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Transition to geostrophic convection: The role of boundary conditions

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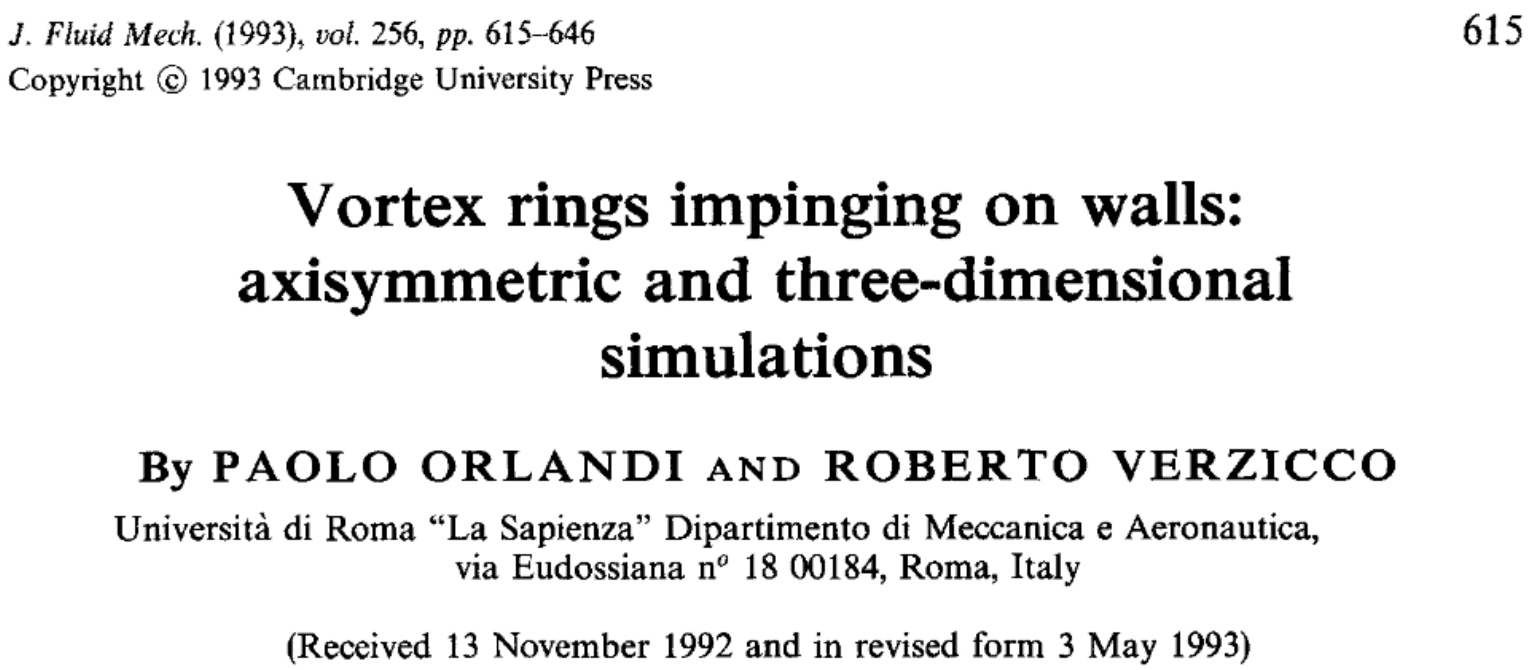
Emil was the Editor of my first JFM paper



J. Fluid Mech. (1993), *vol.* 256, *pp.* 615–646

Vortex rings impinging on walls: axisymmetric and three-dimensional simulations

By PAOLO ORLANDI AND ROBERTO VERZICCO
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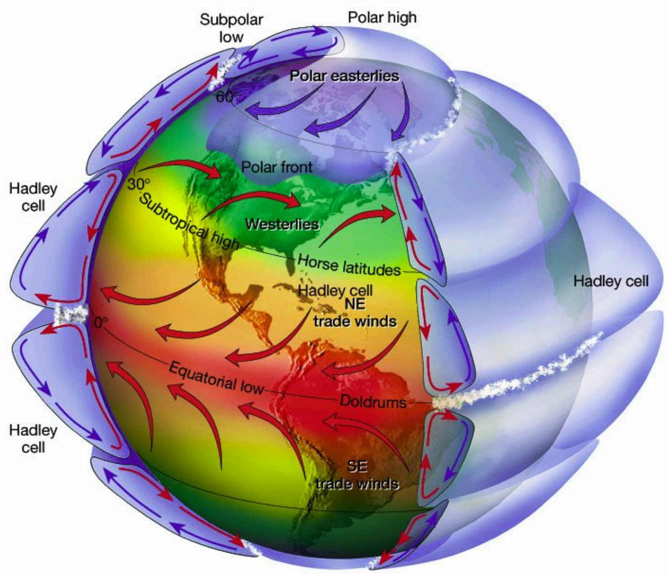
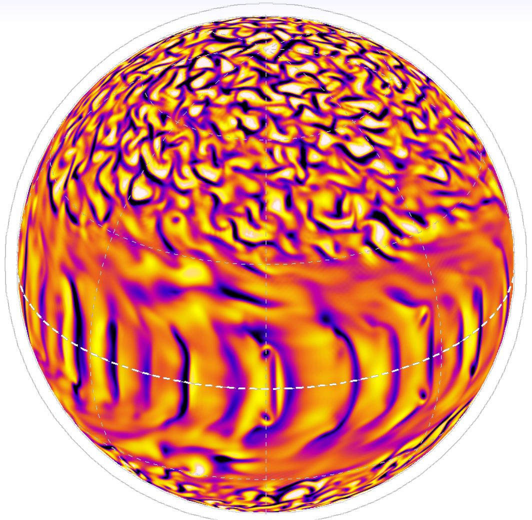
(Received 13 November 1992 and in revised form 3 May 1993)

He was very rigorous but we have to be grateful to him

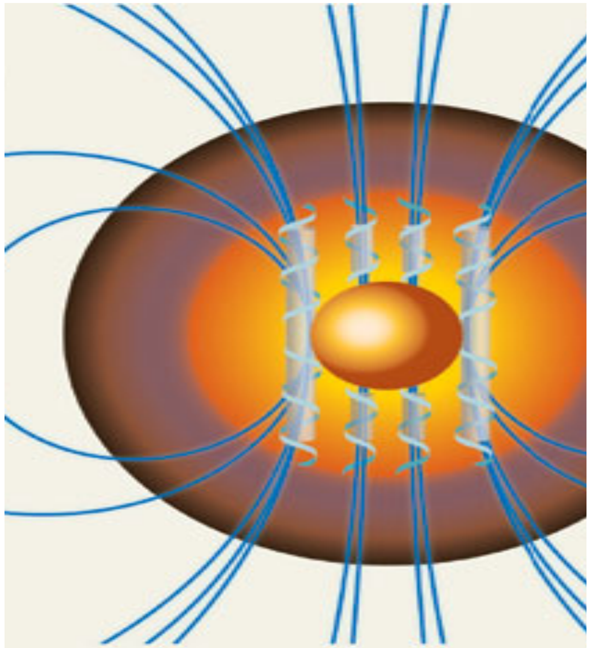
Turbulent *rotating* convection





BP Brown, U. Wisconsin

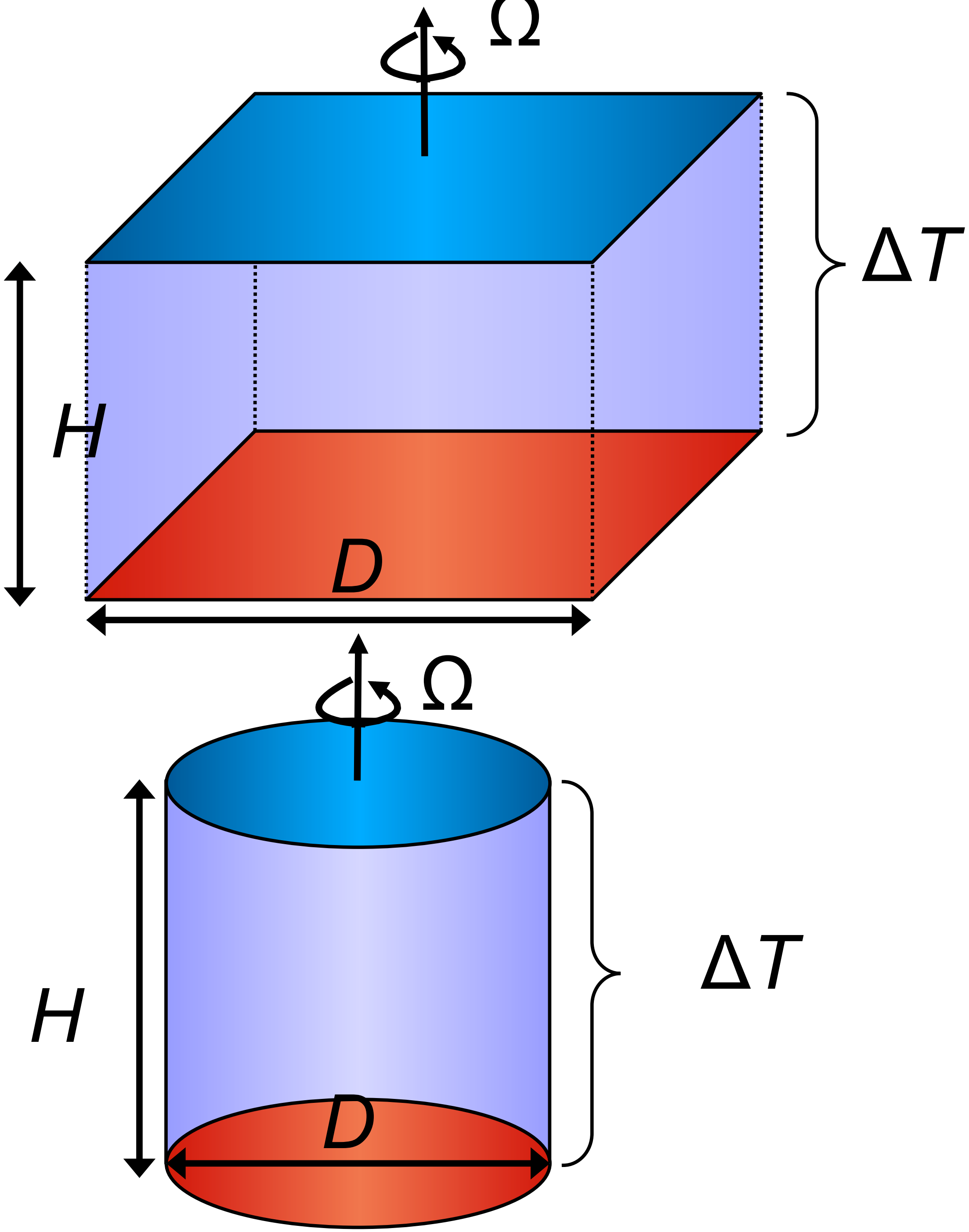
Relevant in the atmosphere of planets, in planets and stellar interiors



UR Christensen, *Nature* **454**, 1058 (2008)

… and in any flow where buoyancy interacts with a background rotation

Rotating thermal convection



Rayleigh: thermal forcing

$$Ra = \frac{g\alpha\Delta T\, H^3}{\nu\kappa}$$

Prandtl: fluid properties

$$Pr = \frac{\nu}{\kappa}$$

Ekman: $\dfrac{\text{viscous}}{\text{Coriolis}}$

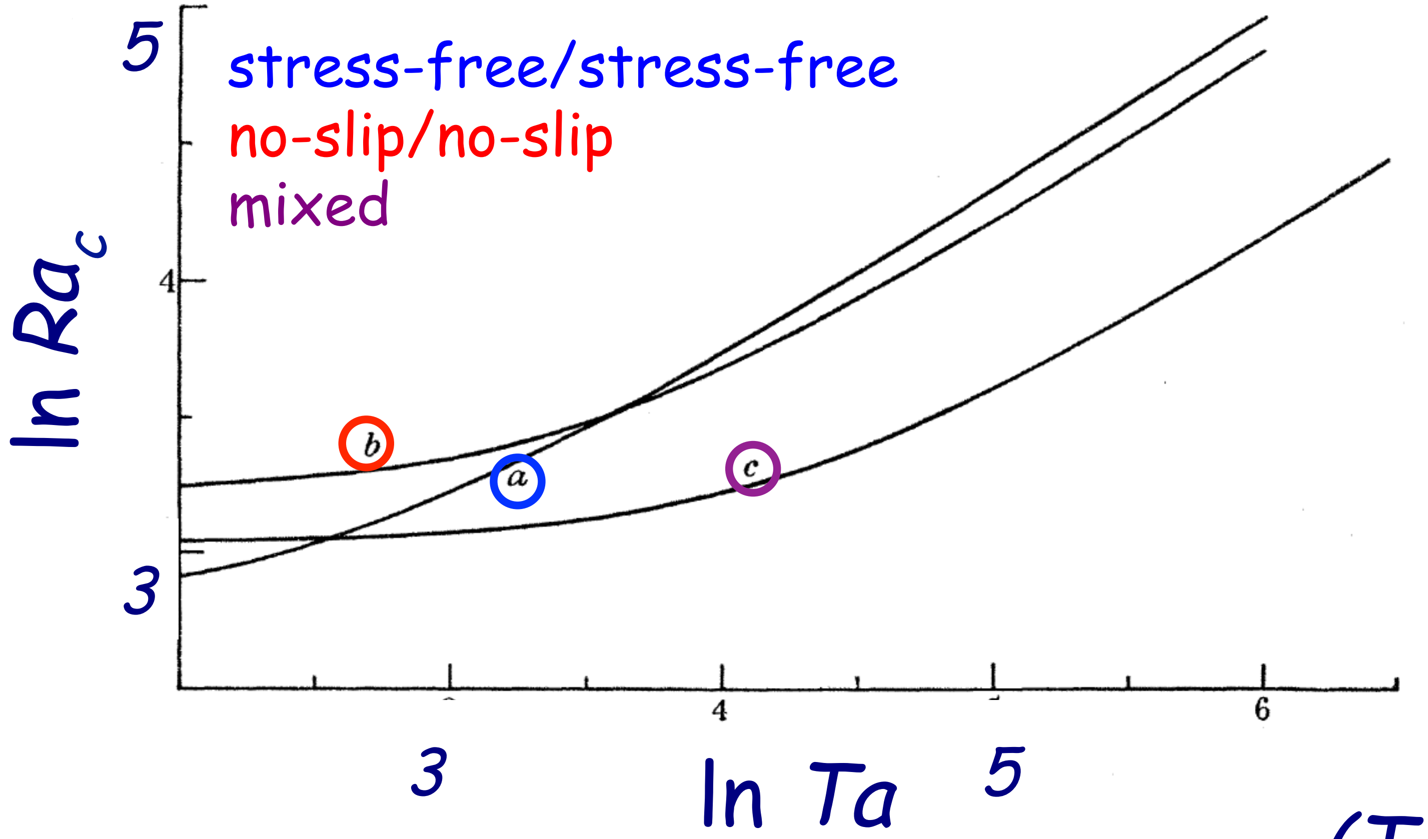
$$Ek = \frac{\nu}{2\Omega H^2}$$

Aspect ratio: geometry

$$\Gamma = \frac{D}{H}$$

Rotation *stabilises* RB convection

S Chandrasekhar, *Proc R Soc Lond A* **217**, 306 (1953)



$(Ta=Ek^{-2})$

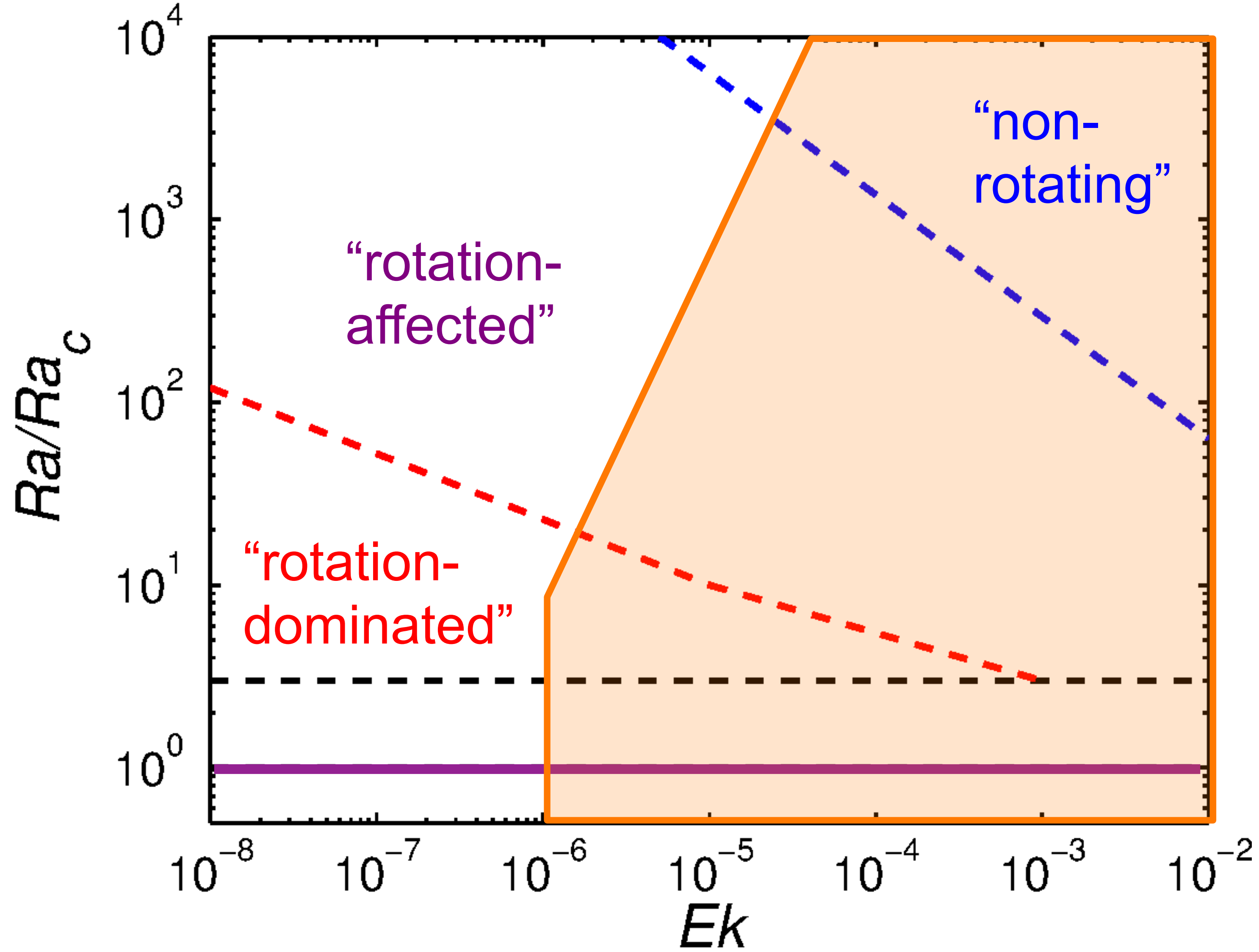
Asymptote for rapid rotation ($Ta \rightarrow \infty$):

$Ra_c = 8.7\ Ta^{2/3} = 8.7\ Ek^{-4/3}$

$L_c = 4.8\ Ta^{-1/6} = 4.8\ Ek^{1/3}$ (most unstable wavelength)

Parameter space (Ra/Ra_c vs Ta)

RE Ecke & JJ Niemela, *Phys Rev Lett* **113**, 114301 (2014)

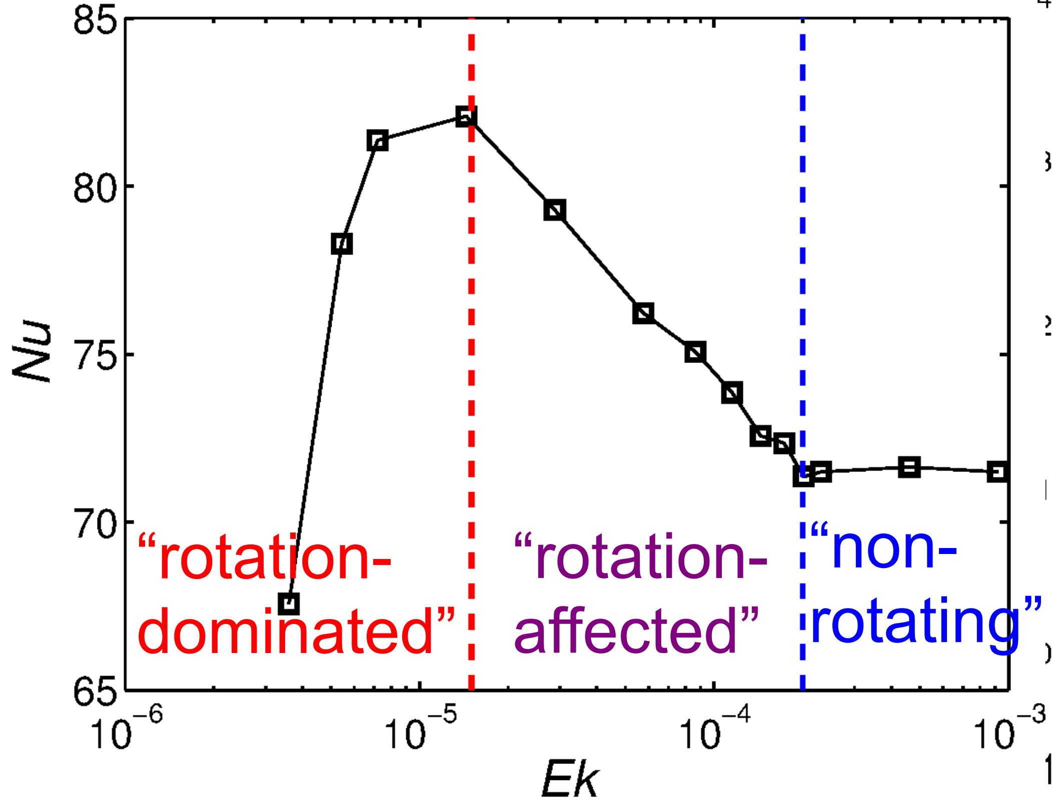


State of the art: DNS and water-based experiments

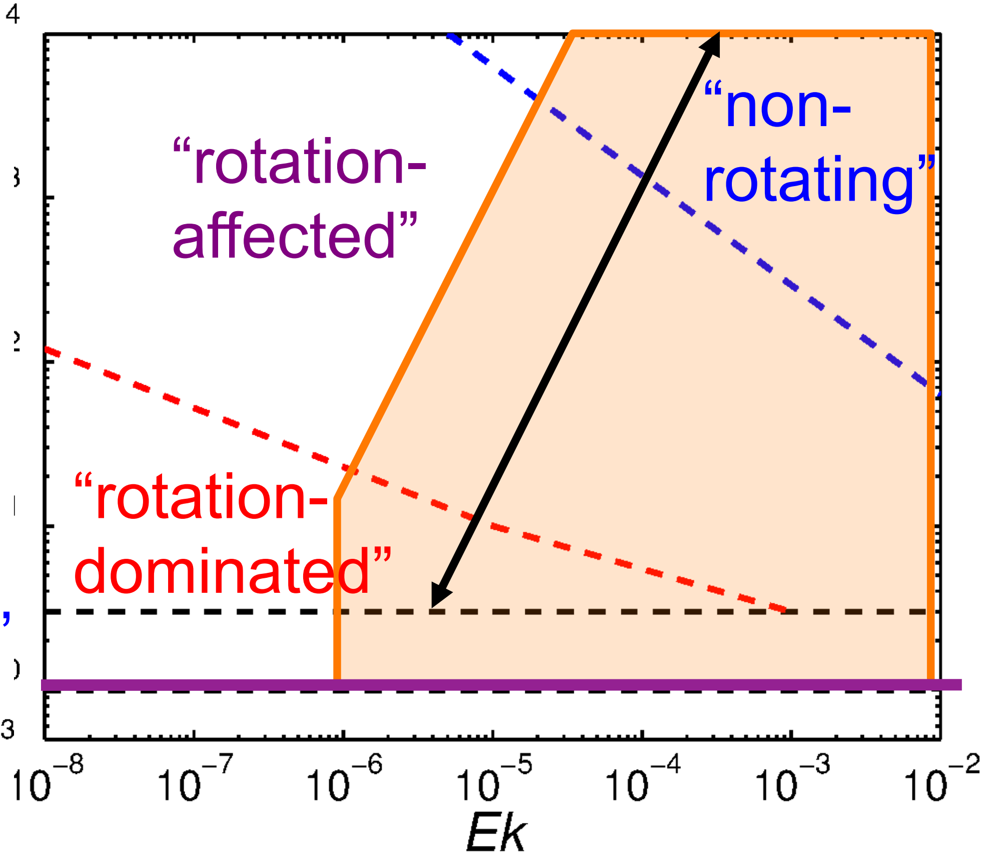
Critical *Ra* (Chandrasekhar)

Parameter space (Ra/Ra_c vs Ta)

Typical result from Kunnen et al. (2008)



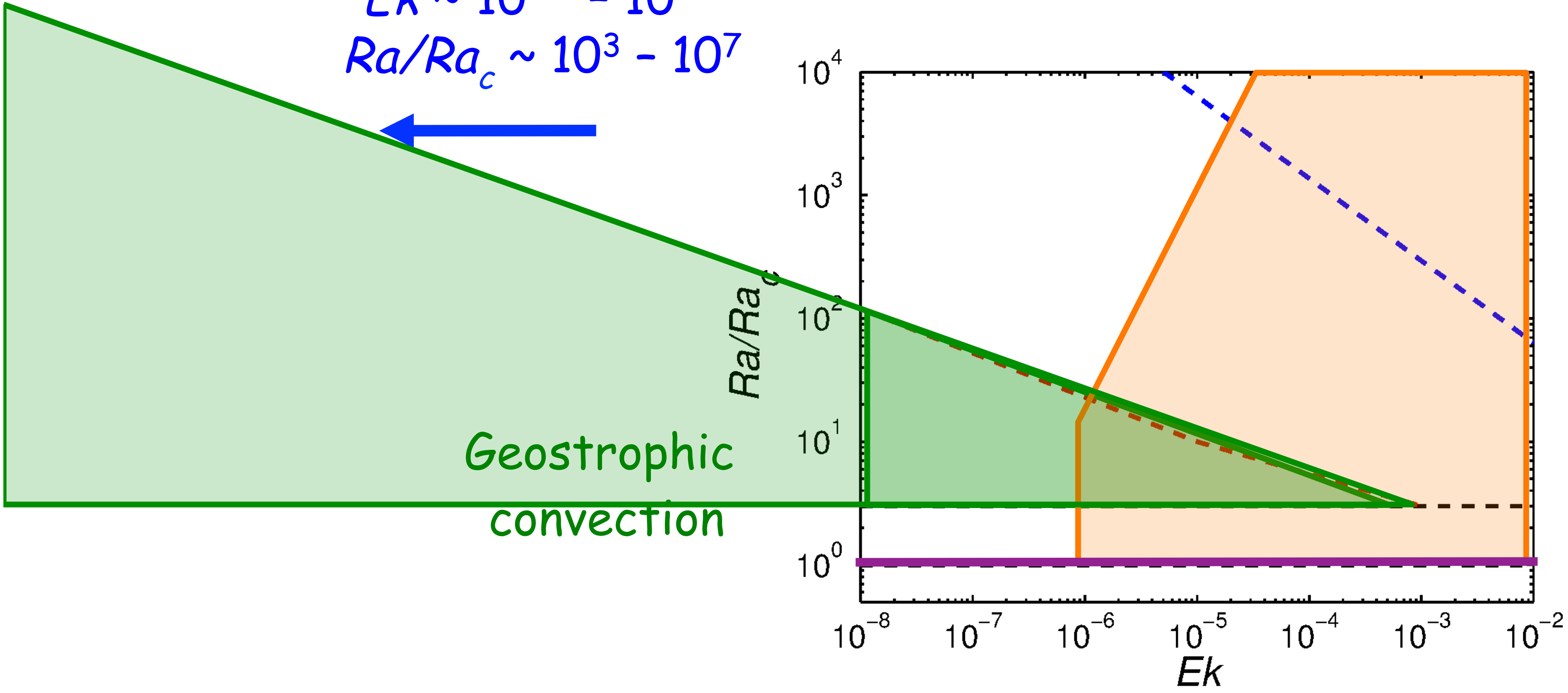
Ra= 10^9, Pr=7



After Ecke & Niemela (2014)

Parameter space (Geostrophic convection)

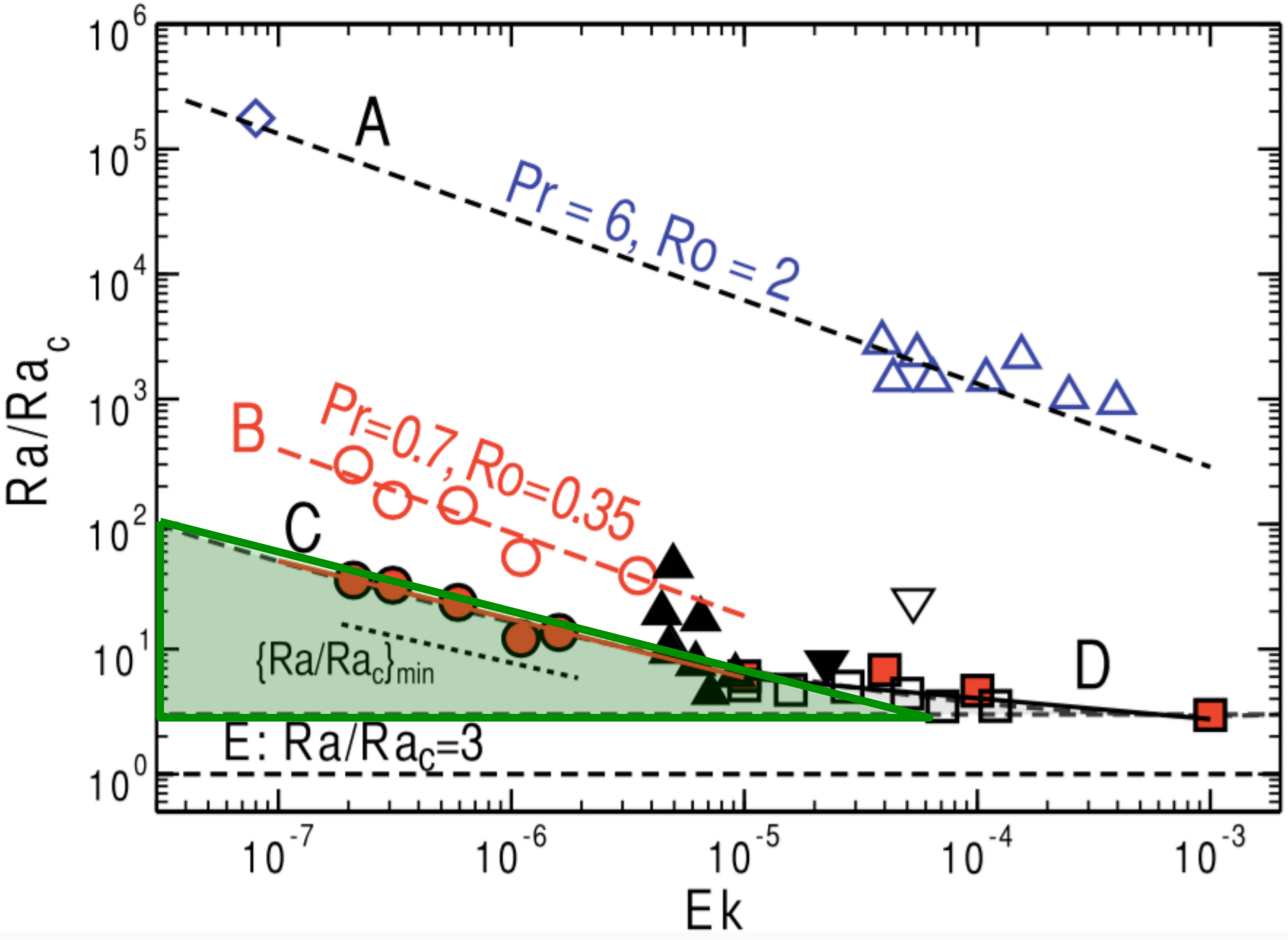
Geo-/astrophysics:
$Ra \sim 10^{20} - 10^{25}$
$Ek \sim 10^{-15} - 10^{-12}$
$Ra/Ra_c \sim 10^{3} - 10^{7}$



Geostrophic convection

Figure from Ecke & Niemela 2014

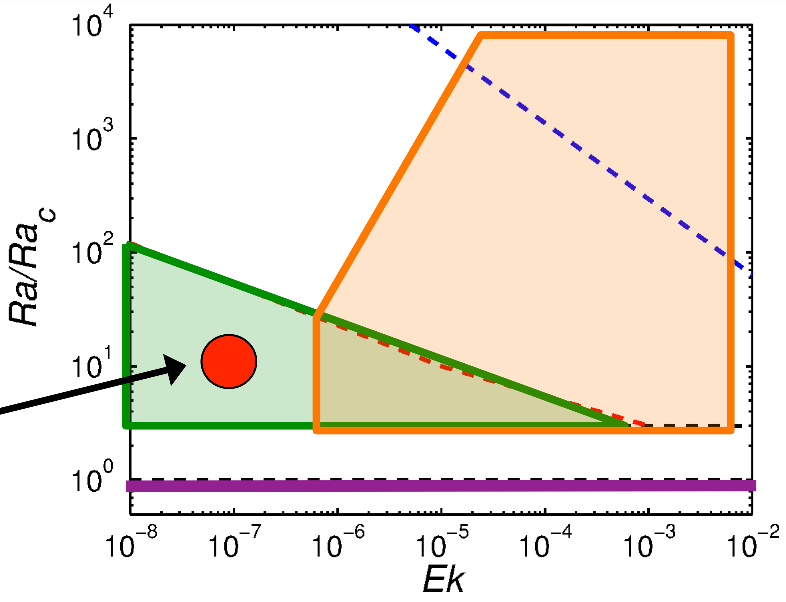
The "interesting" area is quite empty



Data from: Ecke & Niemela 2014, King et al. 2009, Zhong et al. 2009, Liu & Ecke 2009, Zhong & Ahlers 2010, Niemela et al. 2010

Geostrophic convection: parameters

Challenging for
experiment and
numerics!



Example: $Ra/Ra_c = 10$ and $Ek = 1 \times 10^{-7}$

$Ra_c = 8.7\, Ek^{-4/3} = 1.9 \times 10^{10}$ → $Ra \approx 2 \times 10^{11}$!!!

Assume water-filled cell, $H = 1$ m, $\Gamma = 1/2$ → $\Delta T \approx 10$ K

To get to $Ek = 10^{-7}$ → $\Omega = 5$ rad/s = 48 rpm !!!

Extra problem: centrifugal buoyancy $\Omega^2 R\, /\, g = 0.64$

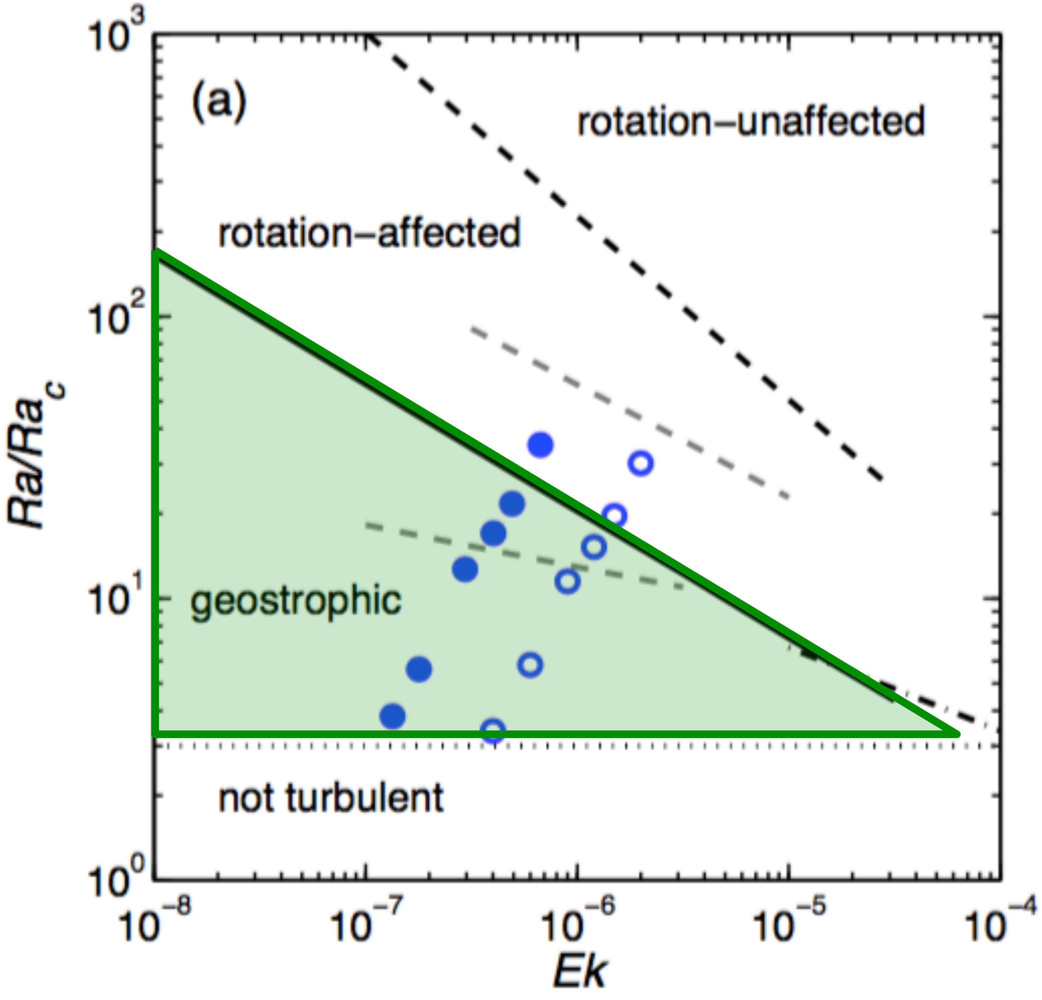
Objective

Fill with some data the "empty" region

Explore transition to the geostrophic regime

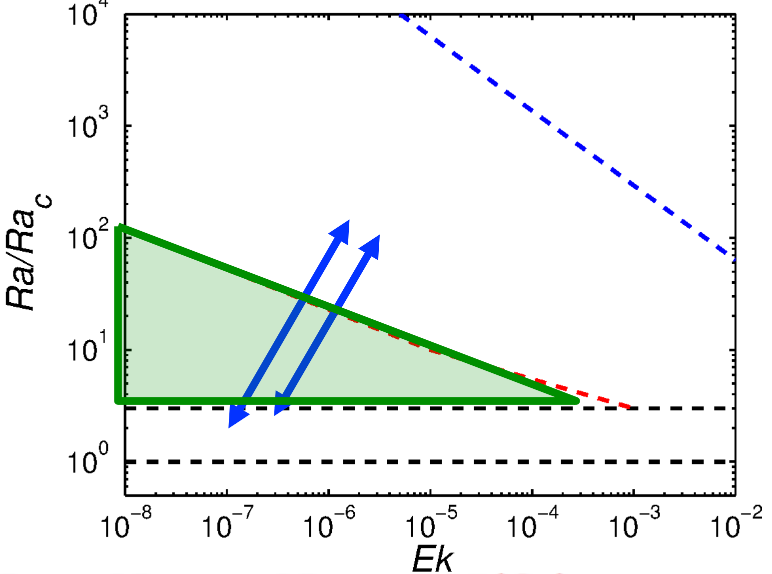
DNS at constant *Ra* = 1 x 10^{10} or 5 x 10^{10}, *Pr* = 1 at various *Ek* numbers

Compare no-slip (NS) and stress-free (SF) plates: Ekman boundary layers present/ absent



Numerical simulations

26 three-dimensional simulations (13 no-slip and 13 stress-free) to study the transition to the geostrophic regime



Γ fits 10 most unstable wavelengths ($L_c=4.82Ek^{1/3}$) (only for NS)

Ra	Ek	Ro	$\widetilde{Ra}$	Γ	$N_x \times N_y \times N_z$	Nu_{SF}	Nu_{NS}	$N(\delta_\nu)$
1×10^{10}	4.00×10^{-7}	0.040	29.5	0.36	$384 \times 384 \times 768$	8.82	21.0	12
1×10^{10}	4.00×10^{-7}	0.040	29.5	0.71	$768 \times 768 \times 768$	9.13	21.0	12
1×10^{10}	6.00×10^{-7}	0.060	50.6	0.41	$384 \times 384 \times 768$	20.7	31.4	15
1×10^{10}	9.00×10^{-7}	0.090	86.9	0.46	$384 \times 384 \times 768$	46.2	50.2	17
1×10^{10}	1.20×10^{-6}	0.12	127.5	0.51	$384 \times 384 \times 768$	68.5	65.2	18
1×10^{10}	1.50×10^{-6}	0.15	171.7	0.55	$384 \times 384 \times 768$	91.0	76.0	20
1×10^{10}	2.00×10^{-6}	0.20	252.0	0.61	$512 \times 512 \times 768$	113.7	83.5	23
5×10^{10}	1.34×10^{-7}	0.030	34.3	0.25	$512 \times 512 \times 1024$	9.20	21.1	12
5×10^{10}	1.79×10^{-7}	0.040	50.4	0.27	$512 \times 512 \times 1024$	18.2	30.8	14
5×10^{10}	2.95×10^{-7}	0.066	98.3	0.32	$512 \times 512 \times 1024$	52.9	61.5	17
5×10^{10}	4.02×10^{-7}	0.090	148.6	0.36	$512 \times 512 \times 1024$	95.0	88.3	19
5×10^{10}	4.92×10^{-7}	0.11	194.2	0.38	$512 \times 512 \times 1024$	117.0	103.5	21
5×10^{10}	6.71×10^{-7}	0.15	293.6	0.42	$512 \times 512 \times 1024$	159.6	119.5	23

AFiD

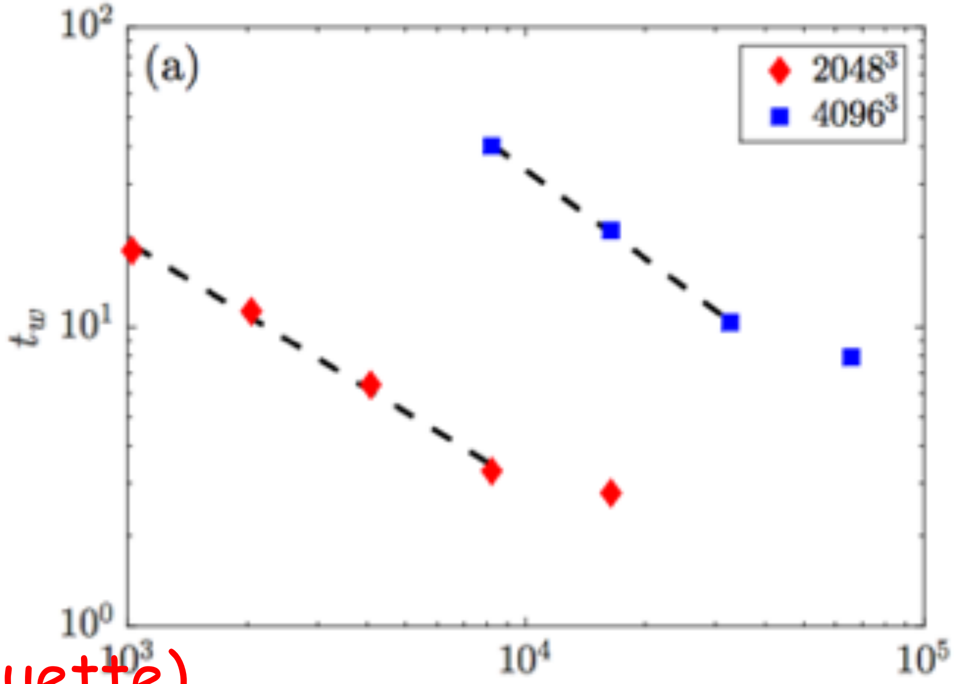
Highly parallel code for wall bounded turbulence

Open-source code available at

www.afid.eu

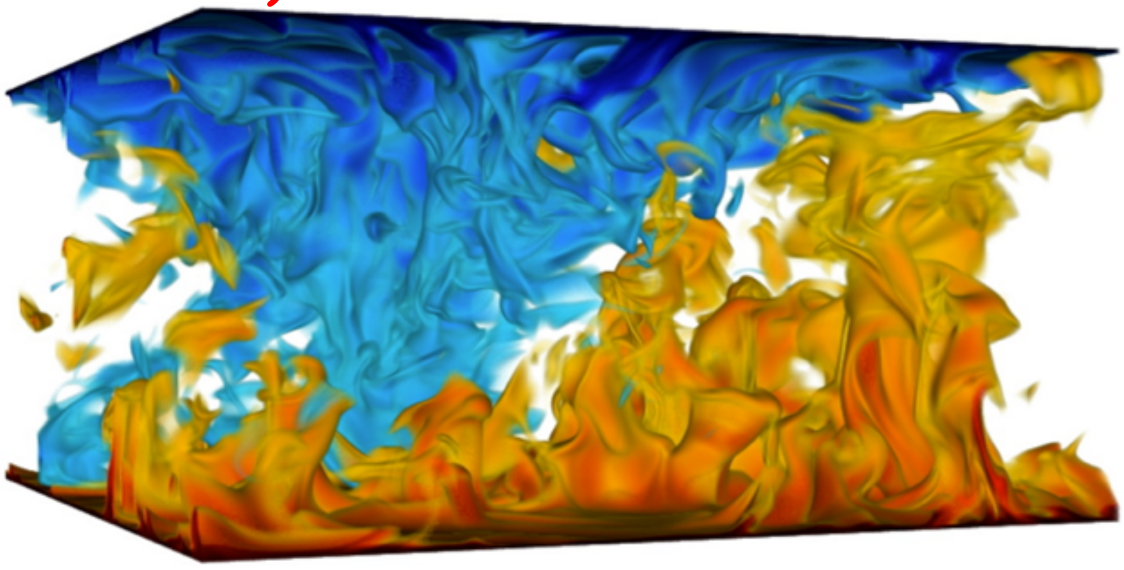
Reference and tutorial:

Van der Poel et al. (2015), Computers & Fluids **116**, "*A pencil distributed finite difference code for strongly turbulent wall-bounded flows*"



Upcoming modules:

- Cylindrical coordinates (Taylor Couette)
- Lagrangian particles
- Double diffusive convection
- GPU architectures



Physics of Fluids

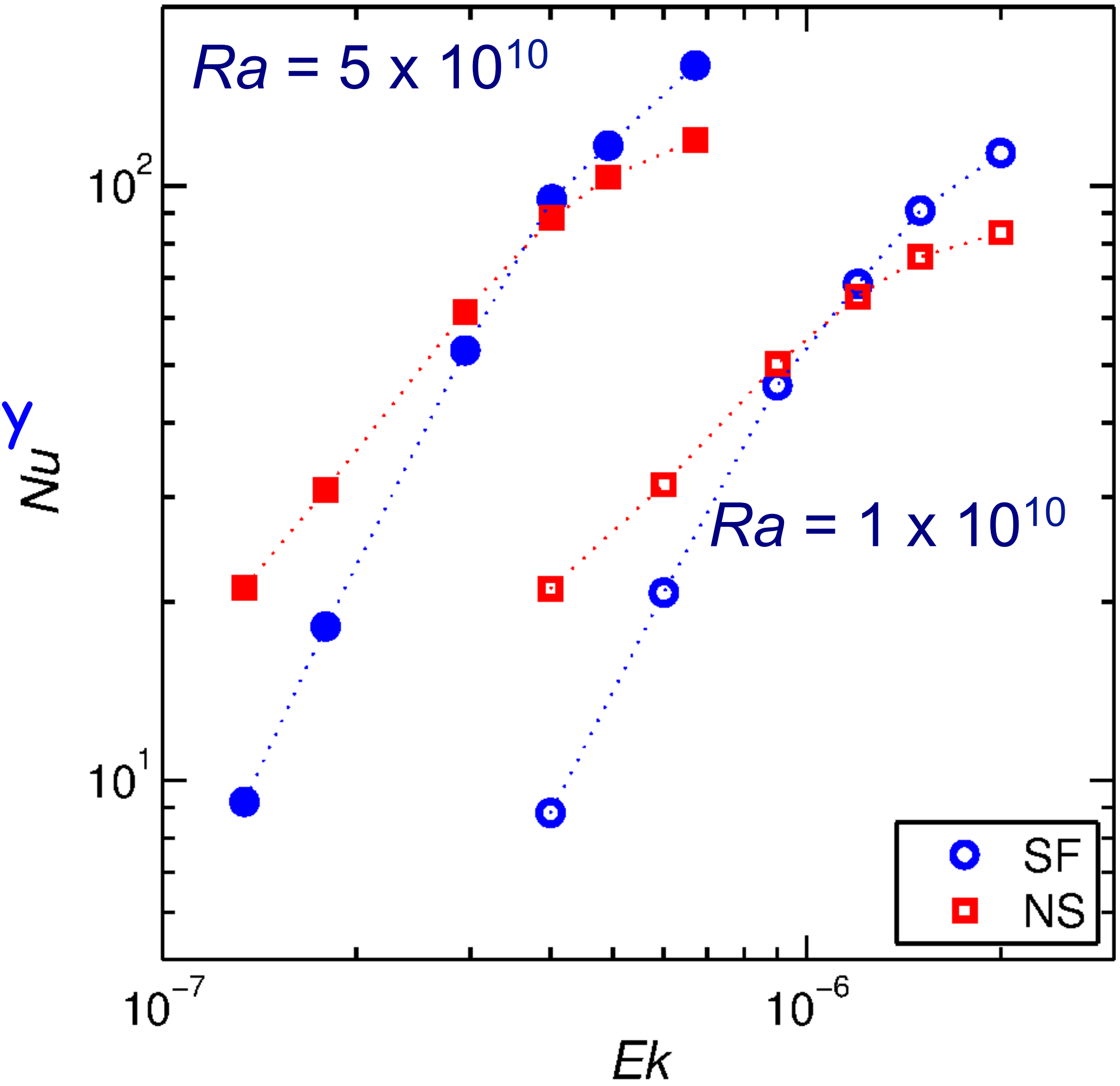
SURF SARA

Heat transfer (Nusselt)

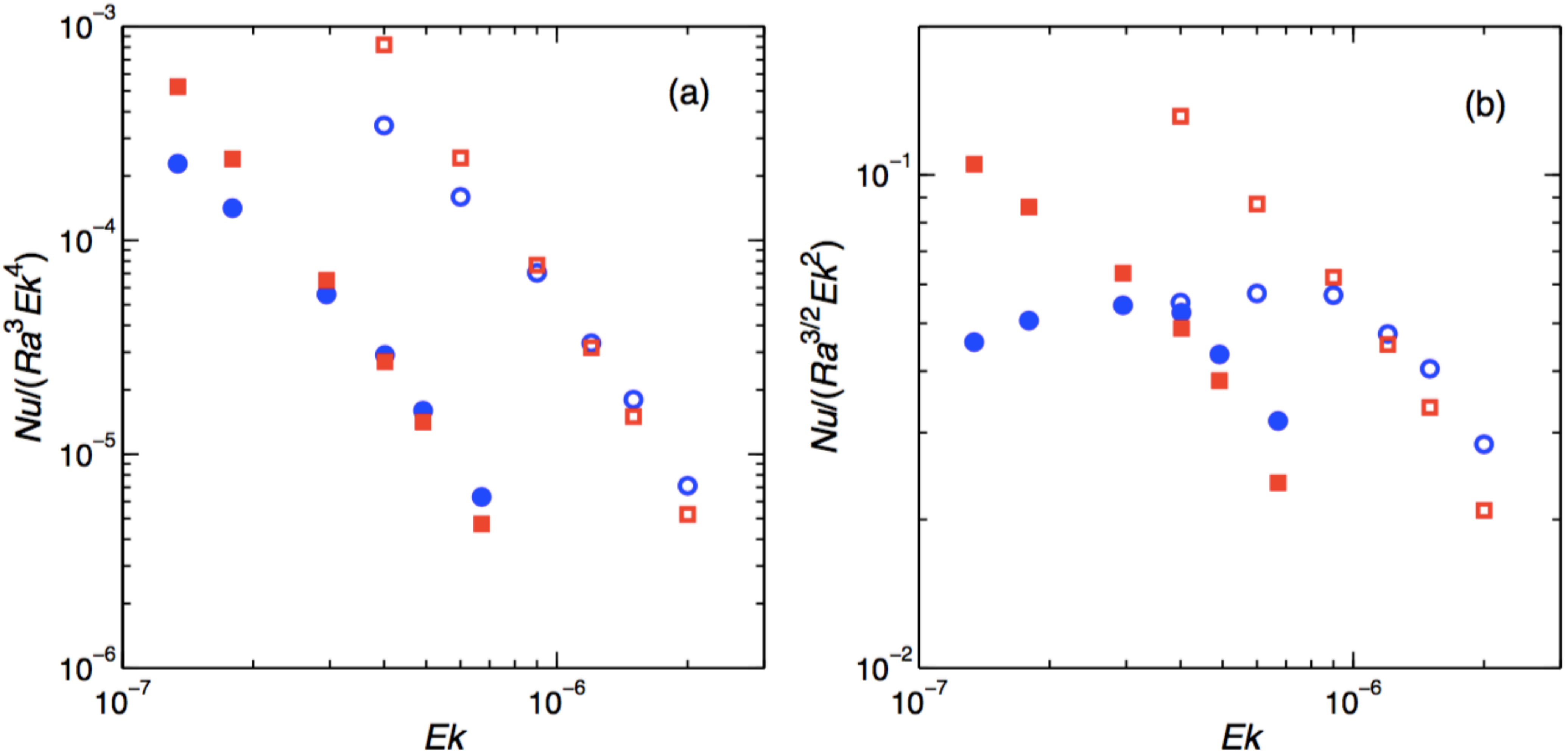
Transition: change of slope *Nu*(*Ek*)

Strong effect of boundary condition (SF/NS), but transition at roughly the same *Ek*

In geostrophic regime *Nu*(NS) > *Nu*(SF) !!



Nusselt vs Ek

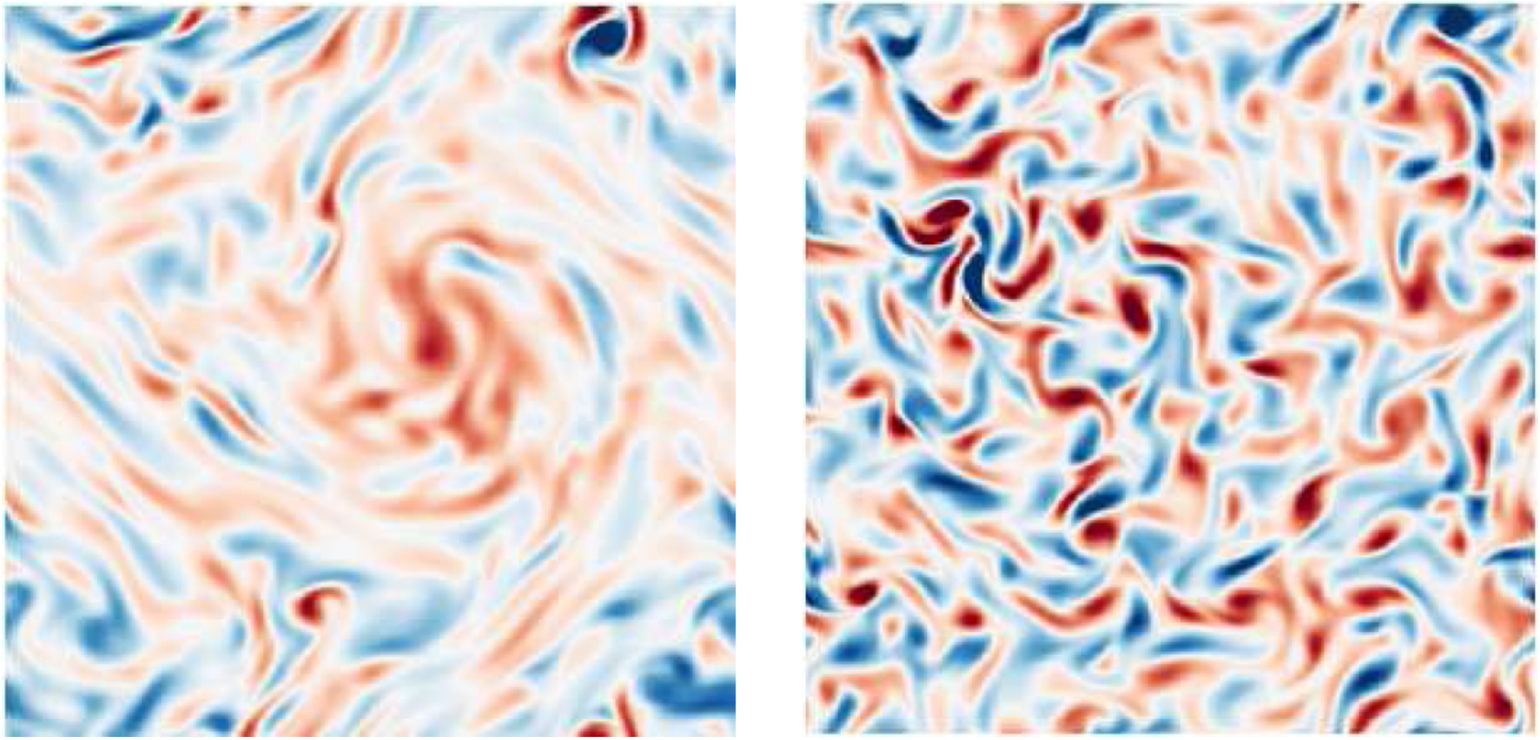


The theoretical prediction by Julien et al (2012) **$Nu \approx Ek^{\alpha}$** with **α=2** agrees fairly well with the SF cases but not with the NS ones

The exponent **α** shows a pronounced dependence on Ra and this might explain the scatter in the observations **$1<\alpha<4$**

Flow snapshots: vorticity

$Ra = 5 \times 10^{10}$, $Ek = 1.34 \times 10^{-7}$



stress-free (SF)

no-slip (NS)

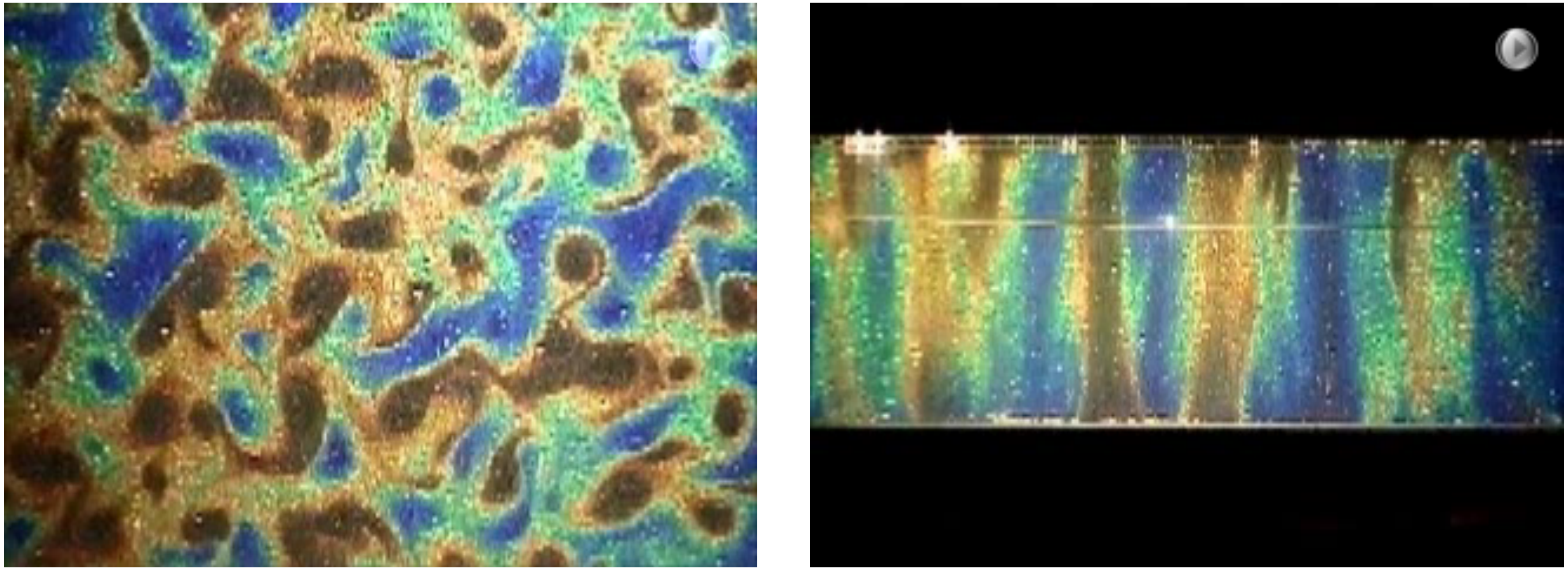
SF: large vortex due to inverse energy cascade (Rubio *et al.*, Favier *et al.*)

NS: Ekman layer: source of intense fluctuations

AM Rubio, K Julien, E Knobloch & JB Weiss, *Phys Rev Lett* **112**, 144501 (2014)
B Favier, LJ Silvers & MRE Proctor, *Phys Fluids* **26**, 096605 (2014)

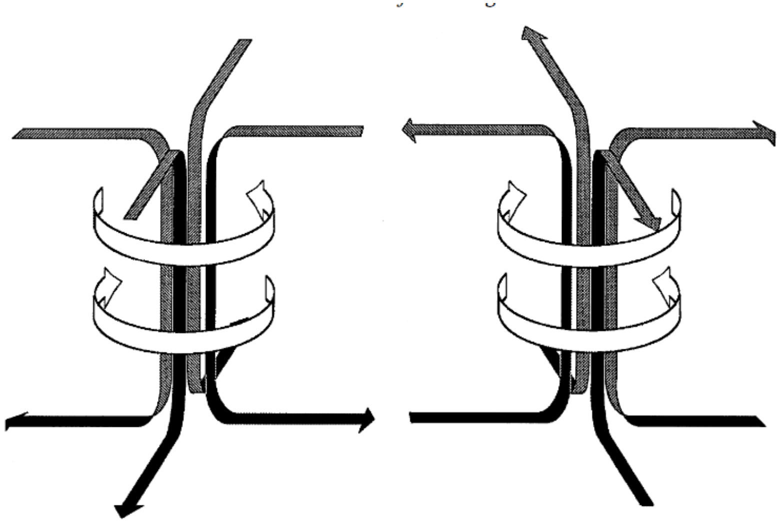
Experimental visualizations (Sakai 1997)

$10^6 < Ra < 10^8$, $Ek < 10^{-12}$



Thermochromic crystals: blue (hot) orange (cold)

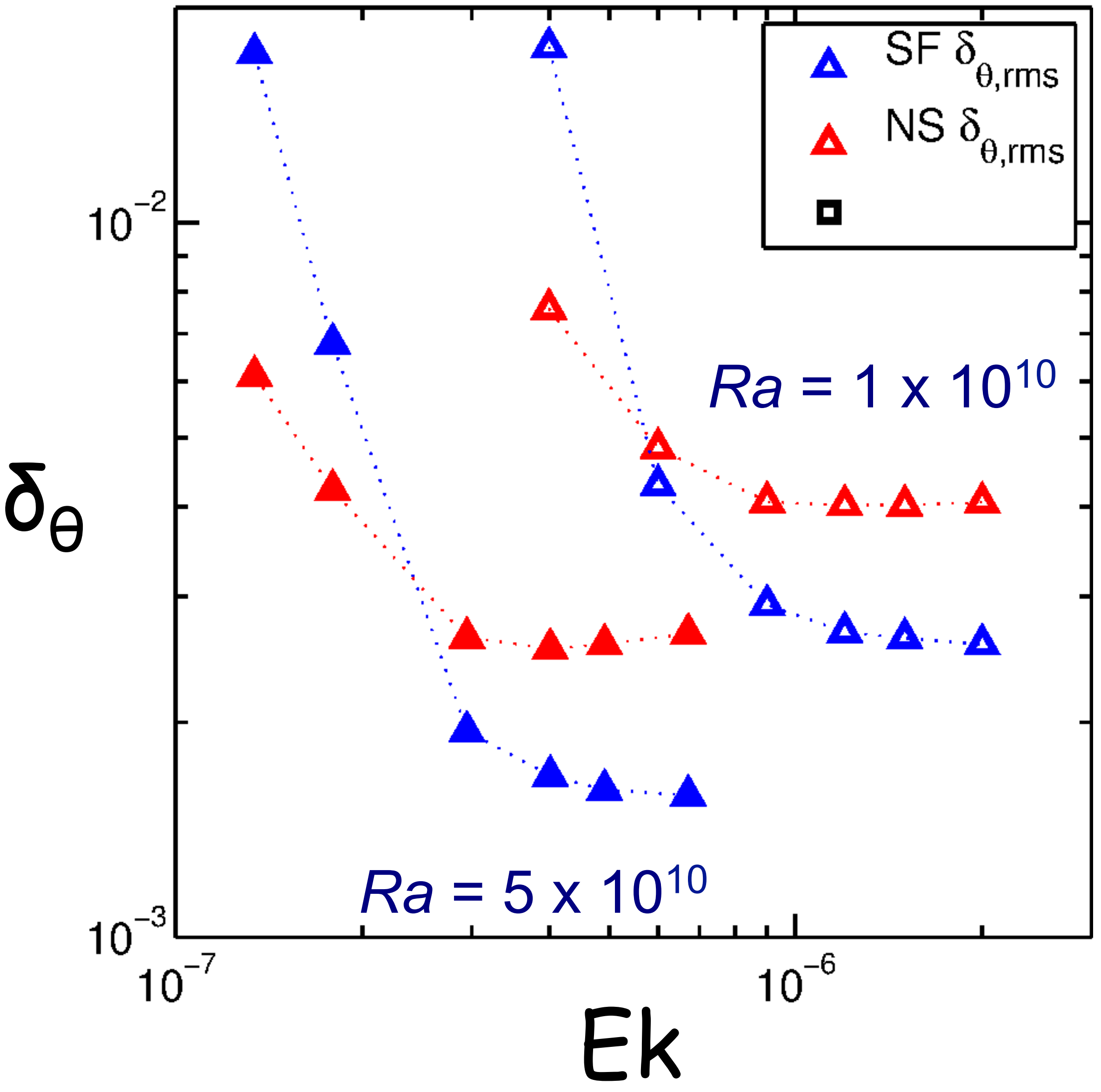
The Ekman layer "pumps" the heat into the vertical columns and this enhances the Nu with respect to the free-slip case



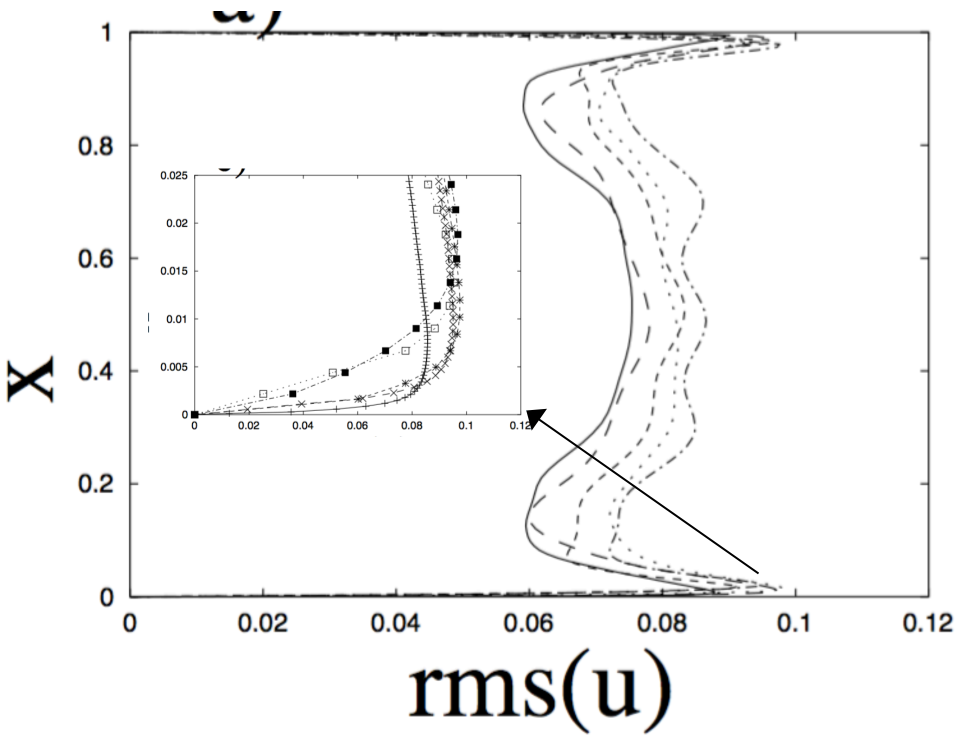
S. Sakai, *J. of Fluid Mech.* **333** (1997)

Thermal boundary layers

Thermal BL:
The transition occurs for, both SF and NS, at similar values of Ek

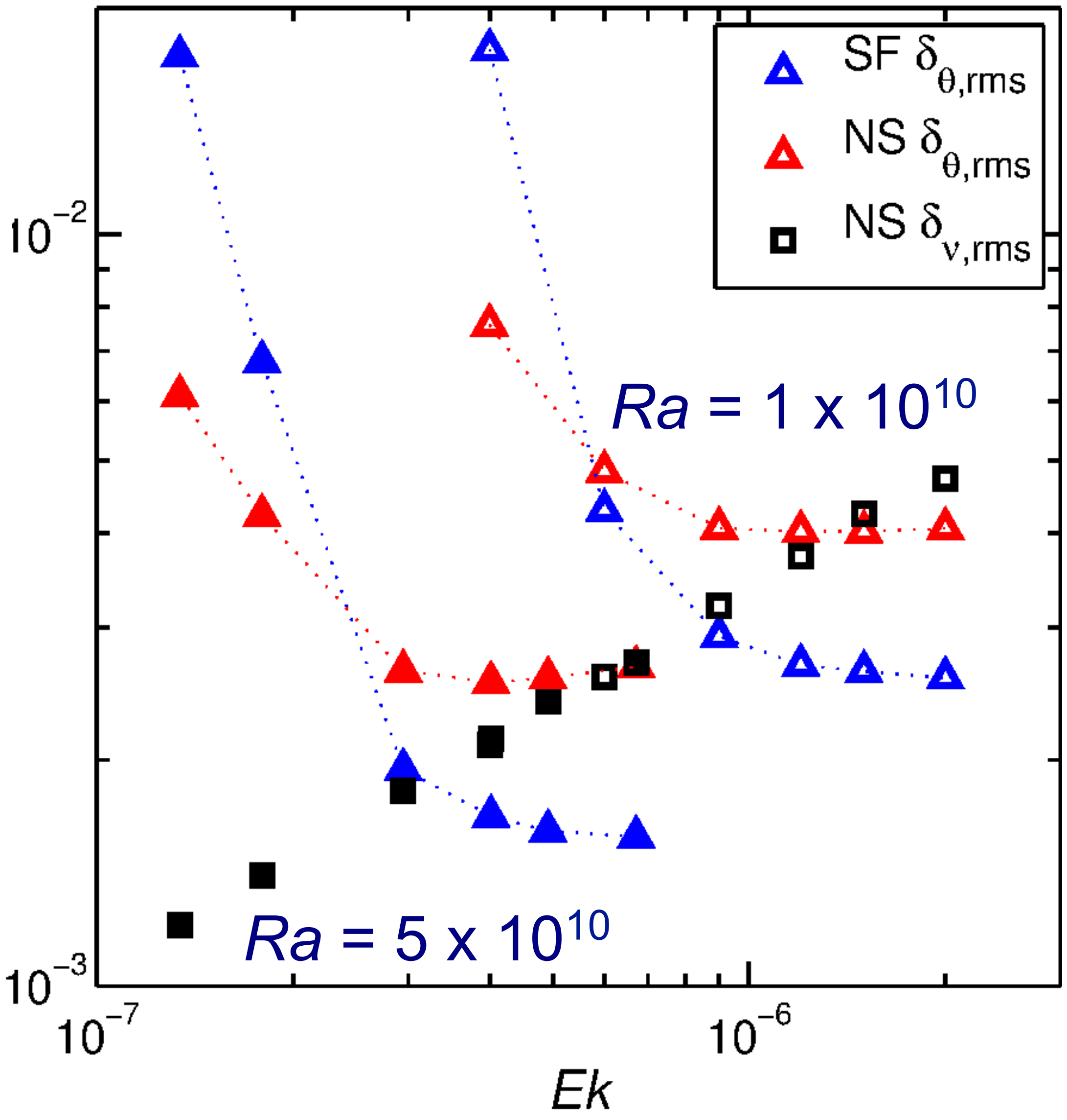


Viscous and thermal boundary layers

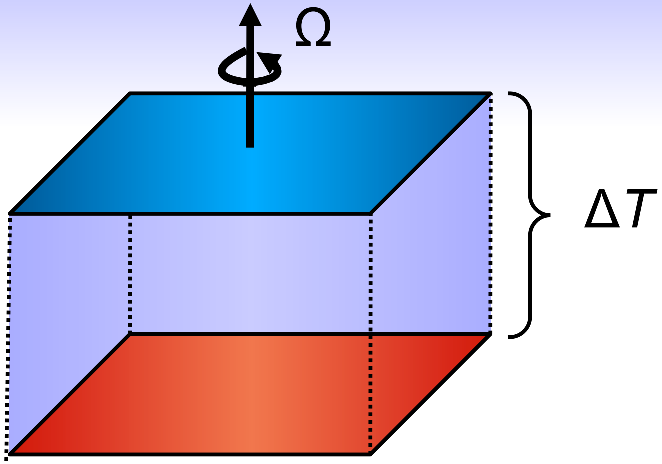


The viscous BL does not show a transition; BL thickness fully determined by *Ek*

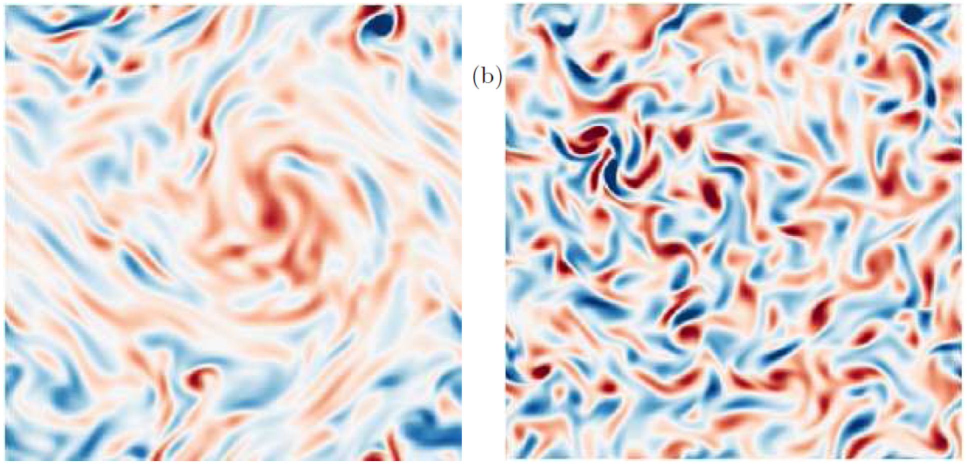
Transition not directly linked to position $\delta_\theta = \delta_v$



Conclusions

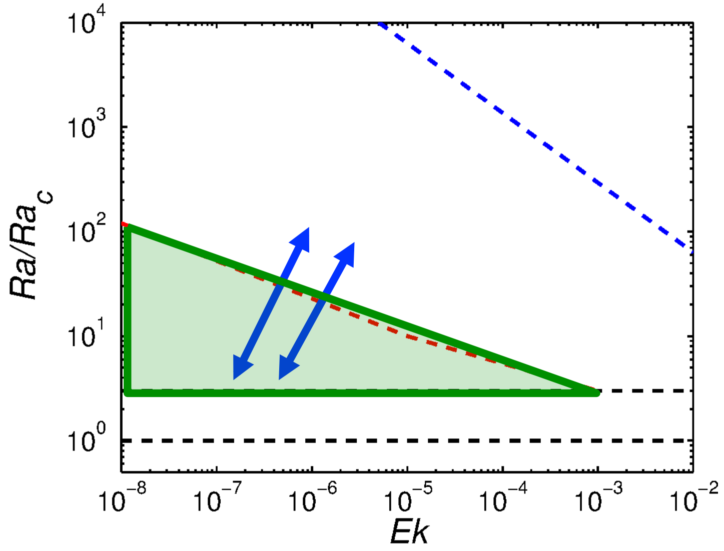


DNS has been used to "populate" an empty geostrophic region of the Phase diagram



The transition to geostrophic convection has been described

Transition found in:
- heat transfer (*Nu*)
- flow phenomenology
- BL thickness



Transition is gradual; found in same *Ek* range for both no-slip and stress-free plates