conditional Thorpe scale :

$$L'_T = \langle d^2 \rangle^{1/2} | d \neq 0$$

$$(L_T = \sqrt{\chi} L'_T)$$



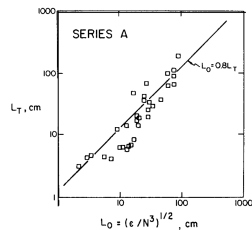
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Osborn, T. Estimates of the local rate of vertical diffusion from dissipation measurements J. Phys. Oceanogr., 1980, 10, 83-89



## On the definition of Thorpe displacement

Thorpe 77 :

*“The vertical displacements are the minimum distances which fluid particles need to be moved **from the observed profile** to produce the synthetic stable profile”*

$$\sigma(z_i) \rightarrow \sigma_s(z_i)$$

$$\sigma(z_i) = \sigma_s(z_i + d_i)$$



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$$\sigma(z_i) \rightarrow \sigma_s(z_i)$$

$$\sigma(z_i) = \sigma_s(z_i + d_i)$$

*The **inverted** vertical displacements are the distances which fluid particles need to be moved from the synthetic stable profile **to produce the observed profile***

$$\sigma(z_i + d_i^*) = \sigma_s(z_i)$$



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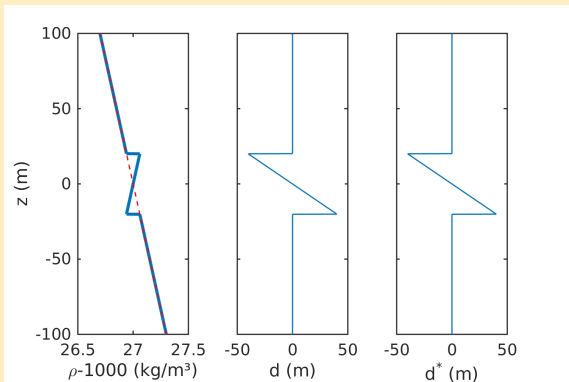
$$\sigma(z_i + d_i^*) = \sigma_s(z_i)$$

$$d_i^* = -d_{p(i)} \text{ where } p \text{ is a permutation operator}$$



# Direct and inverted Thorpe displacements

## Case 1 : solid body rotation

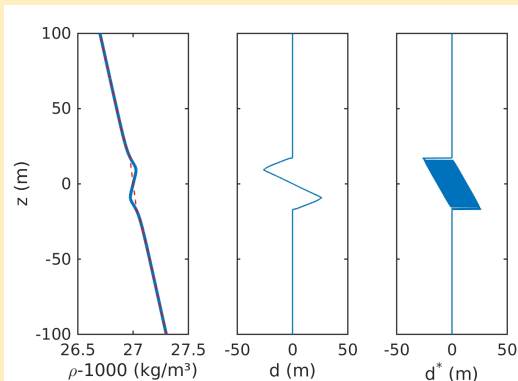


In this case,  $d_i^* = -d_{p(i)} = d_i$



# Direct and inverted Thorpe displacements

## Case 2 : Rankine vortex



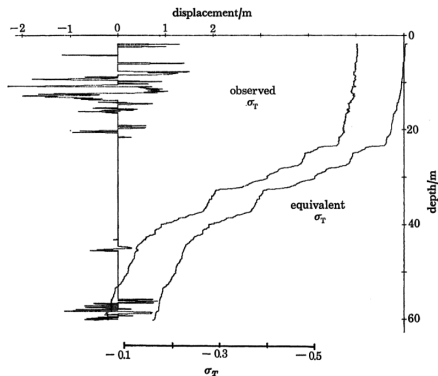
In this case,  $d^*(z)$  is discontinuous



# Direct and inverted displacements in the literature

$$L_T = \langle d^2 \rangle^{1/2}.$$

Thorpe, 1977



Thorpe, S. A. Turbulence and Mixing in a Scottish Loch, Royal Society of London Philosophical Transactions Series A, 1977, 286, 125-181



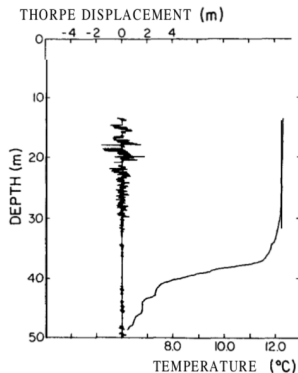


# Direct and inverted displacements in the literature

Thorpe scale

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Dillon, T. M. Vertical Overturns: A Comparison of Thorpe and Ozmidov Length Scales *J. Geophys. Res.*, AGU, 1982, 87, 9601-9613

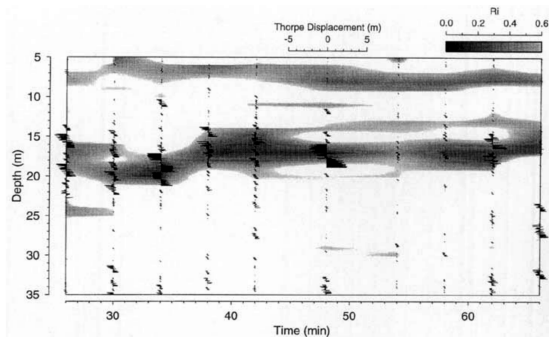


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Galbraith & Kelly, 1996



Galbraith, P. S. & Kelley, D. E. Identifying overturns in CTD profiles *Journal of Physical Oceanography*, 1996, 13, 688-702

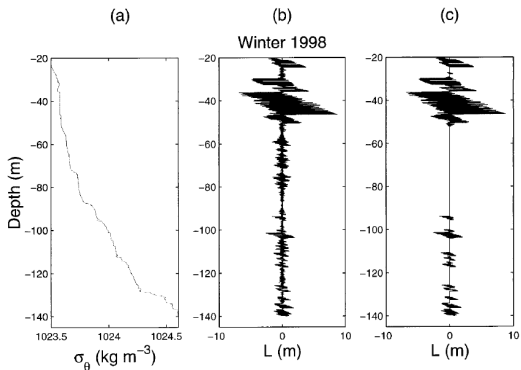


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$$L_T = \langle (d^*)^2 \rangle^{1/2}$$

Stansfield et al. 2004

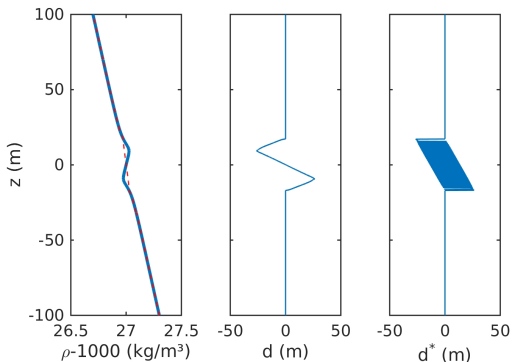


Stansfield, K.; Garrett, C. & Dewey, R. The Probability Distribution of the Thorpe Displacement within Overturns in Juan de Fuca Strait *J. Phys. Oceanogr.*, *Journal of Physical Oceanography*, American Meteorological Society, 2001, 31, 3421-3434



# The signature of the overturn in $d^*$ profiles

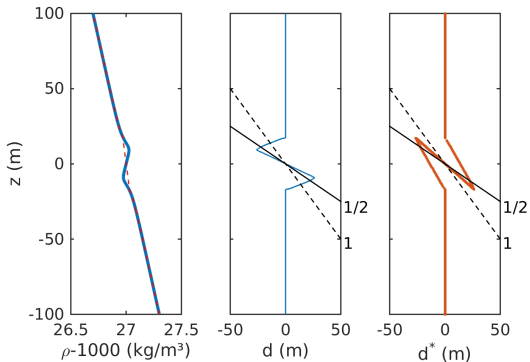
## Rankine Vortex





# The signature of the overturn in $d^*$ profiles

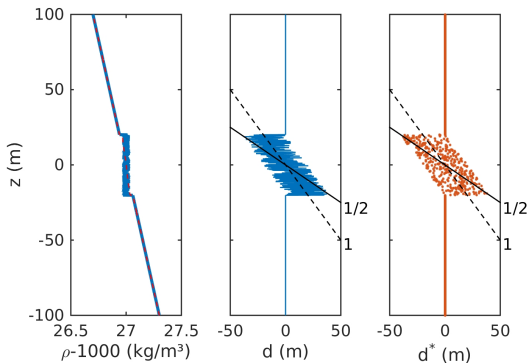
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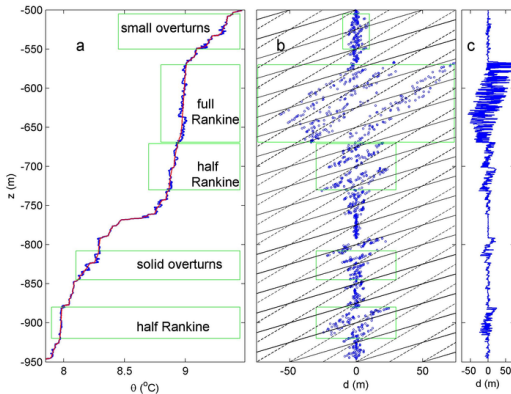
# The signature of the overturn in $d^*$ profiles

## Mixed patch





# Distinguishing overturns on in situ data



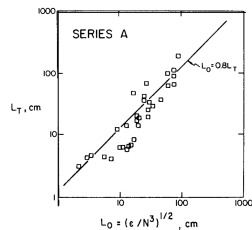
van Haren, H. & Gostiaux, L. Characterizing turbulent overturns in CTD-data Dynamics of Atmospheres and Oceans , 2014, 66, 58 - 76



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# Searching for mixing places

## Importance of boundaries

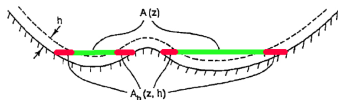


Fig. 1. Schematic of an ocean basin with a bottom boundary layer of thickness  $h$ .  $A_b(z, h)$  is the horizontal area at depth  $z$  that is within the boundary layer, and  $A(z)$  is the interior area at this depth.

$$K_{Vb} = K_V A(z) / A_b(z, h) \quad (45)$$

$$K_{Vb} = K_V A(z) \delta z [A_b(z, \delta z) h]^{-1} \quad (46)$$

Armi [1979a] cites values of about  $3 \times 10^{-4}$  for  $A_b(z, \delta z) / A(z) \delta z$  for the abyssal ocean, so for  $K_V = 10^{-4} \text{ m}^2 \text{ s}^{-1}$  and, say,  $h = 100 \text{ m}$  we require  $K_{Vb} = 3 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$ . Taking this value for  $\kappa_e$  and  $\nu_e$ ,  $N = 10^{-3} \text{ s}^{-1}$ ,  $\sin \theta = 0.1$ , and  $f = 10^{-4} \text{ s}^{-1}$  in (3.3) gives  $g = 0.15$  and  $gh = 15$ . On the basis of this estimate, it seems that the near-boundary mixing that might be caused by internal wave breaking is effective in that the stratification is continually restored by the secondary circulation. Taking a

Armi, L. Some evidence for boundary mixing in the deep Ocean Journal of Geophysical Research: Oceans, 1978, 83, 1971-1979

Garrett, C. The role of secondary circulation in boundary mixing Journal of Geophysical Research, 1990, 95, 3181-3188



# The NIOZ High Sampling-Rate Thermistors

## NIOZ-HST thermistors

- Autonomous sensors (energy and data storage)
- Synchronized 1Hz temperature acquisition
- 1mK relative accuracy
- Duration more than one year
- Deployment depth 6000m

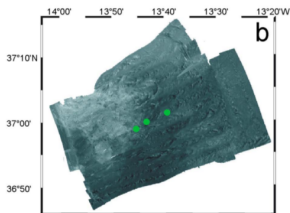
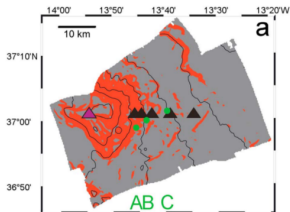
H. van Haren, M. Laan, D.-J. Buijsman, L. Gostiaux, M.G. Smit and E. Keijzer, NIOZ3: independent temperature sensors sampling yearlong data at a rate of 1 Hz, IEEE J. Ocean. Eng., 34, 315-322 (2009).



Hans van Haren,  
NIOZ, The  
Netherlands



# Measurements at Josephine Seamount

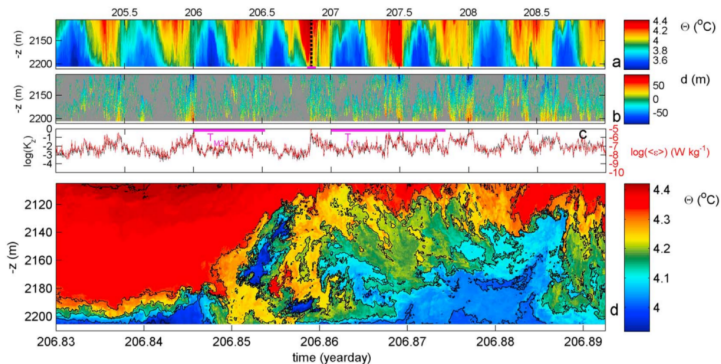


- Deep Atlantic Ocean seamount (submit at 2000m)
- Supercritical to subcritical for M2 tide
- Mooring line with 100 sensors each, 1m vertical resolution, 3 months at 1Hz
- Three different locations (supercritical to subcritical)



# Measurements at Josephine Seamount

Mooring A, 2210m,  $N = 1.4 \cdot 10^{-3} \text{ rad/s}$

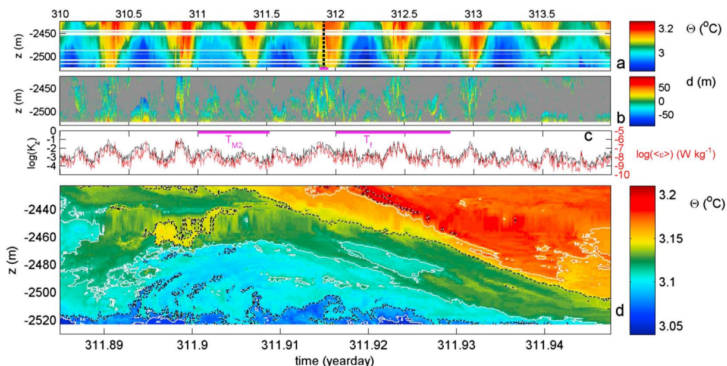


H. van Haren, A. Cimattorus and L. Gostiaux, Where large deep-ocean waves break, Geophysical Research Letters, GL063329, 2015



# Measurements at Josephine Seamount

Mooring B, 2530m,  $N = 1.0 \cdot 10^{-3} \text{ rad/s}$

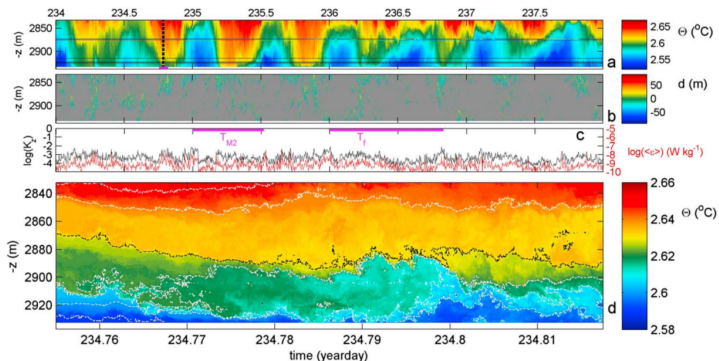


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# Measurements at Josephine Seamount

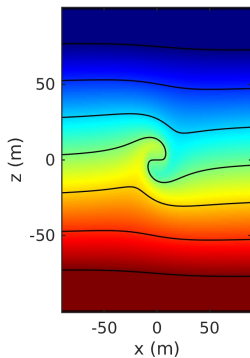
Mooring C, 2937m,  $N = 0.7 \cdot 10^{-3} \text{ rad/s}$



H. van Haren, A. Cimattorus and L. Gostiaux, Where large deep-ocean waves break, Geophysical Research Letters, GL063329, 2015

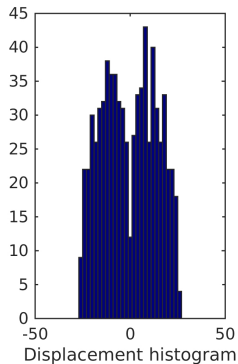
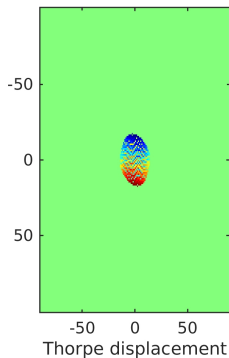


# Analytical overturn





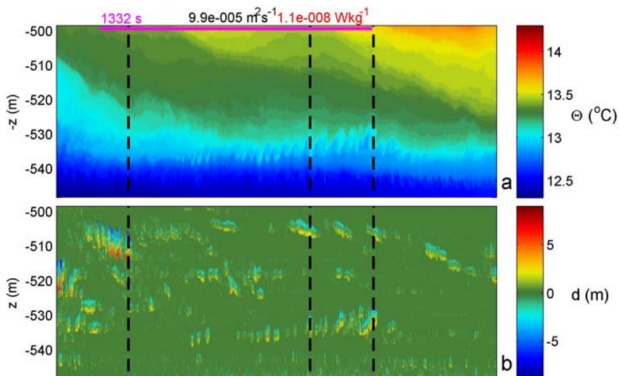
# Analytical overturn





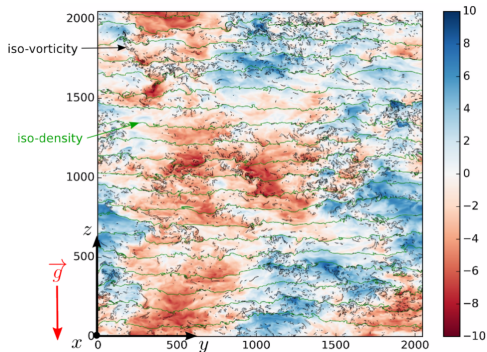


# Deep ocean overturns





# Numerical overturns



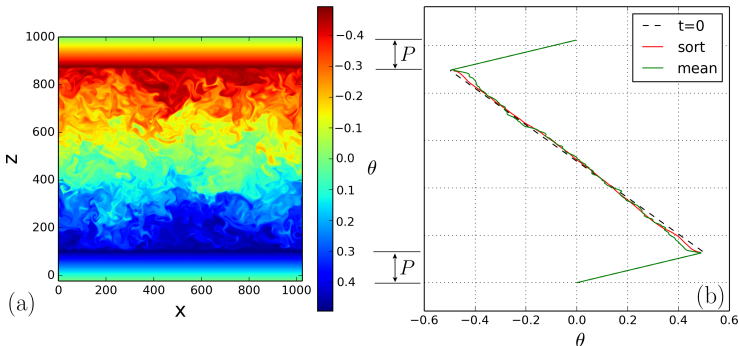
High resolution ( $2048^3$  points,  $Re=1000$   $k_{max}\eta \simeq 3$ ) with pseudospectral method, freely decaying turbulence.

A. Delache, E. Horne, LMFA



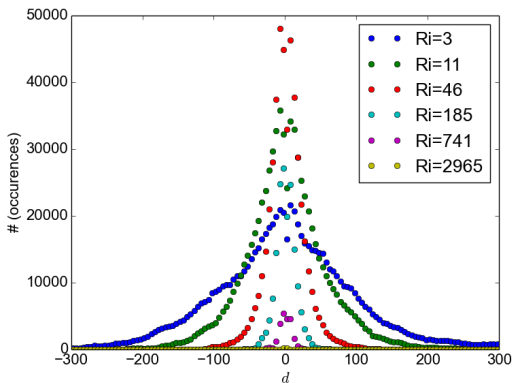
## Statistics of displacement values

Global richardson number  $Ri = N^2 H^2 / u^2$  with  $u$  the rms turbulent velocity and  $H$  the vertical domain size

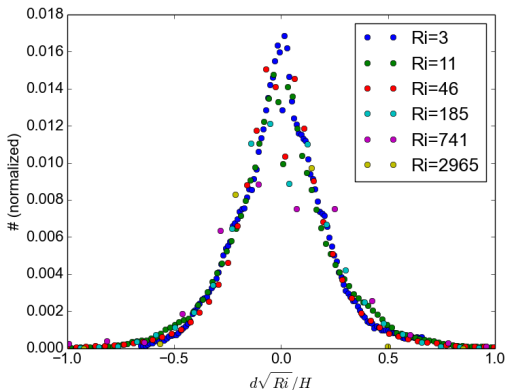




# Statistics of displacement values



# Statistics of displacement values





## Simple energetic scaling

From the energetic point of view, the potential energy in an overturn is  $Ep \simeq N^2 d^2$  (Thorpe 1977)

If an overturn is seen as a transfer of kinetic to potential energy,  $E_c \simeq Ep$  leads to  $d^2 \simeq u^2 / N^2 = H^2 Ri^{-1}$

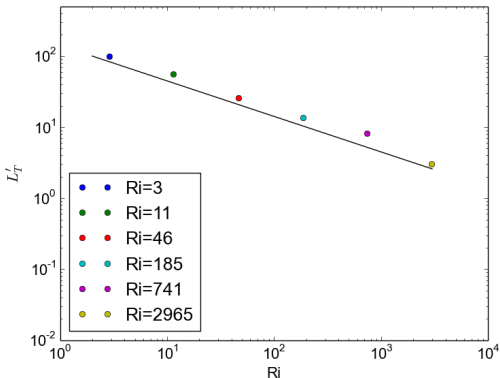
$$d \simeq H / \sqrt{Ri}$$



# Conditional Thorpe scale $L'_T$

$$L'_T = \langle d^2 \rangle^{1/2} \mid d \neq 0$$

$$(L_T = \sqrt{\bar{\chi}} L'_T)$$

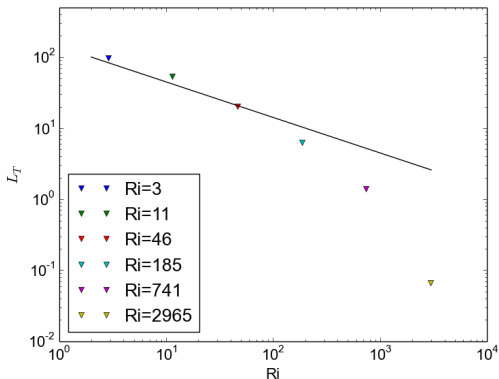




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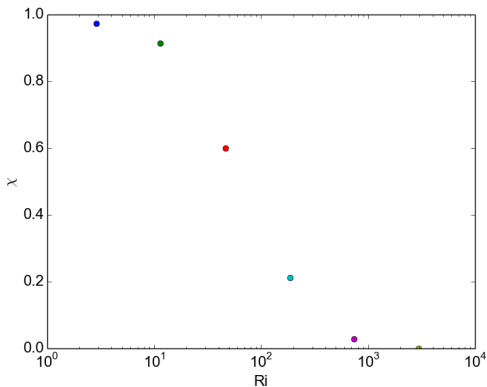






# Effect of intermittency $\chi$

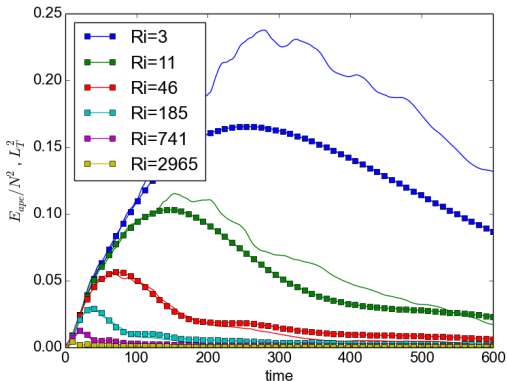
The intermittency influences the Thorpe scale estimate at high Ri





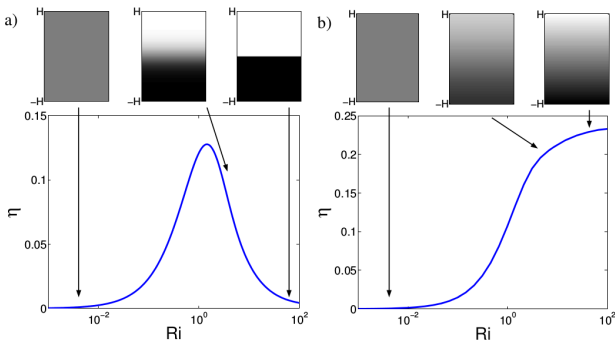
## Energetic approach

On the other hand, the energetic interpretation  $Ep \simeq N^2 d^2$  is more valid with a strong intermittency





# Perspectives



- Comparison with A. Venaille statistical model
- DNS in the case of two-layer stratified turbulence
- Statistics of Thorpe displacements on oceanic data



# The inverted Thorpe displacement

$$\begin{cases} \sigma(z_i) = \sigma_s(z_i + d_i) \\ \sigma(z_i + d_i^*) = \sigma_s(z_i) \end{cases}$$

With  $z_i^* = z_i + d_i^*$ , it follows

$$\sigma(z_i^*) = \sigma_s(z_i^* - d_i^*)$$

But  $[z_i^*]$  is a permutation of  $[z_i]$ , which can be written :

$$\begin{cases} z_i^* = z_{p(i)} \\ d_i^* = -d_{p(i)} \end{cases}$$