PRIMARY ATOMIZATION UNDER THE SIMULTANEOUS ACTION OF RAYLEIGH-TAYLOR AND KELVIN-HELMHOLTZ MECHANISMS

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MOTIVATION

- Atomization - Converting bulk fluid into a multitude of smaller fragments[1]

[1] Lefebvre,1989.

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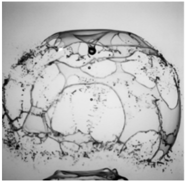

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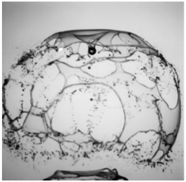
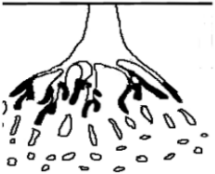
- Breakup of a cylindrical liquid sheet into asymmetric ligaments[3]





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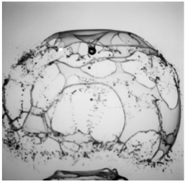

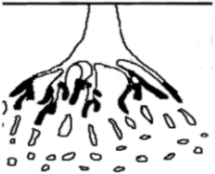

These structures are inherently three–dimensional.

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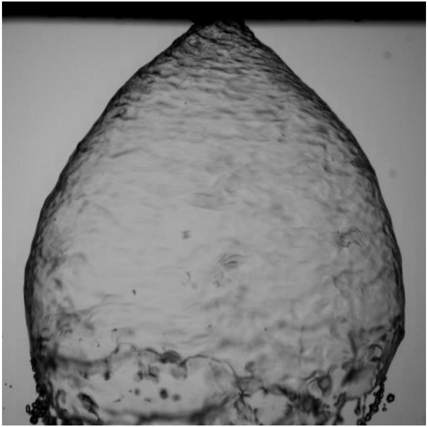
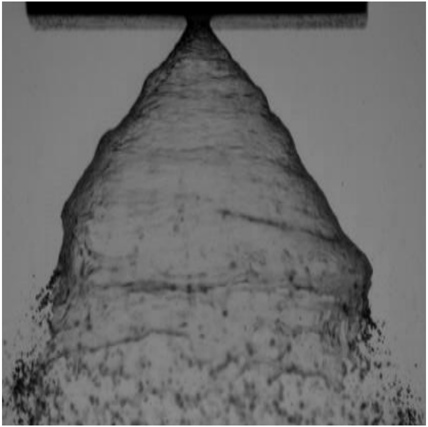
PRIMARY ATOMIZATION

- Acceleration at a two fluid interface

Accelerating inwards



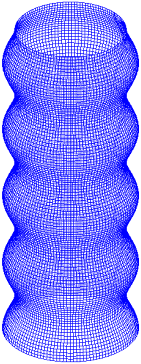
Accelerating outwards



DESTABILIZATION

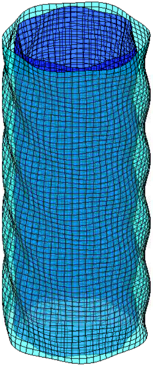
K–H mechanism

Single interface



Yang, 1992

Annular interface

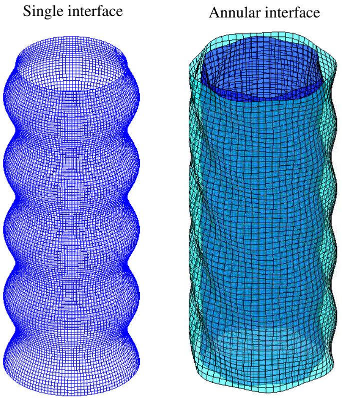


Panchagnula et.al, 1996

$$We = \frac{\rho_1 (\triangle W)^2 R}{\sigma}$$

DESTABILIZATION

K–H mechanism | R–T mechanism



Yang, 1992 | Panchagnula et.al, 1996 | Chen et al, 1997

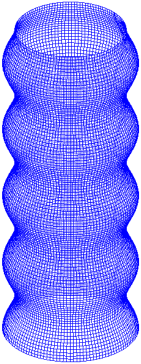
$$We = \frac{\rho_1 (\triangle W)^2 R}{\sigma}$$

$$Bo = \frac{\triangle \rho \ddot{R} R^2}{\sigma}$$

DESTABILIZATION

K–H mechanism

Single interface



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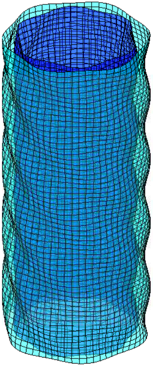

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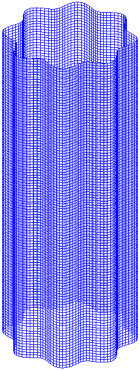
R–T mechanism

Single interface

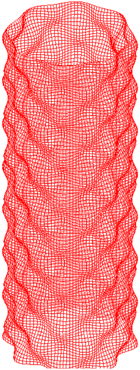


Chen et al, 1997

$$Bo = \frac{\triangle \rho \ddot{R} R^2}{\sigma}$$

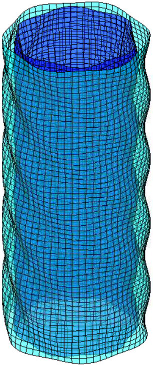
R–T–K–H mechanisms

Single interface



Present study

Annular interface



Present study

$$Bo + We??$$

OBJECTIVE

TO STUDY THE SIMULTANEOUS ACTION OF RAYLEIGH-TAYLOR (R-T) AND KELVIN-HELMHOLTZ (K-H) INSTABILITIES ON A SINGLE AND ANNULAR INTERFACE.

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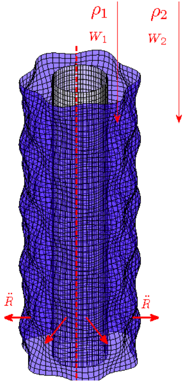
ASSUMPTIONS

- Inviscid
- Incompressible
- Immisicible
- Non-evaporating

SINGLE INTERFACE

MEAN FLOW DESCRIPTION

$$\Phi_j(r,z) = W_j z + R_0 \dot{R}_0 \ln\left(\frac{r}{R_\infty}\right); \qquad j = 1, 2 \tag{1}$$



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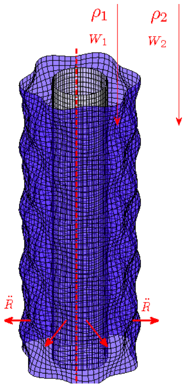


GE:

$$\nabla^2 \phi_j = 0 \tag{2}$$

KBC:

$$\left.\frac{\partial \phi_j}{\partial r}\right|_{r=r_{sj}} = \frac{\partial r_s}{\partial t} + W_j \frac{\partial r_s}{\partial z}; \qquad j = 1,2 \tag{3}$$

DBC:

$$p_1 - p_2 = \sigma \kappa \tag{4}$$

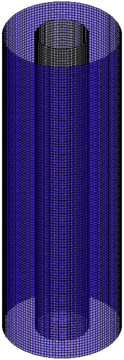
$$p_j(r,t) = P_j(t) - \rho_j \left[\frac{\partial \phi_j}{\partial t} + \frac{1}{2}|\nabla \phi_j|^2\right] \tag{5}$$

RADIAL MOTION

MEAN FLOW DESCRIPTION

$$\Phi_j(r,z) = W_j z + R_0 \dot{R}_0 \ln\left(\frac{r}{R_\infty}\right); \qquad j = 1,2$$

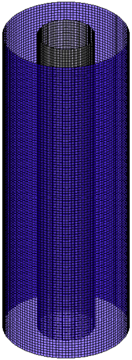
- Cross sectional area of each fluid is constant in time



$$R_0 \dot{R}_0 = R_1 \dot{R}_1 \qquad (6)$$

$$\dot{R}_0^2 + R_0 \ddot{R}_0 = \dot{R_1}^2 + R_1 \ddot{R}_1 \qquad (7)$$

Linear stability analysis



Movement of the interface

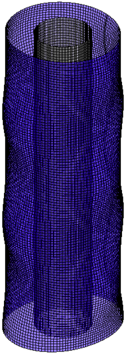
$$r_{sj}(\theta, z, t) = R_j(t) + a_j e^{(\omega t + ikz + im\theta)} \quad (8)$$

Frozen flow approximation

$$a_j(t) << R_j(t)$$
$$\dot{a}_j(t) >> \dot{R}_j(t)$$
$$\ddot{a}_j(t) >> \ddot{R}_j(t)$$

$$\phi_j = \Phi_j(r, z) + \phi_j'(r) e^{(\omega t + ikz + im\theta)} \quad (9)$$

$$\nabla^2 \phi_j = 0 \quad (10)$$



Growth of the disturbance

DISPERSION RELATION - SINGLE INTERFACE

$$|A| = 0$$

$$A = \begin{pmatrix} A_{11} & 0 & \omega + ikW_1 + \frac{\dot{R}}{R} \\ 0 & kK_m^{'}(kR) & \omega + ikW_2 + \frac{\dot{R}}{R} \\ \rho_1 A_{31} & -\rho_2 A_{32} & A_{33} \end{pmatrix}$$

$$A_{11} = I_m^{'}(kR)K_m^{'}(kR_0) - K_m^{'}(kR)I_m^{'}(kR_0)$$
$$A_{31} = -(\omega + ikW_1)B_1 - \dot{R}A_{11}$$
$$A_{32} = (\omega + ikW_2)K_m(kR) - \dot{R}kK_m^{'}(kR)$$
$$A_{33} = (\rho_1 - \rho_2)\ddot{R} - \frac{\sigma}{R^2}(1 - m^2 - k^2R^2)$$
$$B_1 = (I_m(kR)K_m^{'}(kR_0) + K_m(kR)I_m^{'}(kR_0))$$

$I_m^{'}(x)$ and $K_m^{'}(x)$ are the first derivatives of $I_m(x)$ and $K_m(x)$ with respect to its argument.

DISPERSION RELATION - ANNULAR INTERFACE

$$|A| = 0$$

$$A = \begin{pmatrix} -kI_m'(kR_i) & kK_m'(kR_i) & 0 & 0 & 0 & A_{16} & 0 \\ kI_m'(kR_0) & -kK_m'(kR_0) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & kK_m'(kR_o) & 0 & 0 & 0 & A_{37} \\ 0 & 0 & 0 & -kI_m'(kR_i) & kK_m'(kR_i) & A_{46} & 0 \\ 0 & 0 & 0 & -kI_m'(kR_o) & kK_m'(kR_o) & 0 & A_{57} \\ A_{61} & 0 & 0 & A_{64} & A_{65} & A_{66} & 0 \\ 0 & 0 & A_{73} & A_{74} & A_{75} & 0 & A_{77} \end{pmatrix}$$

$$A_{16} = \omega + ikW_i + \frac{\dot{R}_i}{R_i}; A_{37} = \omega + ikW_o + \frac{\dot{R}_o}{R_o}; A_{46} = \omega + ikW_l + \frac{\dot{R}_i}{R_i}$$

$$A_{57} = \omega + ikW_l + \frac{\dot{R}_o}{R_o}; A_{61} = -\rho_i \left((\omega + ikW_1) I_m(kR_i) - \dot{R}_i A_{11} \right)$$

$$A_{64} = -\rho_l \left((\omega + ikW_l) I_m(kR_i) - \dot{R}_i A_{11} \right); A_{65} = -\rho_l \left((\omega + ikW_l) I_m(kR_i) + \dot{R}_i A_{12} \right)$$

$$A_{66} = (\rho_i - \rho_1)\ddot{R}_i - \frac{\sigma_i}{R_i^2}(1 - m^2 - k^2 R_i^2); A_{73} = \rho_o \left((\omega + ikW_o) K_m(kR_o) - \dot{R}_o A_{33} \right)$$

$$A_{74} = -\rho_l \left((\omega + ikW_l) I_m(kR_o) - \dot{R}_o A_{44} \right); A_{75} = -\rho_l \left((\omega + ikW_l) K_m(kR_o) - \dot{R}_o A_{33} \right)$$

$$A_{77} = (\rho_o - \rho_1)\ddot{R}_o + \frac{\sigma_o}{R_o^2}(1 - m^2 - k^2 R_o^2)$$

DISPERSION RELATION

NON-DIMENSIONAL DISPERSION RELATION

$$\mathscr{D}(\omega, k, m) := \mathscr{G}_2\omega^2 + \mathscr{G}_1\omega + \mathscr{G}_0 = 0 \text{ (Single interface)}$$

$$\mathscr{D}(\omega, k, m) := \mathscr{F}_4\omega^4 + \mathscr{F}_3\omega^3 + \mathscr{F}_2\omega^2 + \mathscr{F}_1\omega + \mathscr{F}_0 = 0 \text{ (Annular interface)}$$

$$R_m = R_1; \qquad \alpha = \frac{R_0}{R_m} = 10^{-2}; \qquad \omega = \omega\sqrt{\frac{\sigma}{\rho R_m^3}}; \qquad k = kR_m;$$

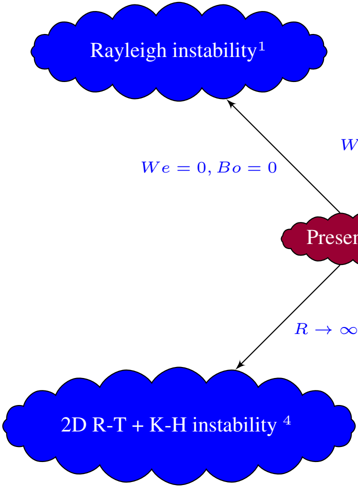
ρ= max(ρ_1 , ρ_2) and $\rho_1 > \rho_2$ for single interface,

$$Bo = \frac{(\rho_2 - \rho_1)\,\ddot{R}R_m^2}{\sigma} \qquad We = \frac{\rho_1\,(W_1 - W_2)^2\,R_m}{\sigma} \qquad Q = \frac{\rho_2}{\rho_1} = 10^{-3}$$

$\rho_{1(i)} < \rho_{2(l)} > \rho_{3(o)}$ for an annular interface, $j = i, o(1,3), 2 = l$

$$Bo = \frac{(\rho_i - \rho_l)\,\ddot{R}_i R_m^2}{\sigma} \qquad We_j = \frac{\rho_l\,(W_j - W_l)^2\,R_m}{\sigma} \qquad Q_j = \frac{\rho_j}{\rho_l} = 10^{-3} \qquad \lambda = \frac{R_i}{R_0} = 0.98;$$

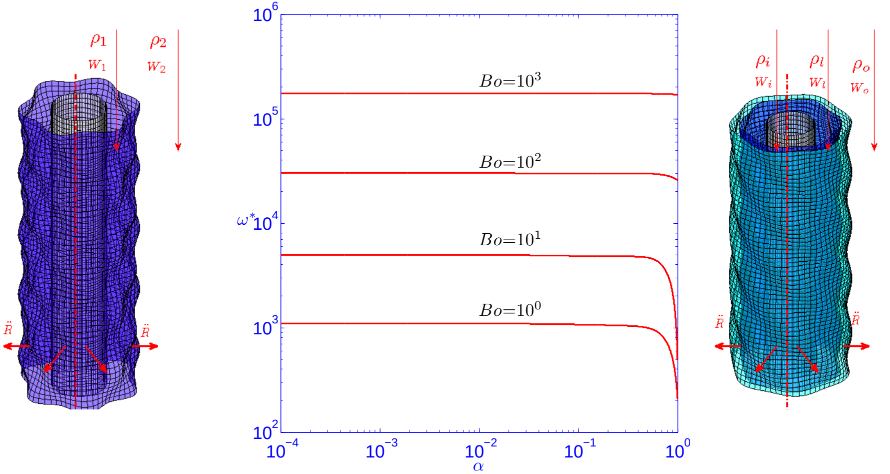
LIMITING CASES



Validation of present study with previous researchers

[1] Rayleigh,1878, [2] Chen et al,1997, [3] Yang,1992, [4] Chandrasekhar ,1961

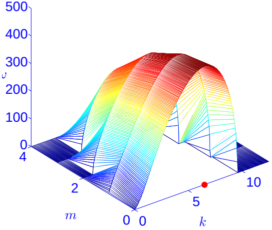
INFLUENCE OF 'INNER' WALL



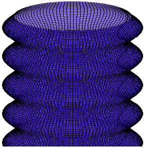
- Inner wall does not affect the stability characteristics

RESULTS - DISPERSION DIAGRAMS (SINGLE INTERFACE)

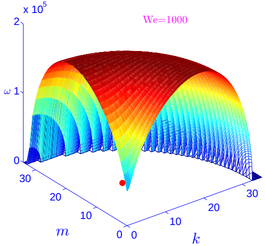
Taylor mode
$(k^* > 0,\ m^* = 0)$
$Bo = 0, We = 1200$



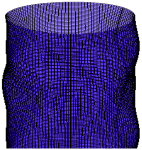
$$\mathcal{L}^* = \frac{2\pi}{k^*}$$



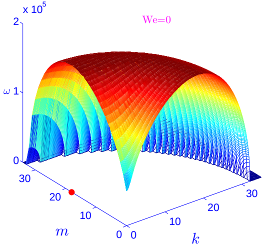
Helical mode
$(k^* > 0,\ m^* > 0)$
$Bo = 1000, We = 1000$



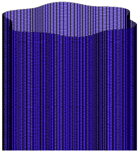
$$\mathcal{L}^* = \frac{2\pi}{k^* m^*}$$



Flute mode
$(k^* = 0,\ m^* > 0)$
$Bo = 1000, We = 0$

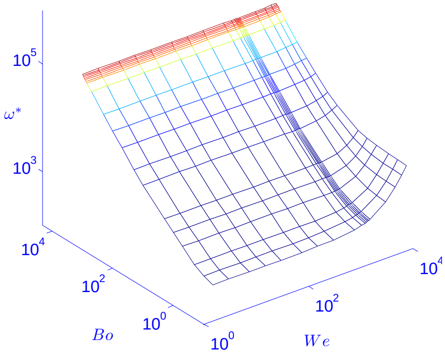


$$\mathcal{L}^* = \frac{2\pi}{m^*}$$

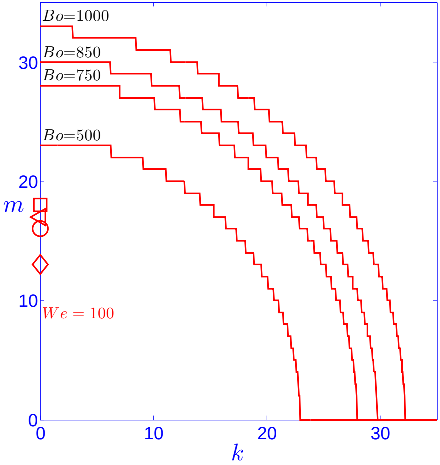


R-T-K-H SINGLE INTERFACE

Growth rate



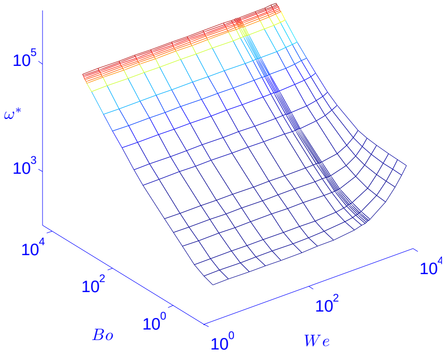
Neutral stability curves



- Neutral stability and growth rate are influenced by Bond number
- Deformation mode is influenced mostly by Weber number

R-T-K-H single interface

Growth rate



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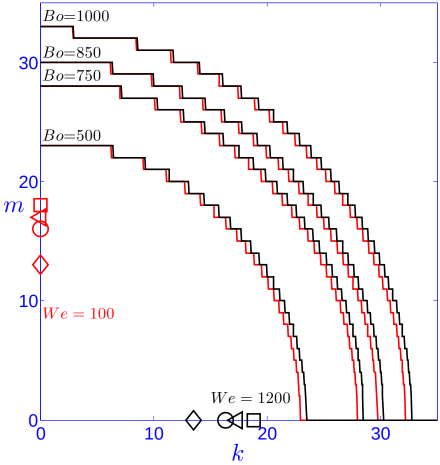

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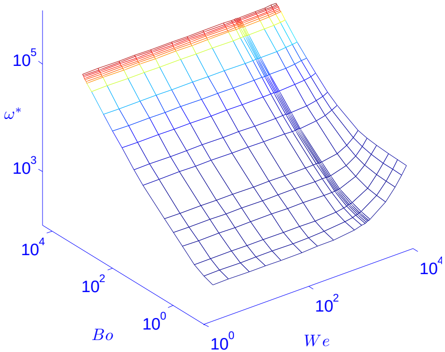

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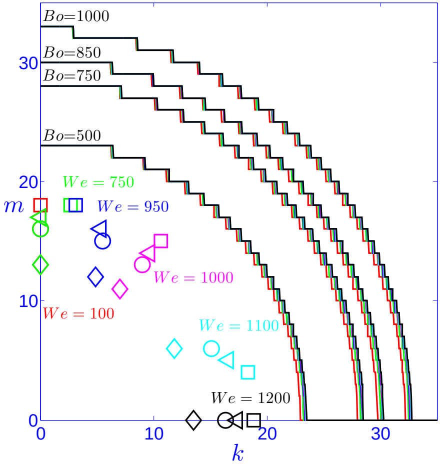

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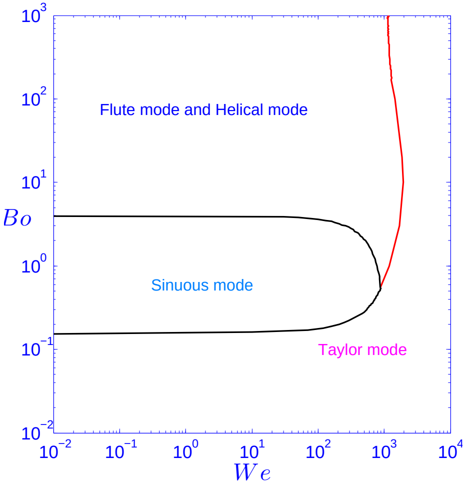
R-T-K-H SINGLE INTERFACE-REGIME CHART



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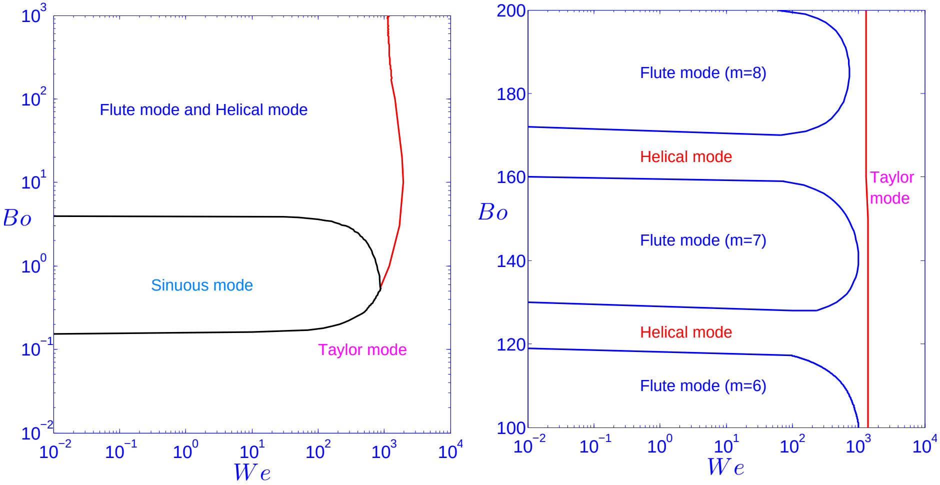


- Shortwave helical wavelength ($800 < We < 1200$)

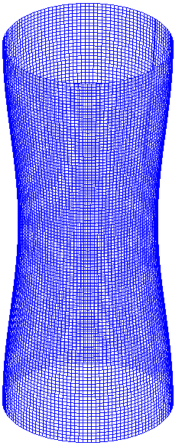
R-T-K-H SINGLE INTERFACE-ENERGY BUDGET ($Bo + We = 1200$)

ENERGY BUDGET

$$\mathscr{E} = (\rho_1 - \rho_2)R\ddot{R} + \rho_1(W_1 - W_2)^2$$

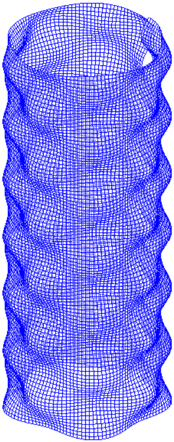
$$R\mathscr{E}/\sigma = Bo + We$$

Taylor mode



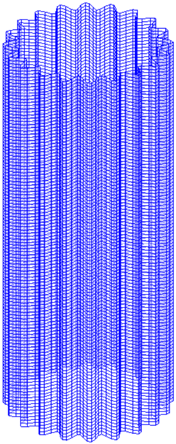
$Bo = 0, We = 1200$
$(k^* = 1.105, m^* = 0)$
$\mathcal{L}^* = 5.6$

Helical mode



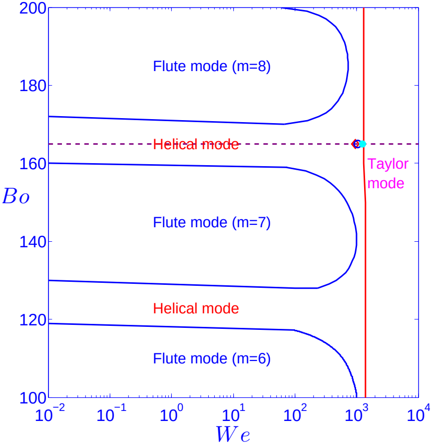
$Bo = 200, We = 1000$
$(k^* = 4.51, m^* = 7)$
$\mathcal{L}^* = 0.19$

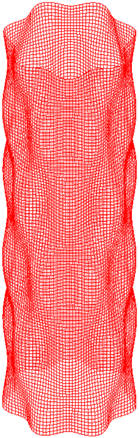
Flute mode



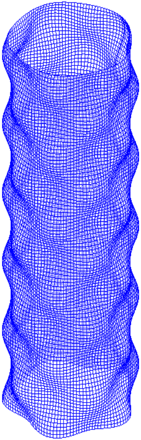
$Bo = 1200, We = 0$
$(k^* = 0, m^* = 20)$
$\mathcal{L}^* = 0.31$

R-T-K-H SINGLE INTERFACE-LENGTH SCALES ($Bo = 165$)

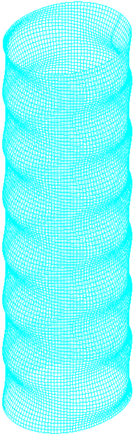




$Bo = 165$, $We = 950$ ($k^* = 2.7$, $m^* = 7$) $\mathcal{L}^* = 0.33$

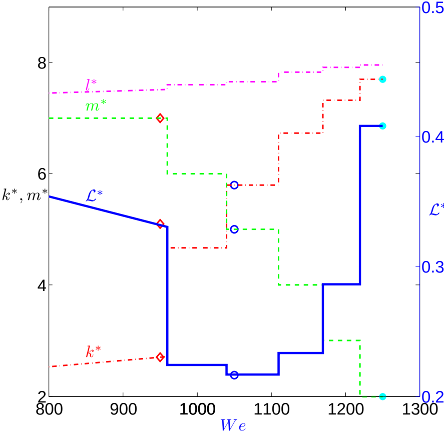


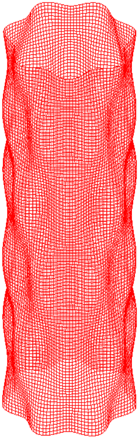
$Bo = 165$, $We = 1050$ ($k^* = 5.8$, $m^* = 5$) $\mathcal{L}^* = 0.21$



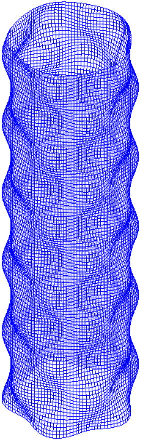
$Bo = 165$, $We = 1250$ ($k^* = 7.7$, $m^* = 2$) $\mathcal{L}^* = 0.40$

R-T-K-H single interface-Length scales ($Bo = 165$)

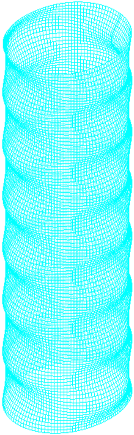




$Bo = 165$, $We = 950$ ($k^* = 2.7$, $m^* = 7$) $\mathcal{L}^* = 0.33$



$Bo = 165$, $We = 1050$ ($k^* = 5.8$, $m^* = 5$) $\mathcal{L}^* = 0.21$



$Bo = 165$, $We = 1250$ ($k^* = 7.7$, $m^* = 2$) $\mathcal{L}^* = 0.40$

- Optimum We occurs when $k^* \approx m^*$

COMBINED R-T-K-H INSTABILITIES OF A CYLINDRICAL INTERFACE - SUMMARY

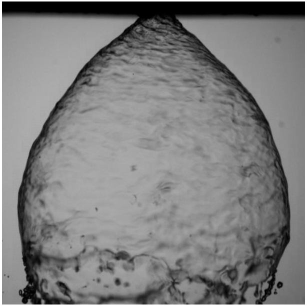
- Neutral stability and growth rate are influenced by Bond number
- Deformation mode is influenced mostly by Weber number
- Optimum Weber number exists for a given Bond number
- Radial acceleration (Bo) based destabilization is significantly more efficient than shear induced (We) destabilization.

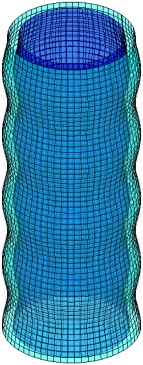
ANNULAR INTERFACE

ANNULAR INTERFACE

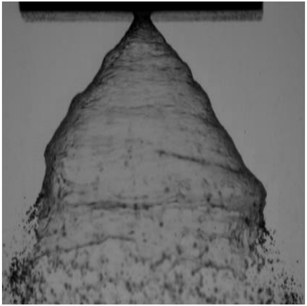
$$Bo = \frac{(\rho_i - \rho_l)\,\ddot{R}_i R_m^2}{\sigma} \quad We_j = \frac{\rho_l\,(W_j - W_l)^2\,R_m}{\sigma} \quad Q_j = \frac{\rho_j}{\rho_l} = 10^{-3} \quad \lambda = \frac{R_i}{R_0} = 0.98;$$

Accelerating inwards



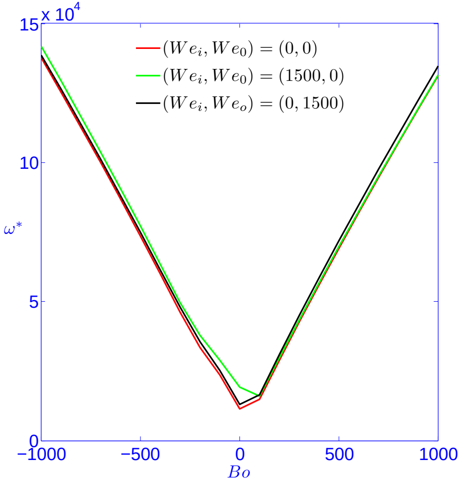


Accelerating outwards



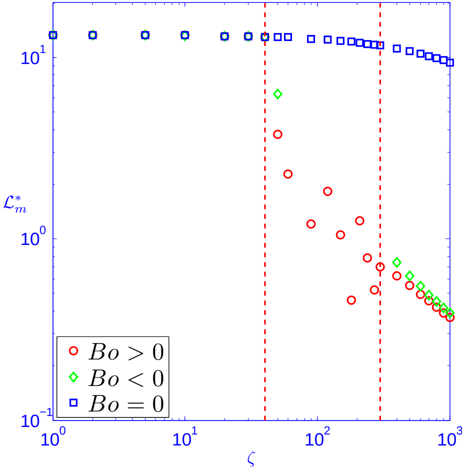
ANNULAR INTERFACE

Growth rate



- $|Bo|$ is significant

$Bo + We_i + We_o = \xi$



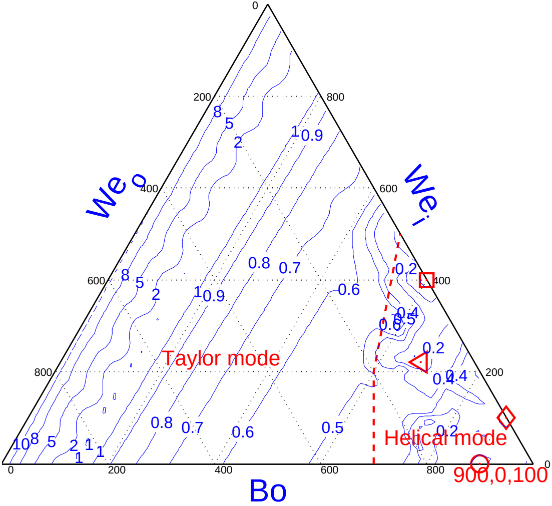
- $|Bo|$ occupies 90 percent of the ξ

ANNULAR INTERFACE, $Bo + We_i + We_o = \xi = 1000$

$\epsilon = (\rho_i - \rho_l) R_m \ddot{R}_i + \rho_l (W_l - W_i)^2 + \rho_l (W_l - W_o)^2$; $R_m \epsilon / \sigma = Bo + We_i + We_o = \xi$

Annular interface, $Bo + We_i + We_o = \xi = 1000$

$\epsilon = (\rho_i - \rho_l) R_m \ddot{R}_i + \rho_l (W_l - W_i)^2 + \rho_l (W_l - W_o)^2$; $R_m \epsilon / \sigma = Bo + We_i + We_o = \xi$



- Highest Bond number yields lowest length scale

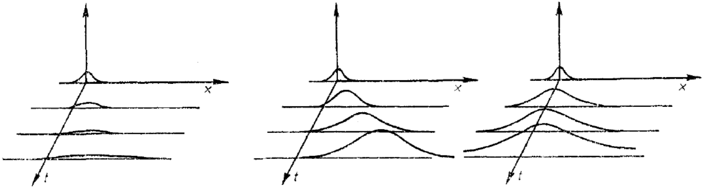
COMBINED R-T-K-H INSTABILITIES OF AN ANNULAR INTERFACE - SUMMARY

- Neutral stability and growth rate are influenced by Bond number
- Deformation mode is influenced mostly by Weber number
- Radial acceleration (Bo) based destabilization is significantly more efficient than shear induced (We_j) destabilization.
- A novel principle of primary atomization is proposed

ABSOLUTE/CONVECTIVE INSTABILITY

ABSOLUTE AND CONVECTIVE INSTABILITY

- If the disturbances spread both upstream and downstream
- If the disturbances are swept downstream or upstream



Stable Convectively unstable Absolutely unstable*

* Huerre et al, 1990.

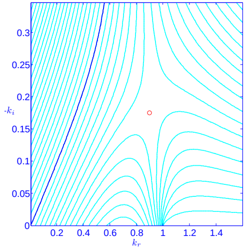
BRIGGS–BERS CRITERIA

$$D_m(k_o, \omega_o) = 0 \qquad \frac{\partial D_m}{\partial k}(k_o, \omega_o) = 0 \qquad \frac{\partial^2 D_m}{\partial k^2}(k_o, \omega_o) \neq 0$$

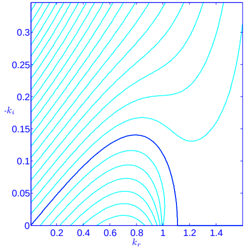
Briggs (1964) Bers (1975)

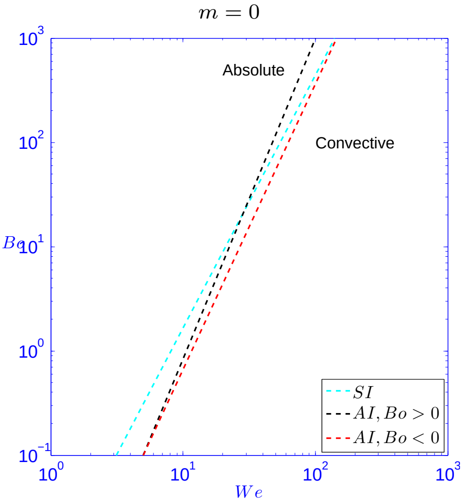
ABSOLUTE/CONVECTIVE INSTABILITY

Absolutely unstable ($We = 1, Bo = 0, Q = 10^{-3}$)

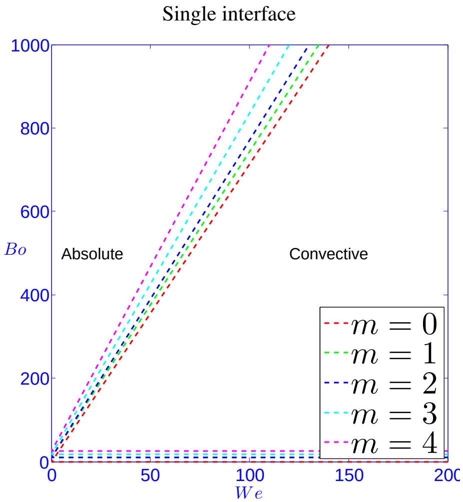


Convectively unstable ($We = 4, Bo = 0, Q = 10^{-3}$)

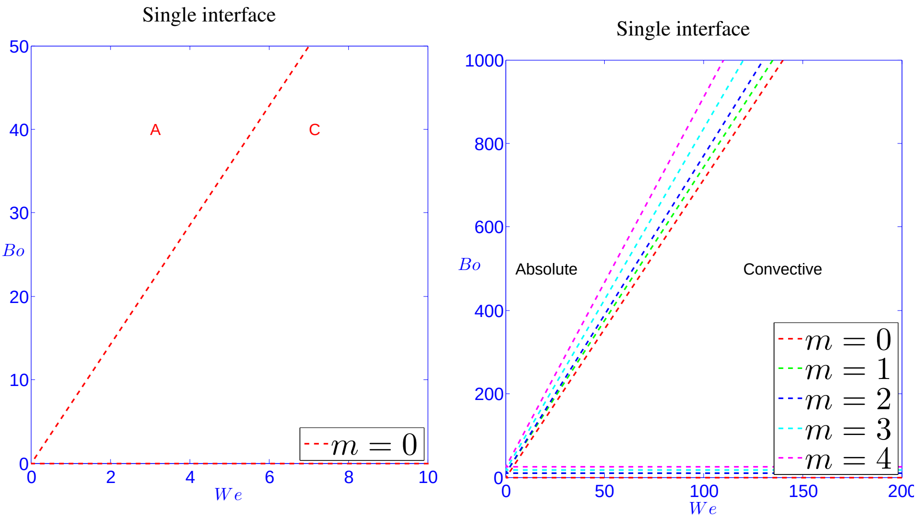




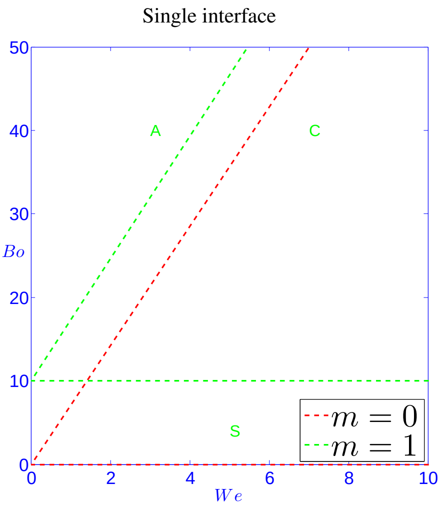
ABSOLUTE/CONVECTIVE INSTABILITY

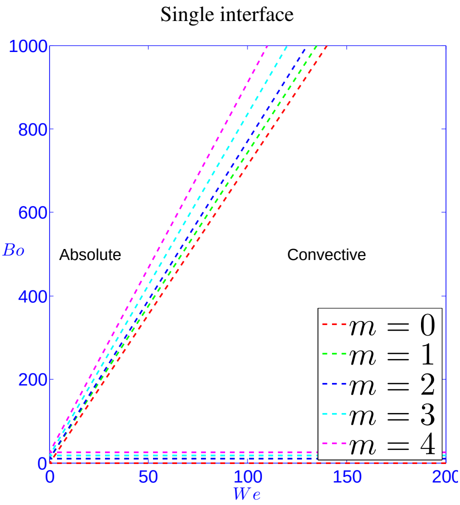


Absolute/Convective instability

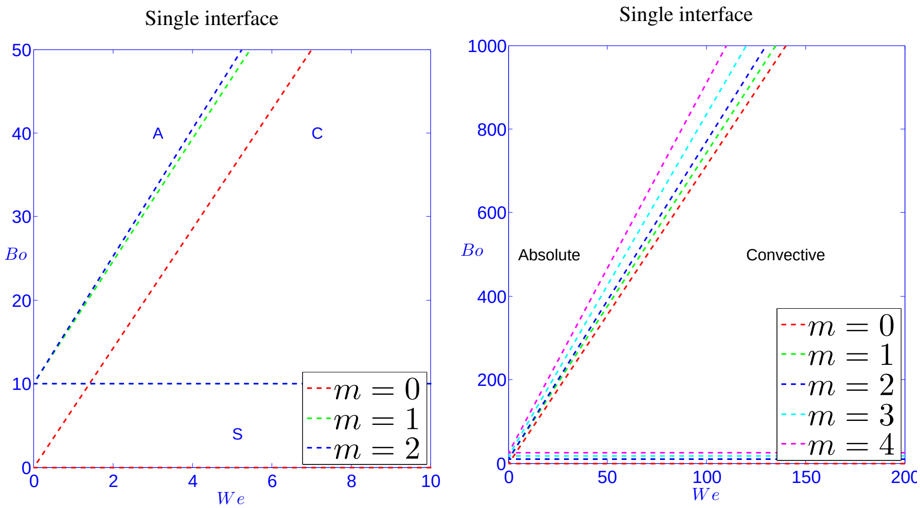


Absolute/Convective instability

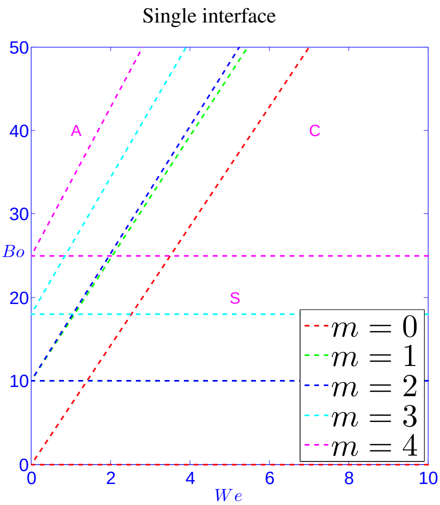


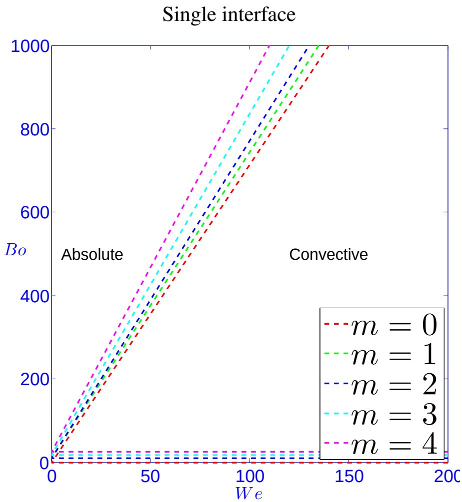


Absolute/Convective instability



ABSOLUTE/CONVECTIVE INSTABILITY





SUMMARY

COMBINED R-T-K-H INSTABILITIES

- Helical modes (Ligaments) are observed
- Radial acceleration based destabilization is significantly more efficient than shear induced destabilization.
- A novel principle of primary atomization is proposed.
- Nature of absolute convective transition is identified.