The turbulent/non-turbulent interface in a plume

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The stars

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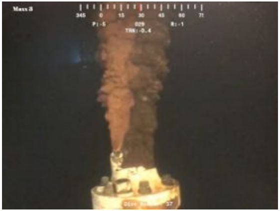
Plumes in the environment



Questions

Part I

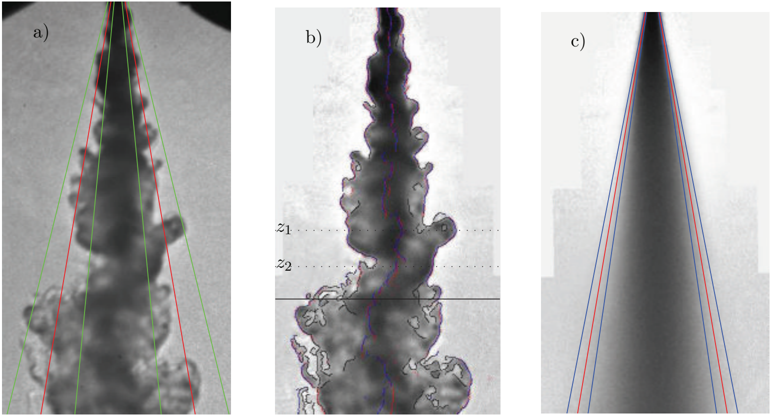
1. Can we determine the buoyancy flux in a plume from visual observations?
2. How is the speed of observable structures related to the speed in the plume?

Part II

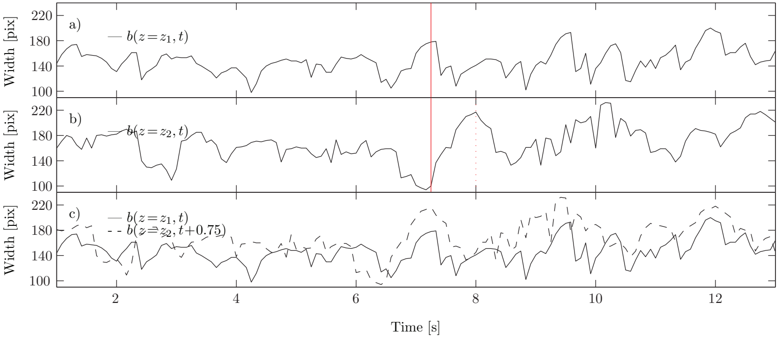
1. What is the flow in the ambient fluid outside the plume?
2. How is this fluid entrained?
3. Why is the average buoyancy width greater than the velocity width?

Laboratory shadowgraph

Shadowgraph images



Time series



a)
— $b(z=z_1,t)$
b)
— $b(z=z_2,t)$
c)
— $b(z=z_1,t)$
- - $b(z=z_2,t+0.75)$
Width [pix]
220
180
140
100
2
4
6
8
10
12
Time [s]

Plume equations

$$\text{Volume flux} \qquad Q = 2\pi \int_0^\infty r\, w(r,z)\, \mathrm{d}r$$

$$\text{Momentum flux} \qquad M = 2\pi \int_0^\infty r\, w(r,z)^2\, \mathrm{d}r$$

$$\text{Buoyancy flux} \qquad F = 2\pi \int_0^\infty r\, w(r,z)\, g'(r,z)\, \mathrm{d}r$$

Plume equations Morton, Taylor & Turner 1956

$$\frac{\mathrm{d}Q}{\mathrm{d}z} = 2\pi^{1/2}\alpha M^{1/2} \propto 2\alpha\, b\, w\,, \quad \frac{\mathrm{d}M}{\mathrm{d}z} = \frac{QF}{M} \propto b^2\, g'\,, \quad \frac{\mathrm{d}F}{\mathrm{d}z} = 0\,,$$

Entrainment constant α

Plume scales

Volume length $L_Q = \dfrac{Q_0}{M_0^{1/2}}$,

Jet length $L_M = \dfrac{M_0^{3/4}}{F_0^{1/2}} = \sqrt{\dfrac{8\alpha_T}{5}} \dfrac{L_Q}{\Gamma_0^{1/2}}$,

Plume parameter $\Gamma = \dfrac{5}{8\alpha_T} \dfrac{Q^2 F}{M^{5/2}}$,

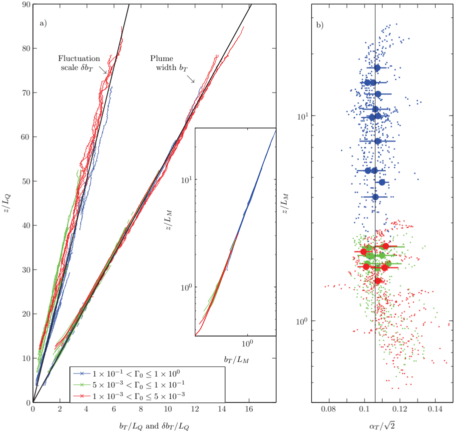
Pure plume $\Gamma_0 \sim 1$

Moderately forced $\Gamma_0 \sim 0.1$

Forced $\Gamma_0 \sim 0.01$

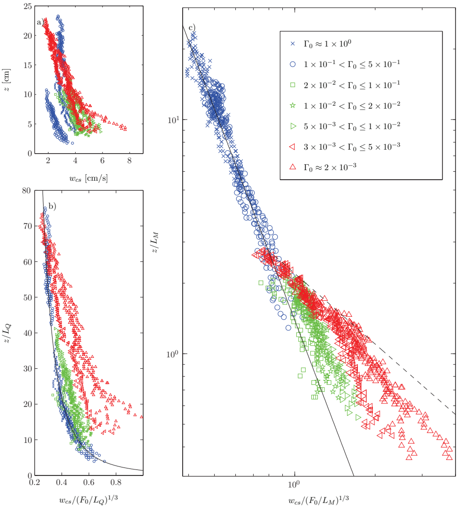
Plume width

Pure plume   Moderately forced   Forced

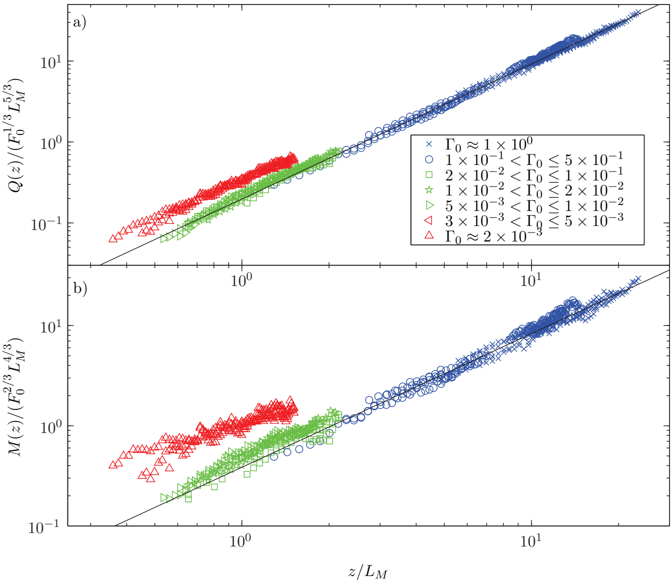


Entrainment constant $\alpha_T = 0.106\sqrt{2}$

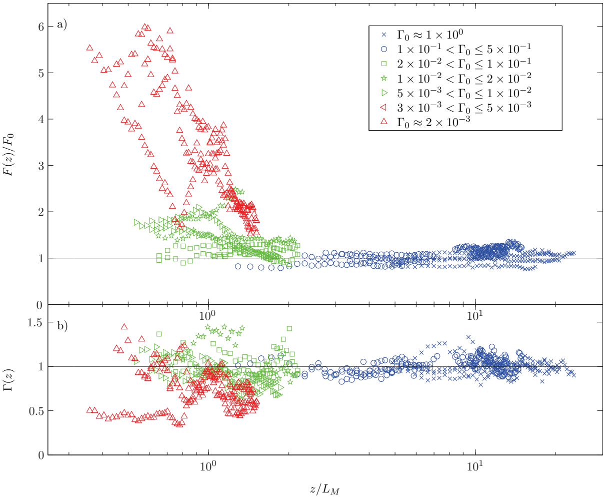
Vertical velocity scaling



Volume flux


a)
b)
$Q(z)/(F_0^{1/3}L_M^{5/3})$
$M(z)/(F_0^{2/3}L_M^{4/3})$
10^1
10^0
10^{-1}
z/L_M
$\Gamma_0 \approx 1 \times 10^0$
$1 \times 10^{-1} < \Gamma_0 \leq 5 \times 10^{-1}$
$2 \times 10^{-2} < \Gamma_0 \leq 1 \times 10^{-1}$
$1 \times 10^{-2} < \Gamma_0 \leq 2 \times 10^{-2}$
$5 \times 10^{-3} < \Gamma_0 \leq 1 \times 10^{-2}$
$3 \times 10^{-3} < \Gamma_0 \leq 5 \times 10^{-3}$
$\Gamma_0 \approx 2 \times 10^{-3}$

Buoyancy flux and plume parameter

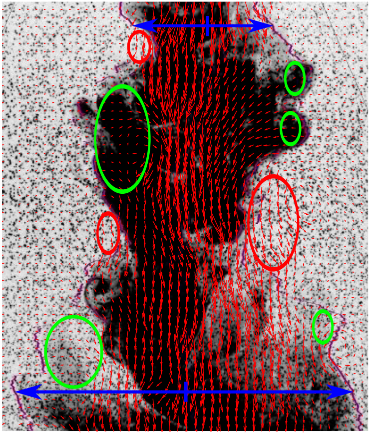


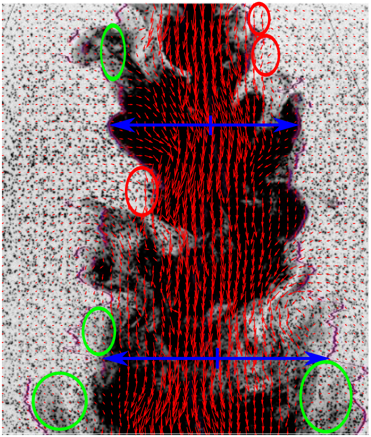
Conclusions I

- Velocity of coherent structures is 30% of centreline velocity (60% of top-hat velocity)
- Motion of visible coherent structures on the edge of a plume can be used to predict the buoyancy (15%) and volume (5%) in a pure plume

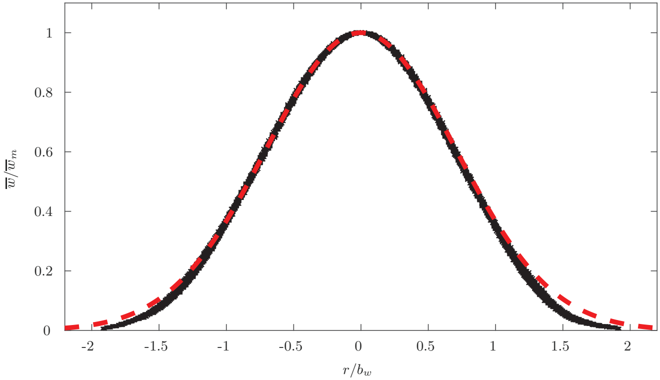
Planar video

Structures

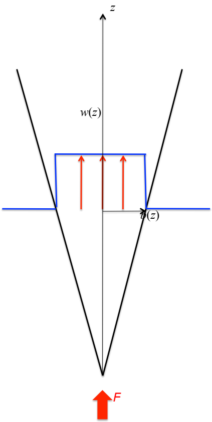




Velocity profile - Gaussian



Top-hat variables



$$\overline{w(r,z)} = \frac{1}{T}\int_0^T w(r,z,t)\,\mathrm{d}t$$

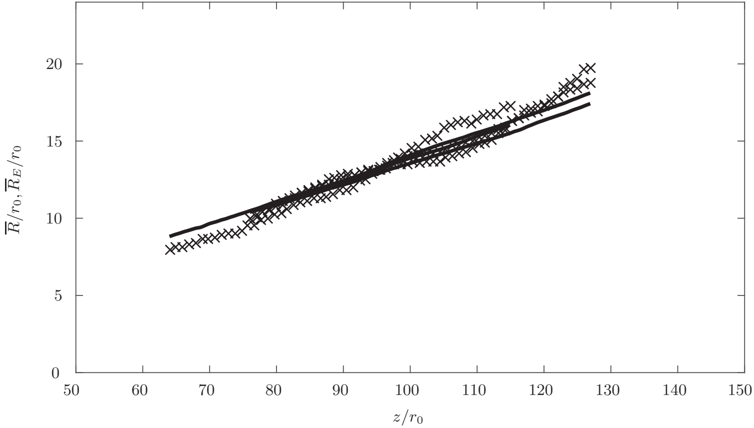
$$\overline{Q} = \int_{-\infty}^{\infty} r\,\overline{w(r,z)}\,\mathrm{d}r$$

$$\overline{M} = \int_{-\infty}^{\infty} r\,\overline{w(r,z)}^2\,\mathrm{d}r$$

$$\overline{R} = \frac{\overline{Q}}{\overline{M}^{1/2}}$$

$$\overline{W} = \frac{\overline{M}}{\overline{Q}}$$

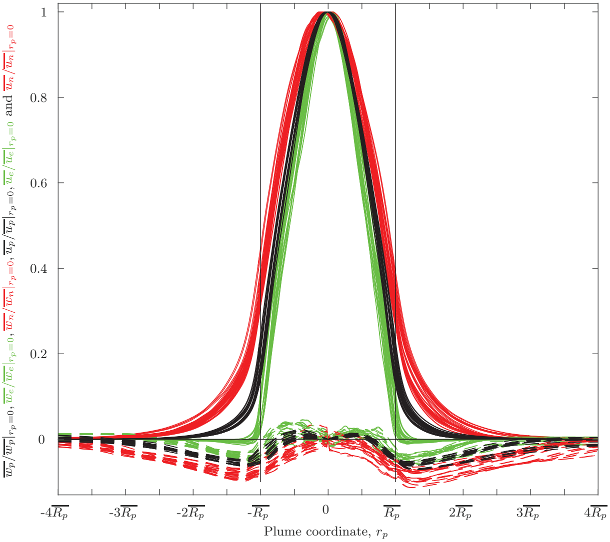
Plume width



Crosses are edge values, lines are flux values

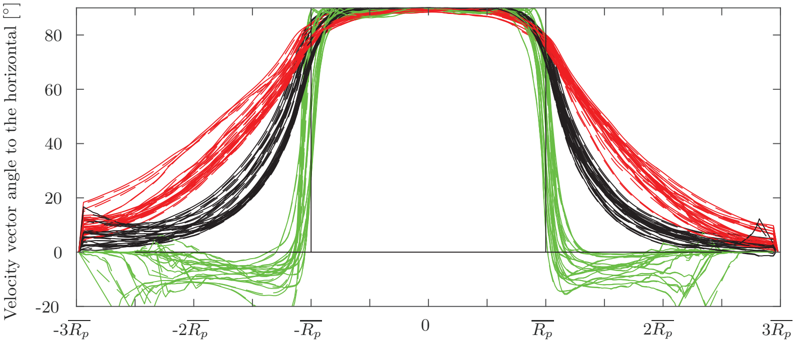
Conditional velocities

Time average — Eddies present — Eddies absent

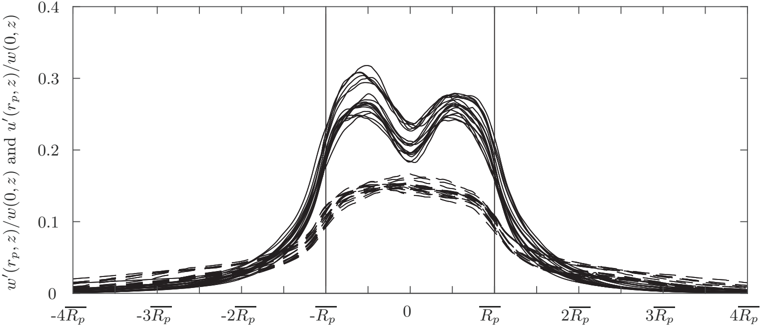


Conditional angles

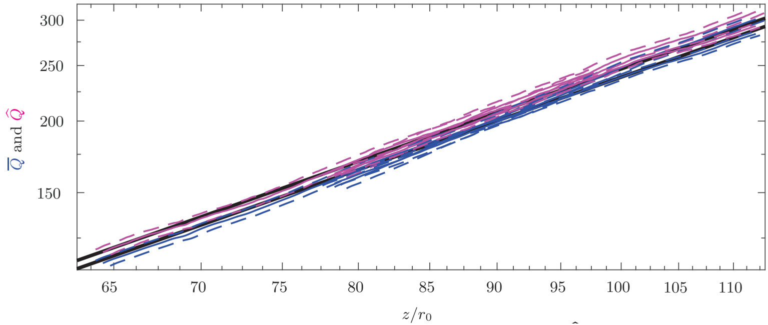
Time average　　Eddies present　　Eddies absent



RMS fluctuations



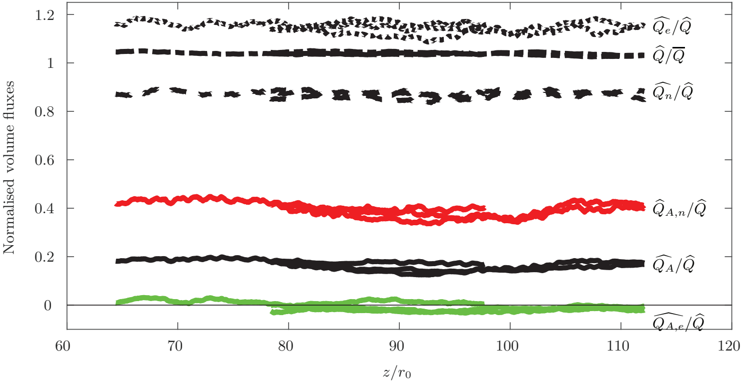
Volume fluxes



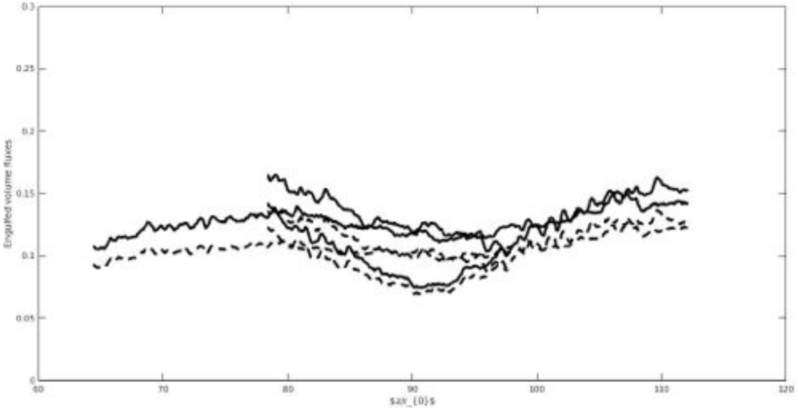
$$\widehat{Q} = \frac{1}{T}\int_0^T \int_{-\infty}^{\infty} r_p\, w(r_p, z, t)\, \mathrm{d}r_p\, \mathrm{d}t$$

Volume fluxes

Time average Eddies present Eddies absent

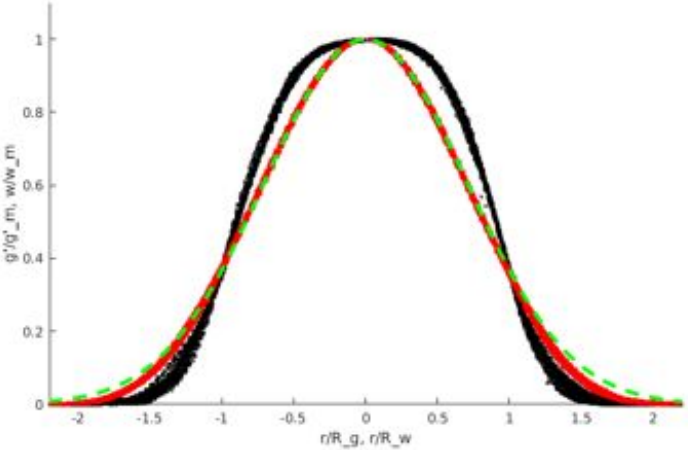


Engulfed fluid

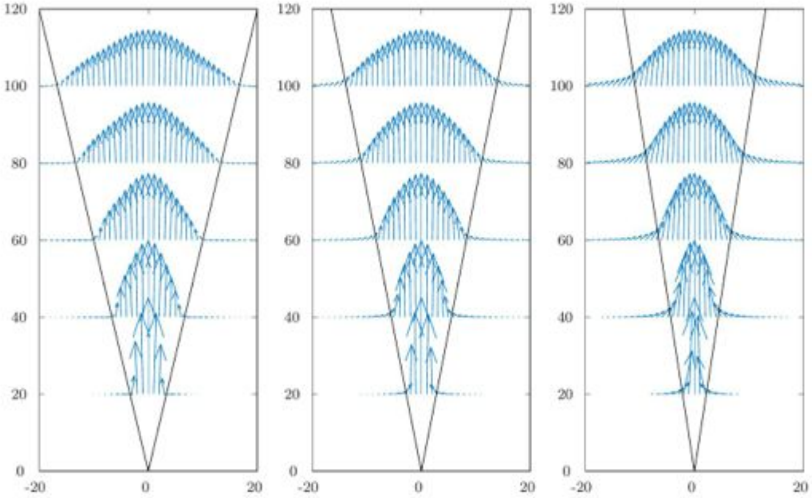


About 10 – 15 % of fluid in plume is engulfed

Average fields



Average flow fields



3D tomography

Conclusions

- Velocity of coherent structures is 30% of centreline velocity (60% of top-hat velocity)
- Motion of visible coherent structures on the edge of a plume can be used to predict the buoyancy (15%) and volume (5%) in a pure plume
- Significant differences in plume behaviour when eddies are present or absent
  1. larger vertical velocities in the external flow
  2. larger velocities into the plume
  3. larger external vertical volume flux
- Larger average width of buoyancy compared with velocity due to presence of engulfed fluid