Evidence of Görtler vortices in katabatic jet along a convexly curved slope.

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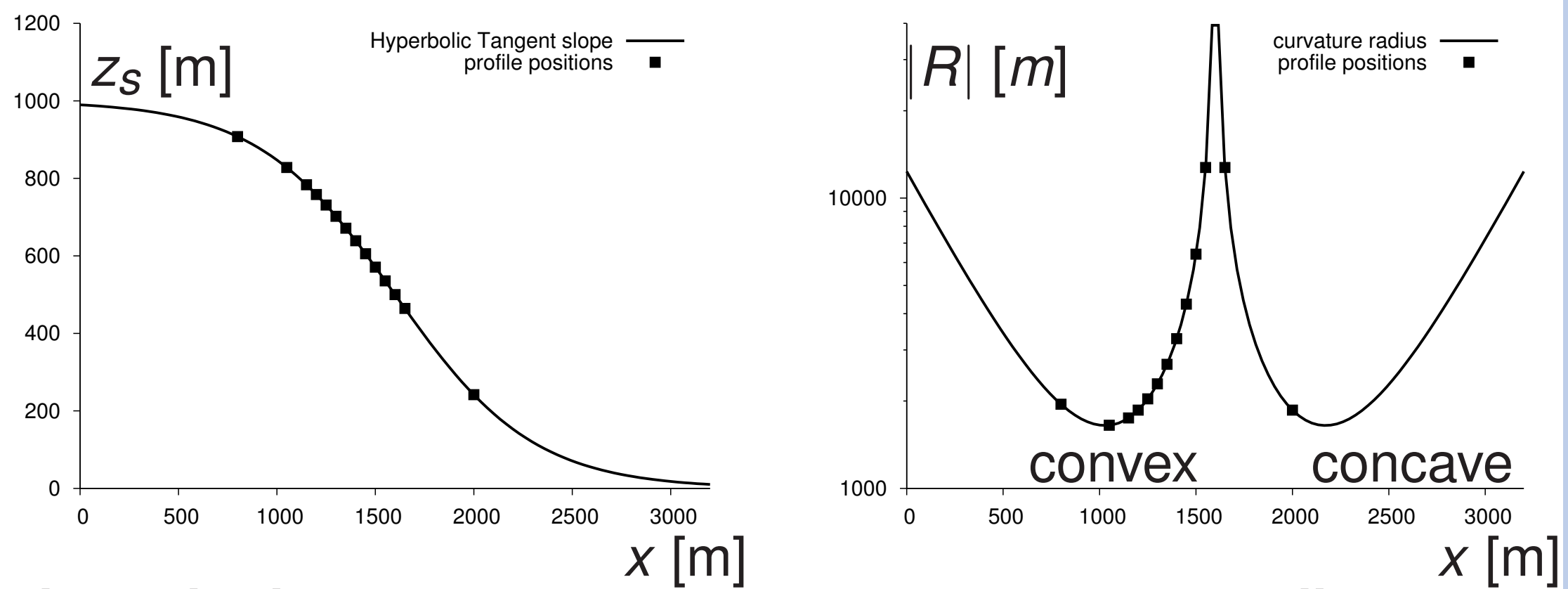
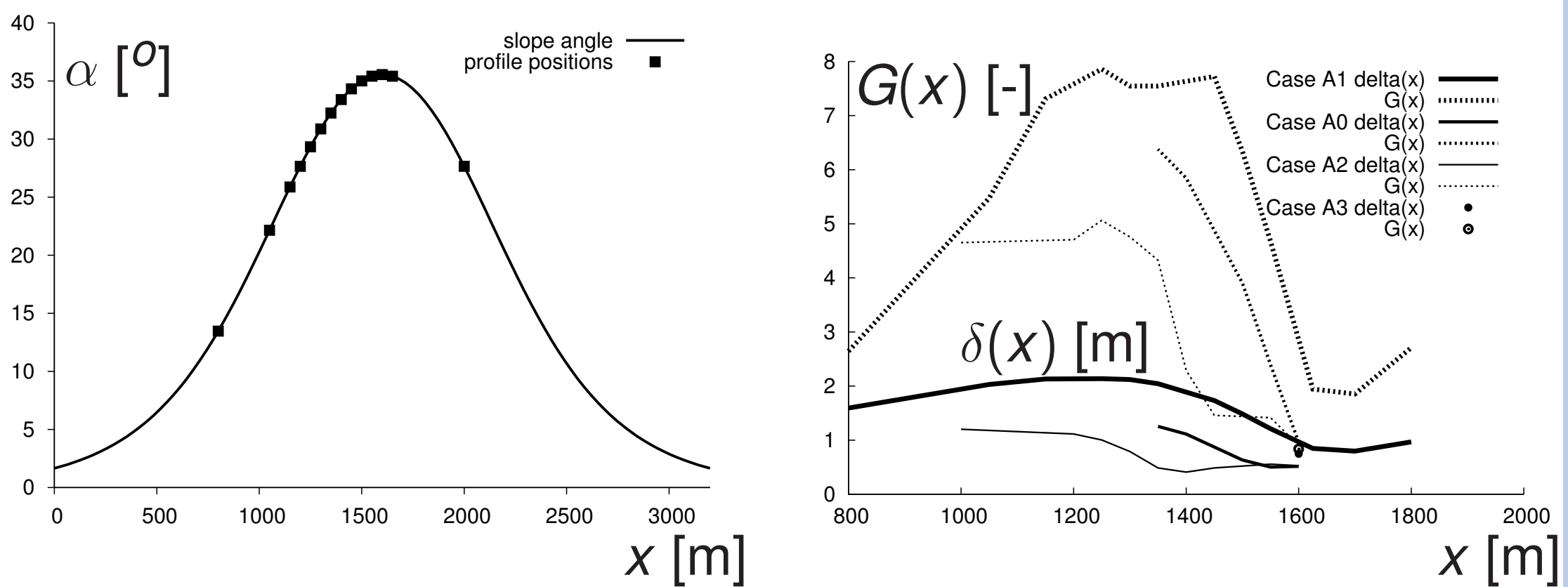
Abstract

Large Eddy Simulation of katabatic flow along a convexly curved slope is performed. A special focus is given on the outer-layer shear of the katabatic jet. Both a statistical quantitative analysis and a qualitative description of vortical structures are used to describe the present turbulent flow. It is shown that Görtler vortices oriented in the streamwise downslope direction and with a vertical mushroom shape develop in the shear layer. They play a specific role with respect to local turbulent mixing in the ground surface boundary layer. Such curved slope constitutes a realistic model for alpine orography. We provide a novel procedure based on local turbulence anisotropy to track Görtler vortices for *in situ* measurements.

Ideal topography



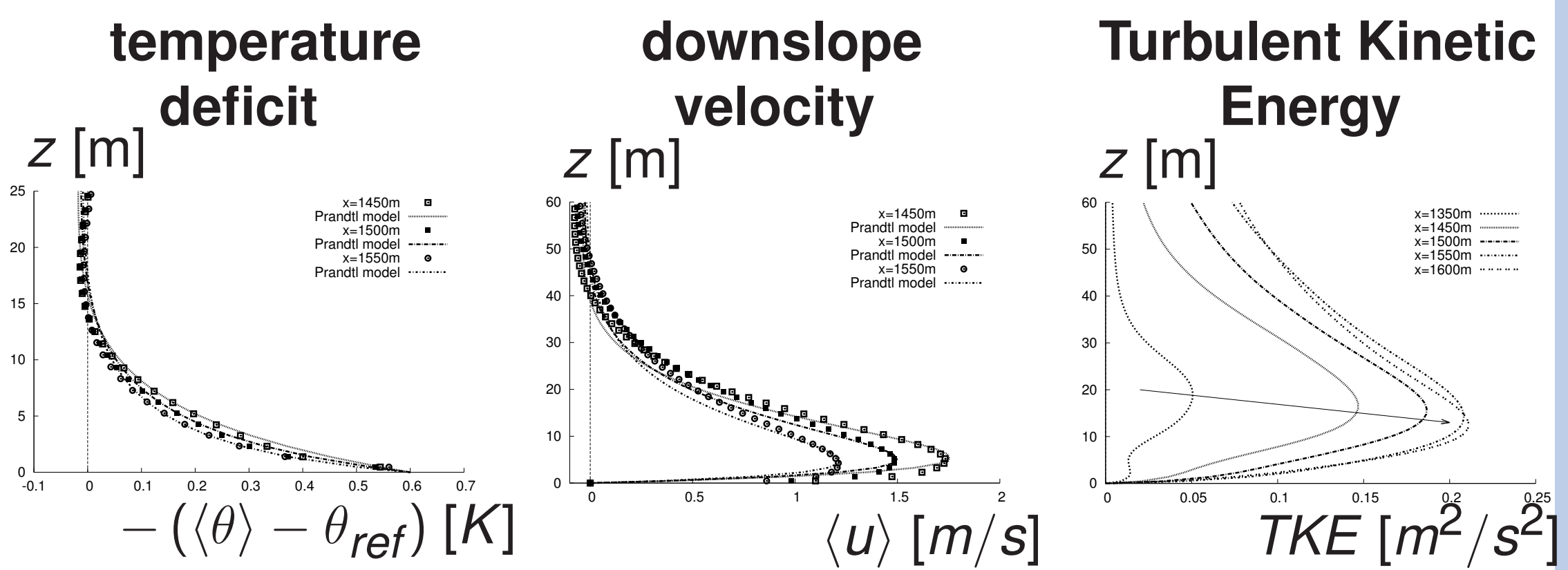
hyperbolic tangent slope **curvature radius**



slope angle **Görtler Number G**

$$G(x) = \frac{\langle u\rangle_{max}(x)\delta(x)}{\nu_{LES}(x)}\sqrt{\frac{\delta(x)}{R_{min}}}$$

Statistical Results



CASE A1

Prandtl Model

Analytical solution for the Prandtl model:

$$u_p(z) = V_o sin(z/L_o)e^{-z/L_o}$$
$$\theta_p(z) - \theta_{ref}(z) = \Theta_o cos(z/L_o)e^{-z/L_o}$$

Three characteristic scales L_o, V_o and Θ_o have to be prescribed from the boundary and ambiant conditions. For heat flux boundary conditions, replacing viscous by turbulent quantities, and assuming mixing coefficients K_m and K_h constant along z, one gets

$$L_o = \frac{1}{Pr_t^{1/4}}\sqrt{\frac{2\ K_m}{N_{ref} sin(\alpha)}}$$
$$V_o = Pr_t^{1/4}\frac{\sqrt{2}\ g\ F_s}{\theta_o\sqrt{K_m N_{ref}^3 sin(\alpha)}}$$
$$\Theta_o = Pr_t^{3/4}\frac{\sqrt{2}\ F_s}{\sqrt{K_m N_{ref} sin(\alpha)}}$$

where $F_s = \frac{H_s}{\rho C_p}$ is the heat flux at the ground surface.
Present model adaptation with non constant diffusion:

$$K_m^{L_o} = K_m^{min}(1 + \frac{\partial K_m}{\partial z}|^{L_o} z)$$
$$K_m^{V_o} = K_m^{min}(1 + \frac{\partial K_m}{\partial z}|^{V_o} z)$$
$$K_m^{\Theta_o} = K_m^{min}(1 + \frac{\partial K_m}{\partial z}|^{\Theta_o} z)$$

Prandtl model	K_m^{min}	$\frac{\partial K_m}{\partial z}\|^{L_o}$	$\frac{\partial K_m}{\partial z}\|^{V_o}$	$\frac{\partial K_m}{\partial z}\|^{\Theta_o}$
$x = 1,450m$	$0.15m^2/s$	$0.04m/s$	$0.14m/s$	$0.17m/s$
$x = 1,500m$	$0.15m^2/s$	$0.05m/s$	$0.25m/s$	$0.33m/s$
$x = 1,550m$	$0.15m^2/s$	$0.06m/s$	$0.42m/s$	$0.58m/s$

CASE A1

Aknowledgements

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Numerical Model

Meso-NH Model: CNRM & LA Toulouse France

- Pseudo-incompressible Navier-Stokes equations
- Anelastic approximation
- Buoyancy effects (gravity)
- No Coriolis effects
- Dry air (perfect gas)
- LES: e_{SGS} equation & mixing length closure
- Grid vertical refinement near the ground surface
- 5 M grid points on 128 MPI proc. of IBM-SP6
- Initial conditions: air at rest with a constant $\frac{\partial\theta_{ref}}{\partial z} = N_{ref}^2\frac{\theta_o}{g}$
- Ground surface cooling $H_s < 0$

Resolution/Boundary conditions

L_x (m)	L_y (m)	L_z (m)	n_x	n_y	n_z
3, 200	1, 280	7, 250	128	128	300
Δx (m)	Δy (m)	Δz_{wall} (m)	Δz_{top} (m)	H (m)	α_{max}
25	10	1	120	1, 000	35.5^o

Case	u_*^{max} (m/s)	N_{ref} (s^{-1})	H_s (W/m^2)	$\langle u\rangle_{max}$ (m/s)
A0	0.19	0.011	-10	1.4
A1	0.24	0.013	-30	2.1
A2	0.18	0.013	-10	1.2
A3	0.15	0.020	-30	–
B1	0	0.013	-30	–

Anisotropy Invariant Map

Anisotropy tensor:

$$a_{ij} = \frac{\langle u'_i u'_j\rangle}{2\ TKE} - \frac{1}{3}\delta_{ij}$$

second and third invariants:

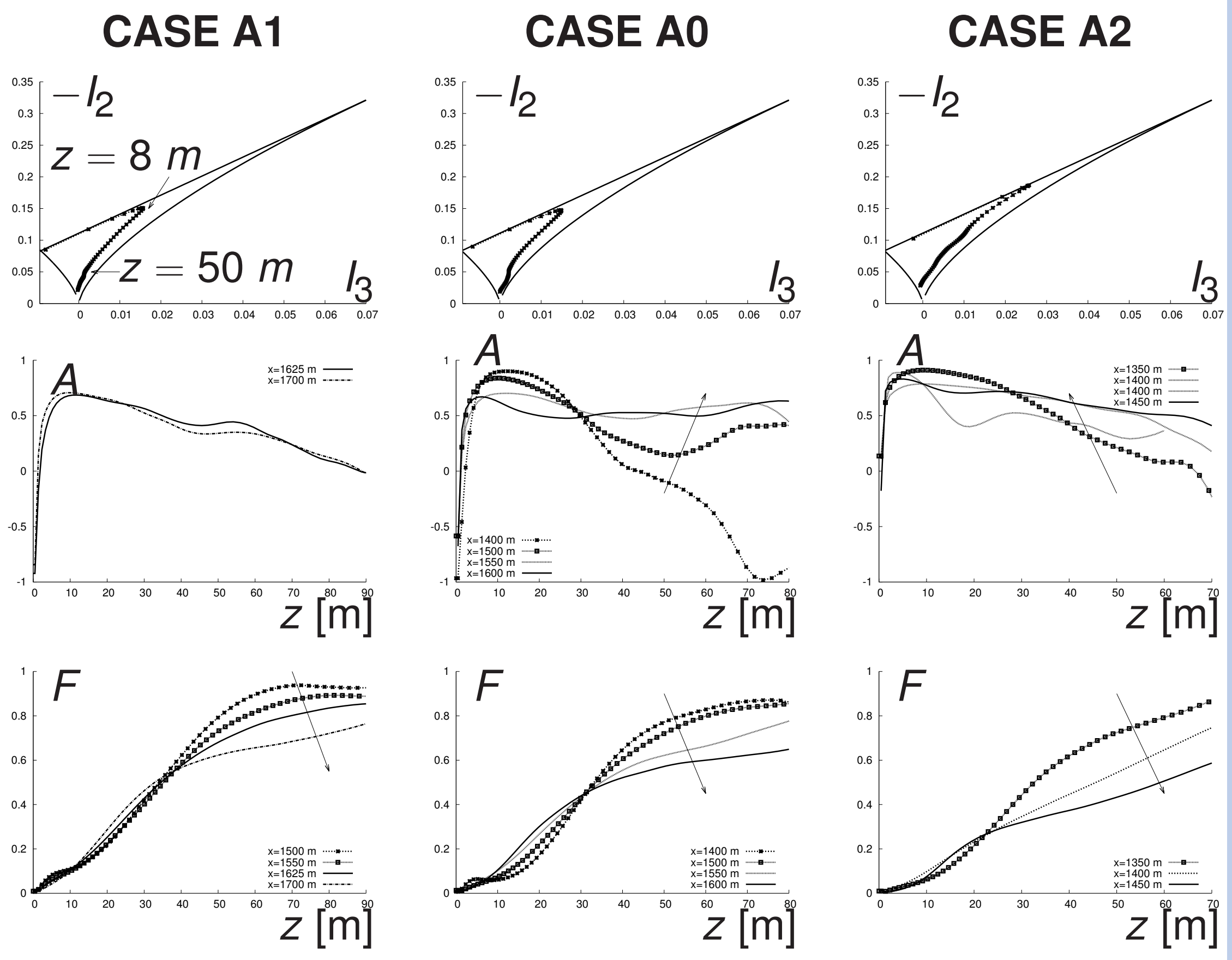
$$I_2 = -\frac{1}{2}A^2 = -\frac{a_{ij}a_{ji}}{2}$$
$$I_3 = \frac{1}{3}A^3 = \frac{a_{ij}a_{jk}a_{ki}}{3}$$

axisymmetric parameter:

$$A(z) = \frac{I_3}{2(-I_2/3)^{3/2}}$$

departure from isotropic turbulence:

$$F(z) = 1 + 9I_2 + 3I_3$$



TKE Budget

Turbulent Kinetic Energy

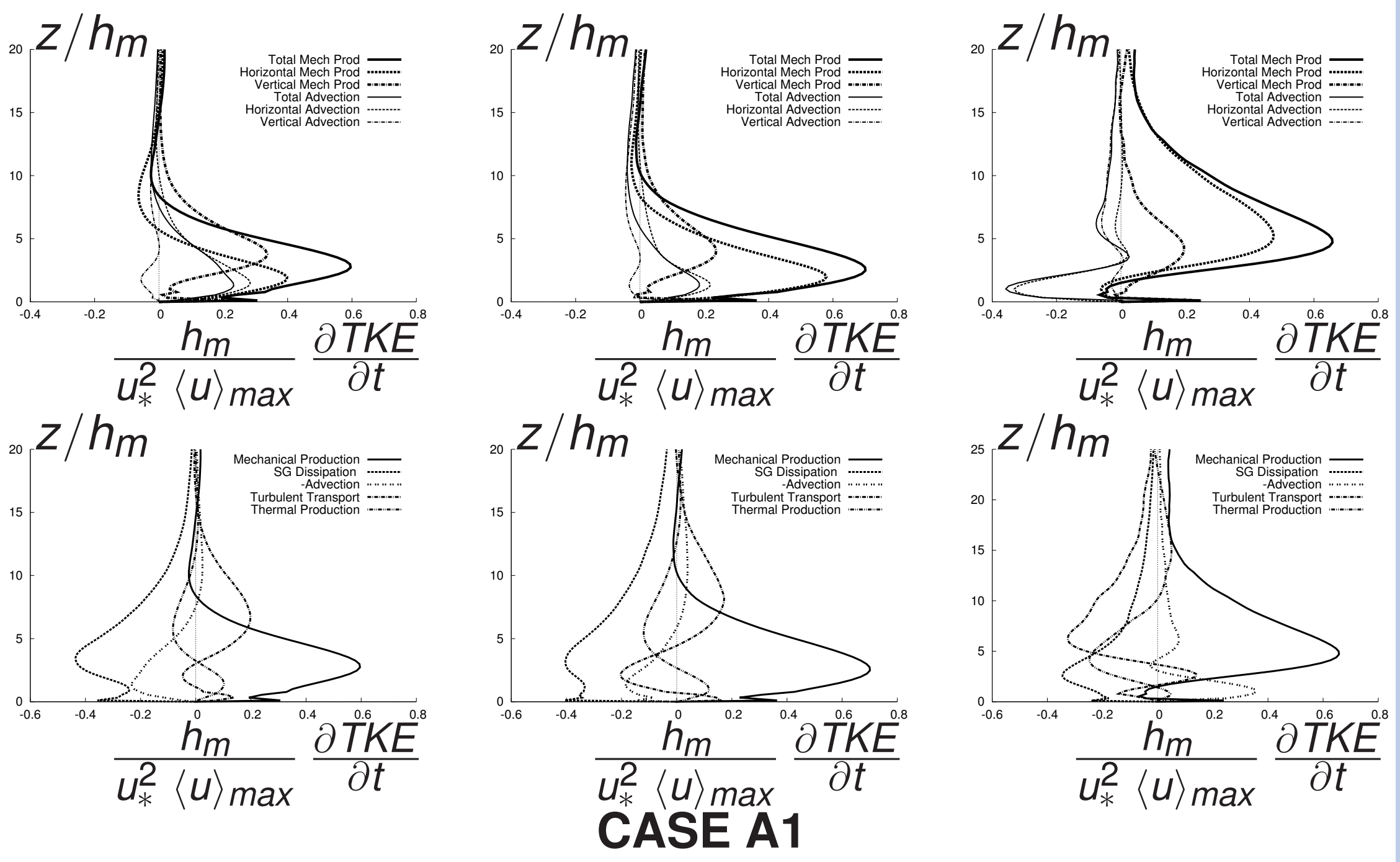
$$TKE = \frac{1}{2}\left(\langle u'^2\rangle + \langle v'^2\rangle + \langle w'^2\rangle\right)$$

Horizontal and vertical production

$$\Pi_x = -\langle u'w'\rangle\frac{\partial\langle w\rangle}{\partial x} - \langle u'^2\rangle\frac{\partial\langle u\rangle}{\partial x}$$
$$\Pi_z = -\langle u'w'\rangle\frac{\partial\langle u\rangle}{\partial z} - \langle w'^2\rangle\frac{\partial\langle w\rangle}{\partial z}$$

horizontal and vertical Advection

$$Adv_x = \langle u\rangle\frac{\partial TKE}{\partial x} \qquad Adv_z = \langle w\rangle\frac{\partial TKE}{\partial z}$$



CASE A1

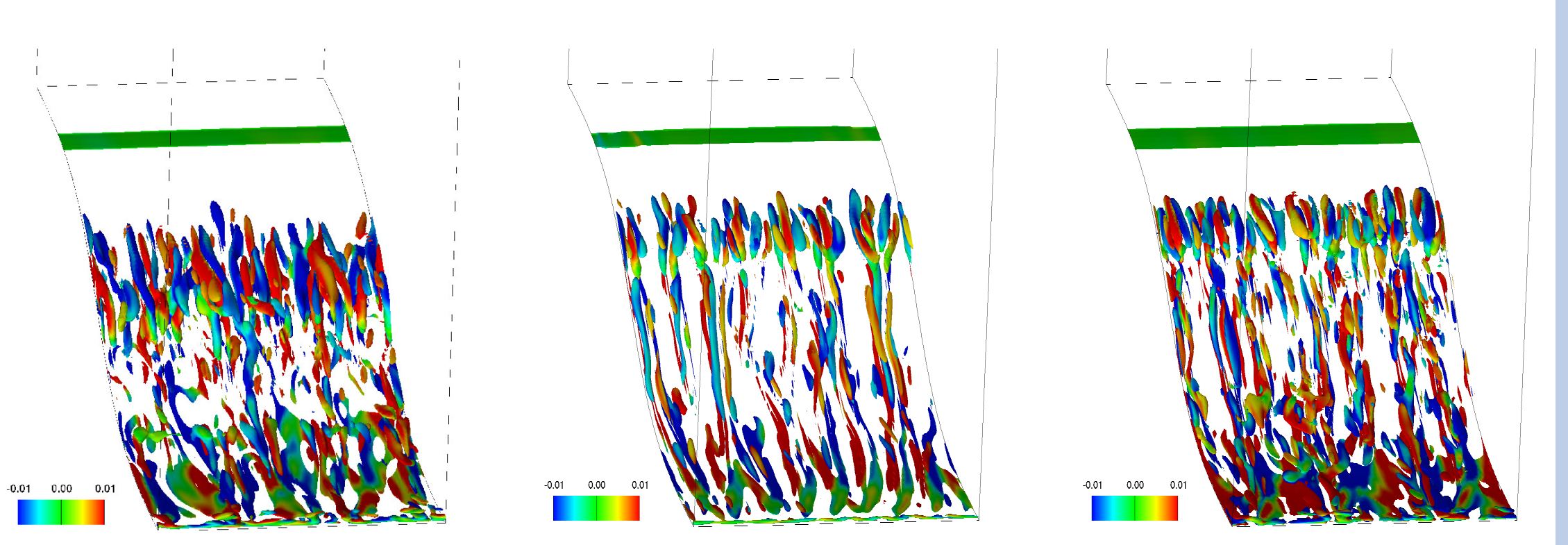
Communications

- Brun & Chollet Turbulent and Shear Flow Phenomena 2009
- Blein PhD Thesis 2016
- Brun, Blein & Chollet J. Atmospheric Sciences (submitted)

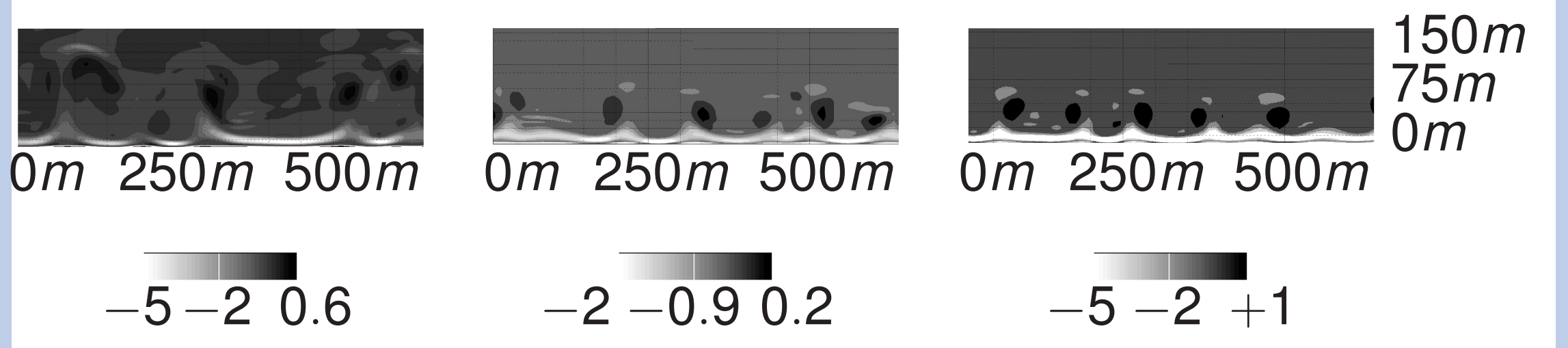
Flow Visualisation

- Transition to turbulence in the outer shear layer zone: $Ri \leq Ri_c = 0.2$
- Convex slope $- >$ Görtler instability : spanwise wavelength $\lambda_{\mathbf{y}} = \mathbf{110 - 130}$ **m**
- Strong streamwise counter-rotating (red/blue) vortices
- Local turbulent mixing enhanced

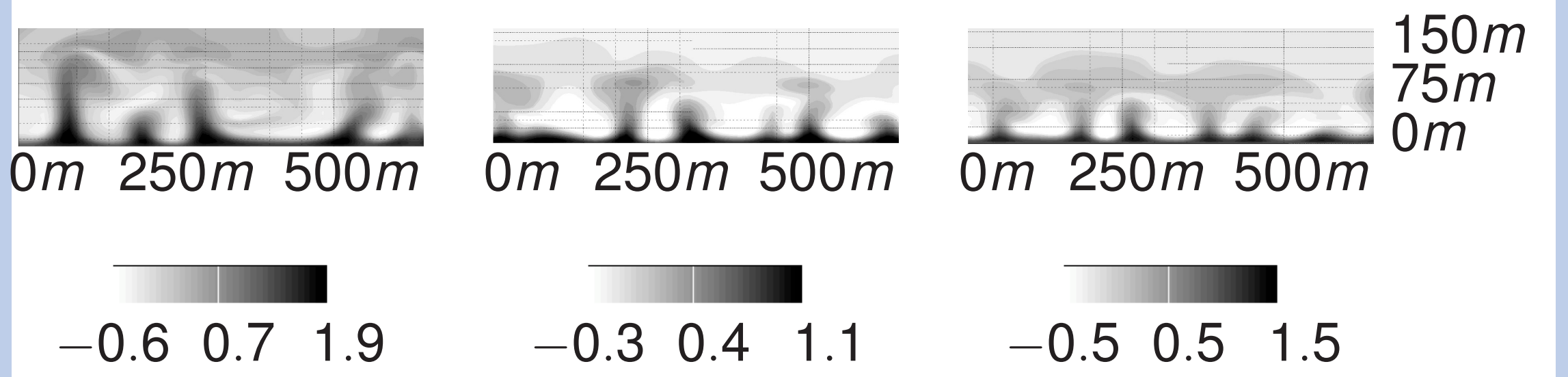
Q criterion $Q = \langle u\rangle^2_{max}/H^2$



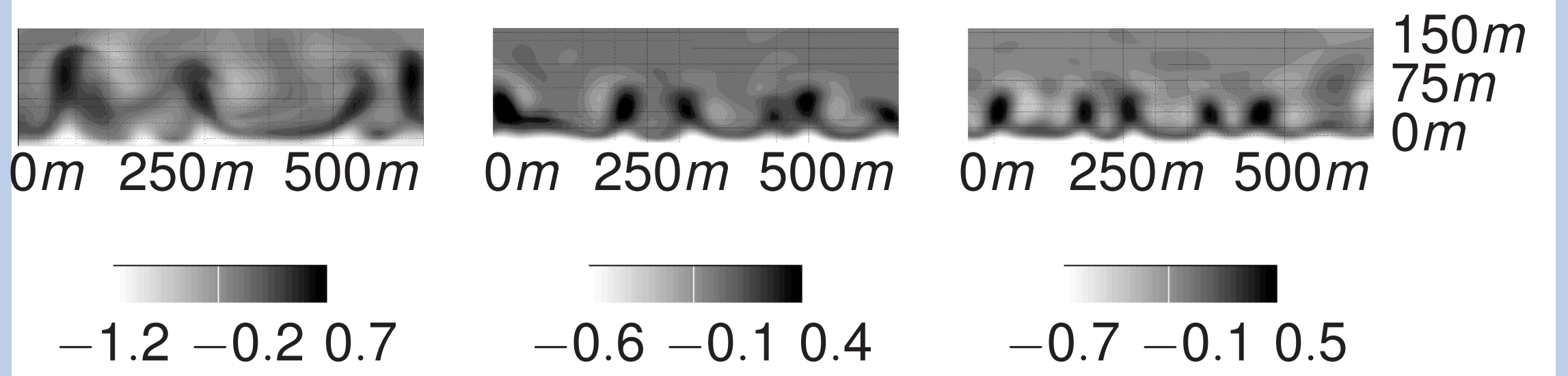
Q criterion Q $[10^{-3}$ $s^{-2}]$



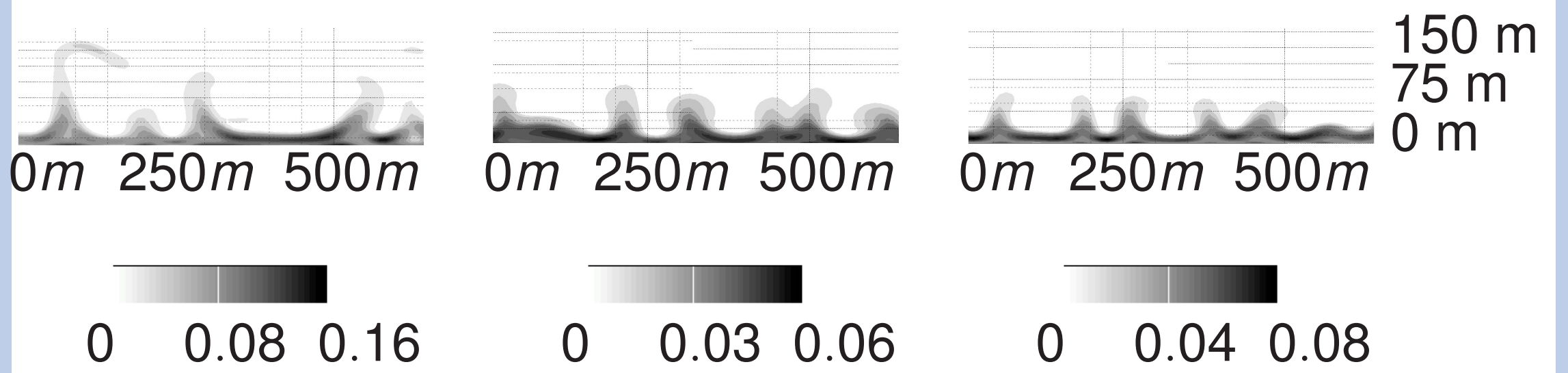
Streamwise velocity u_1 $[m.s^{-1}]$



vertical velocity (mixing) u_3 $[m.s^{-1}]$

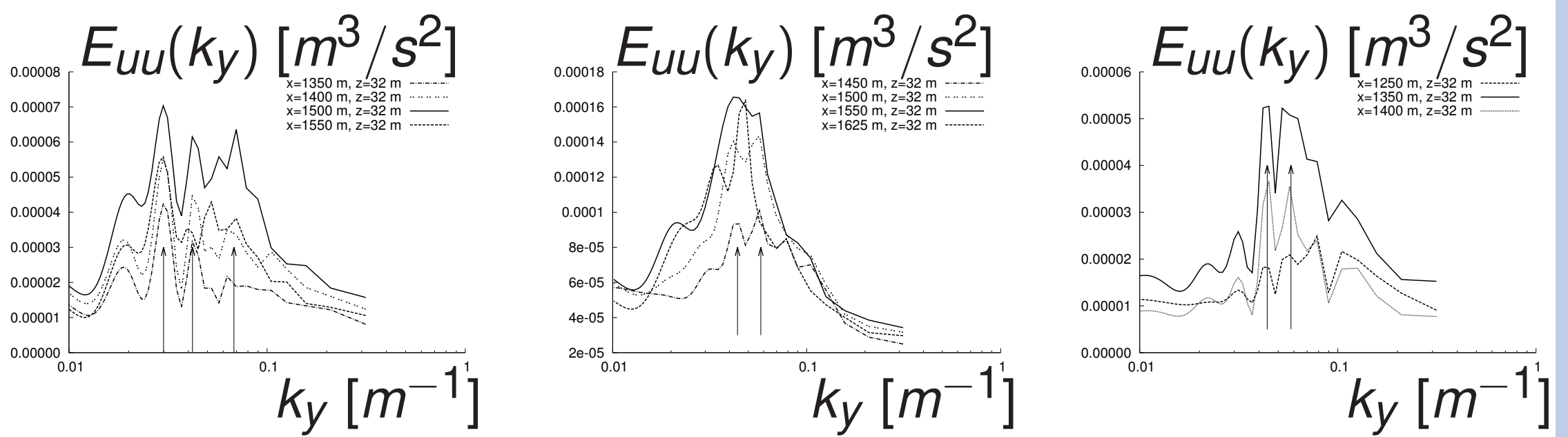


SGS Kinetic Energy (Turbulent shear layer) e $[m^2/s^2]$



Case A1 **Case A2** **Case A3**

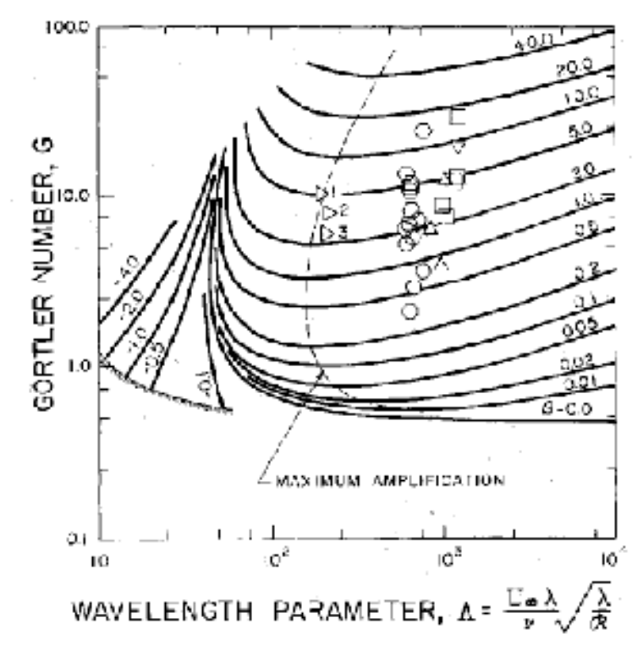
Spanwise Energy spectra



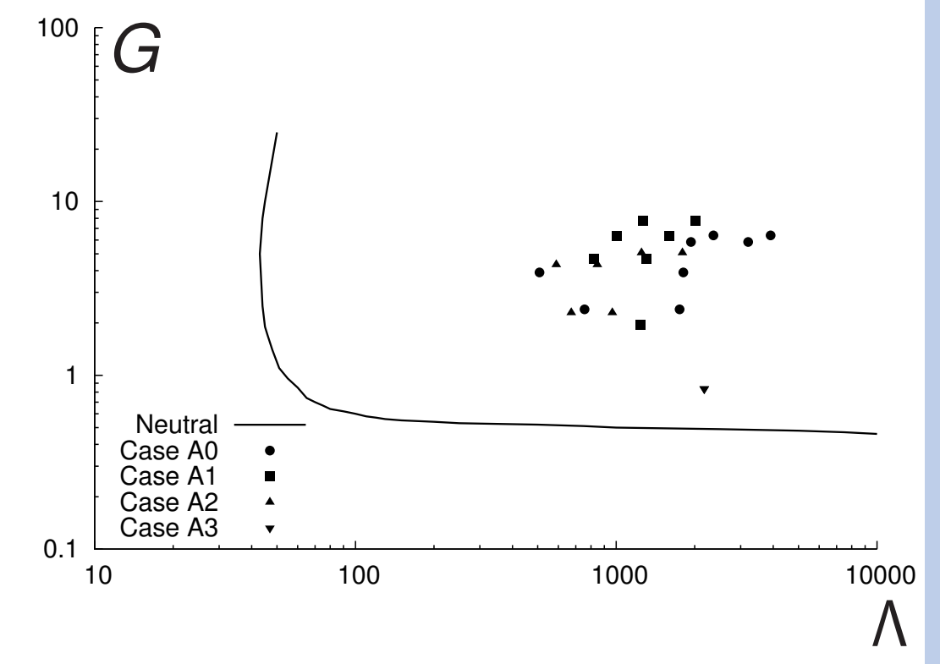
Case A0 **Case A1** **Case A2**

Görtler Stability Analysis

Boundary Layer along a concave wall



Katabatic jet along a convex surface



wavelength parameter:

$$\Lambda(x) = \frac{\langle u\rangle_{max}(x)\lambda_y(x)}{\nu_{LES}(x)}\left(\frac{\lambda_y(x)}{R_{min}}\right)^{0.5}$$

References

- Saric, Görtler vortices. *Ann. Rev. Fluid Mech.*, 1994
- Grisogono & Oerlemans, Katabatic flows : Analytical solution for gradually varying eddy diffusivities. *J. Atmospheric Sciences*, 2001
- Skyllingstad, Large-eddy simulation of katabatic flows. *Boundary Layer Meteor.*, 2003.
- Cuxart & Jimenez , Mixing processes in a nocturnal low-level jet: an LES study. *J. Atmospheric Sciences*, 2006
- Fedorovich & Shapiro, Structure of numerically simulated katabatic and anabatic flows along steep slopes. *Acta Geophysica*, 2009.
- Axelsen & van Dop, Large- eddy simulation of katabatic winds. Part 1 : comparison with observations. Part 2 : sensitivity study and comparison with analytical models. *Acta Geophysica*, 2009.

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