Surface and subsurface dynamics of two vortex patches

Mikhail A. Sokolovskiy[a,b], Jacques Verron[c] and Xavier Carton[d]
[a] *Water Problems Institute of RAS, Moscow, Russia*
[b] *P.P. Shirshov Institute of Oceanology of RAS; Moscow, Russia*
[c] *Laboratoire de Glaciologie et Géophysique de l'Environnement, CNRS/UJF, Grenoble, France*
[d] *Laboratoire de Physique des Océans, UBO, Brest, France*

Instead of introduction

Emil Hopfinger played a determining role in the formation of a research team which presents now this work. So, in 1990, Emil in absentia introduced Mikhail and Jacques, and this acquaintance has led to their receiving a first INTAS grant. In the framework of this project,

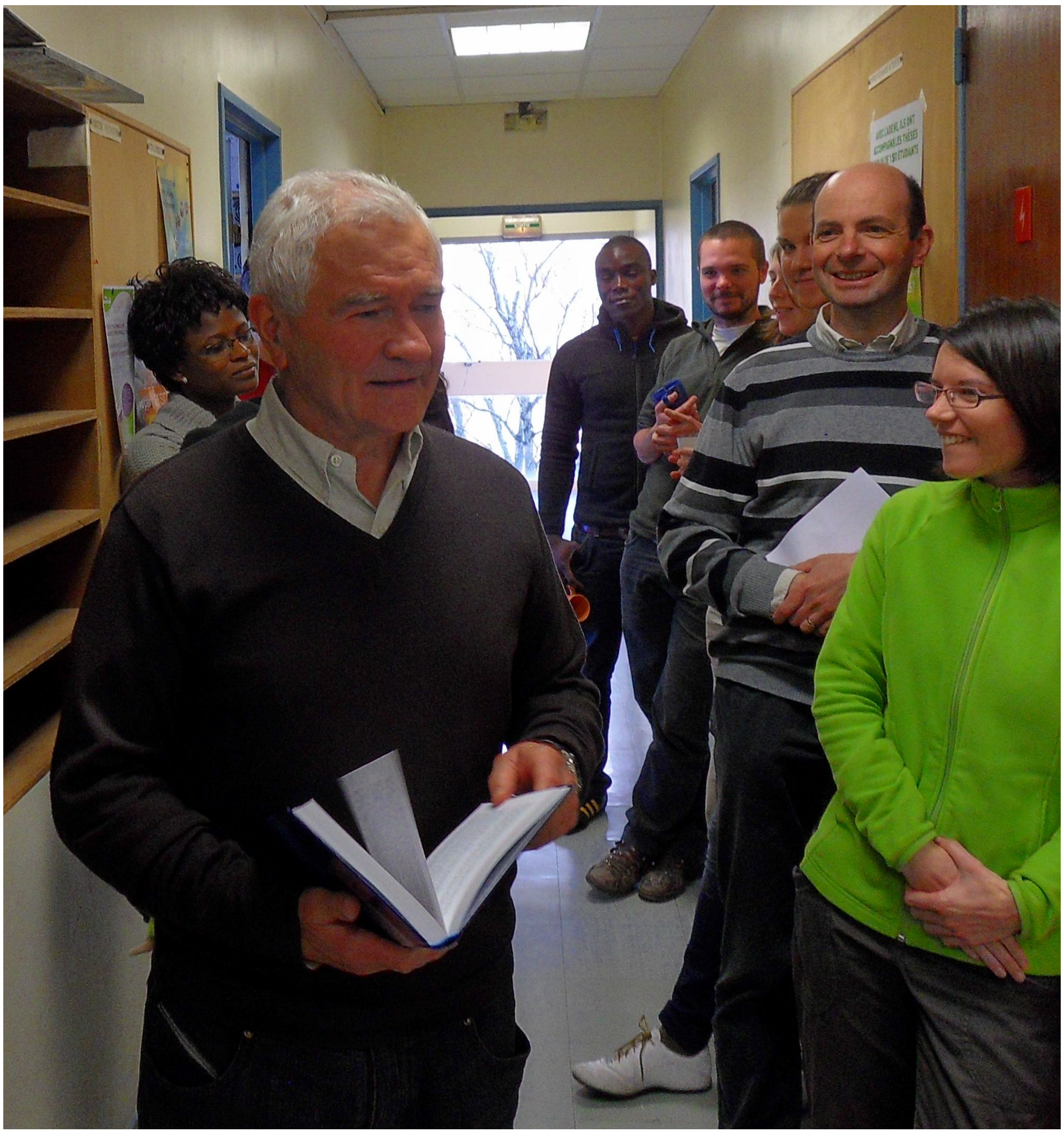

Figure 1: Emil Hopfinger on the presentation of the book [1] (Grenoble, LEGI, March 2012).

at the turn of the century, we have started our cooperation. Later Xavier joined this work. Now, the authors are working together on the next, already the fifth project program. Two monographs, the first in Russian [1] and second (much expanded) in English [2], MS and JV have dedicated to his mentors, Professors Vadim Kozlov and Emil Hopfinger. For the second book, Emil kindly agreed to write an application [3] with the results of experimental laboratory studies of the interaction of two-layer vortices.

The present work is going on with our long-term research on the dynamics of baroclinic geostrophic vortices.

Interaction of two upper layer' vortex patches

First, we would like to note that the problem of interaction of two identical vortex patches is a traditional and well-studied area of fluid dynamics [4–8, e. g.]. This work is aimed to determine the dynamics of two vortices in the upper layer of the two-layer rotating fluid with passive and deep bottom layer, what is typical of oceanic conditions.

In the framework of quasi-geostrophic approximation for a two-layer fluid we study the interaction between two initially circular vortex patches located in the relatively thin upper layer. Contour dynamics method allows obtaining numerically diagrams of various states of vortex structures, depending on the upper layer thickness and the stratification parameter.

The results of these calculations are shown in Figure 2. The dimensionless variable R is plotted on the vertical axis, it corresponds to half the distance between the initially circular vortex patches of unit radius; the parameter γ is plotted on the horizontal axis, it is equal to the ratio of the characteristic horizontal scale to the Rossby radius of deformation. Thus, γ being increased, as the stratification of the two-layer fluid decreases. The calculations were performed for two values of the dimensionless thickness of the upper layer $h = 0.02$ and $h = 0.10$. For an average ocean depth of 4 km, the thickness of the upper layer in the first case is 80 m and 400 m in the second one.

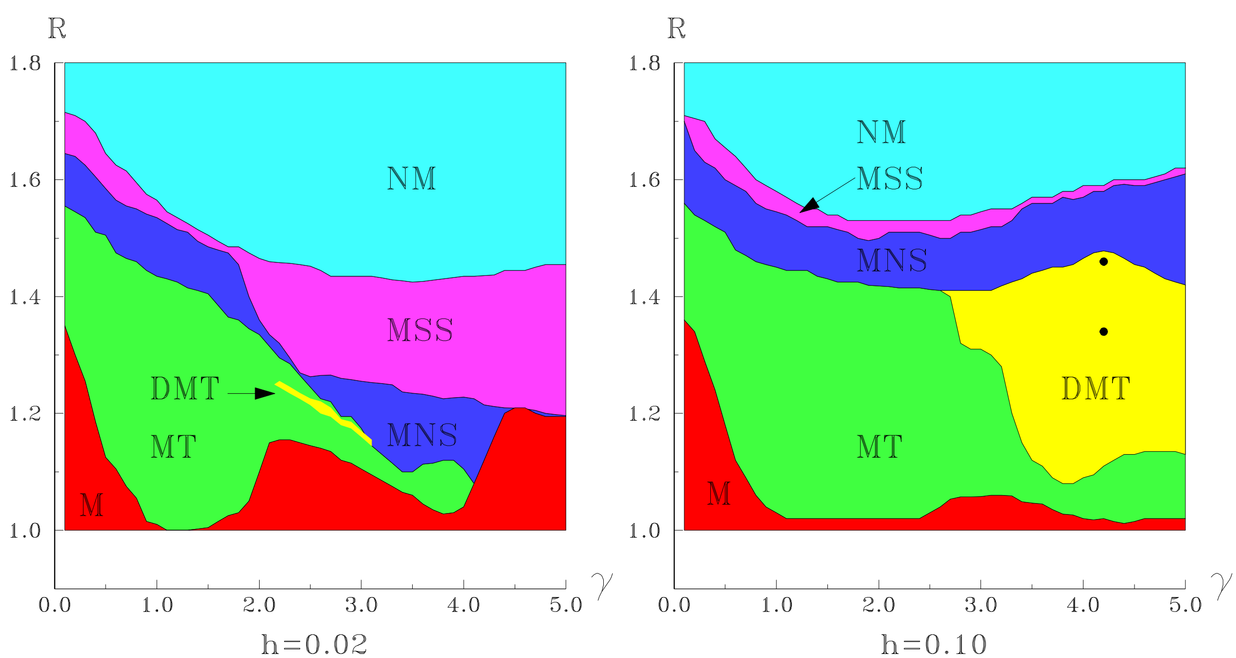

Figure 2: Diagrams in the rectangular domain ($\gamma \in [0.1; 5.0]; R \in [1.0; 1.8]$) of possible states of two circular vortices of upper layer with the specified values of the upper–layer thickness. Black round markers in the right MDT area indicate the parameters used for calculations shown in Figures 3 and 4. The explanations of symbols are given in the text.

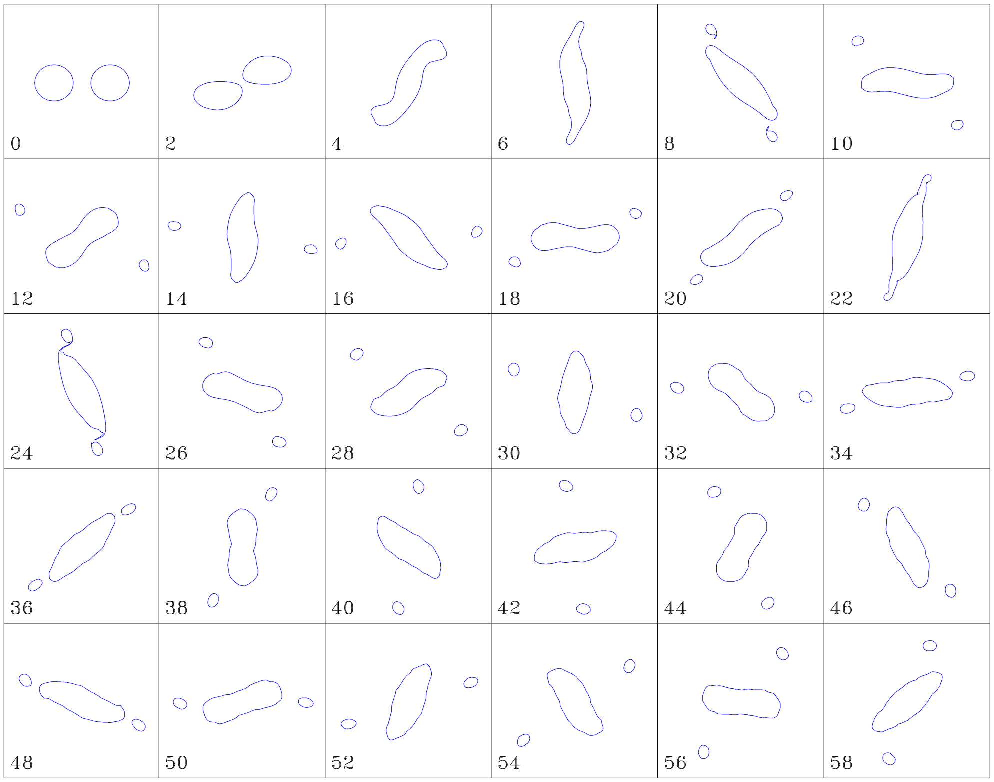

Figure 3: The instantaneous configuration of vortex patches during the interaction of two identical upper-layer circular cyclones with $h = 0.10, \gamma = 4.2$ and $R = 1.46$ (**DMT**–regime) in the given moments of dimensionless time.

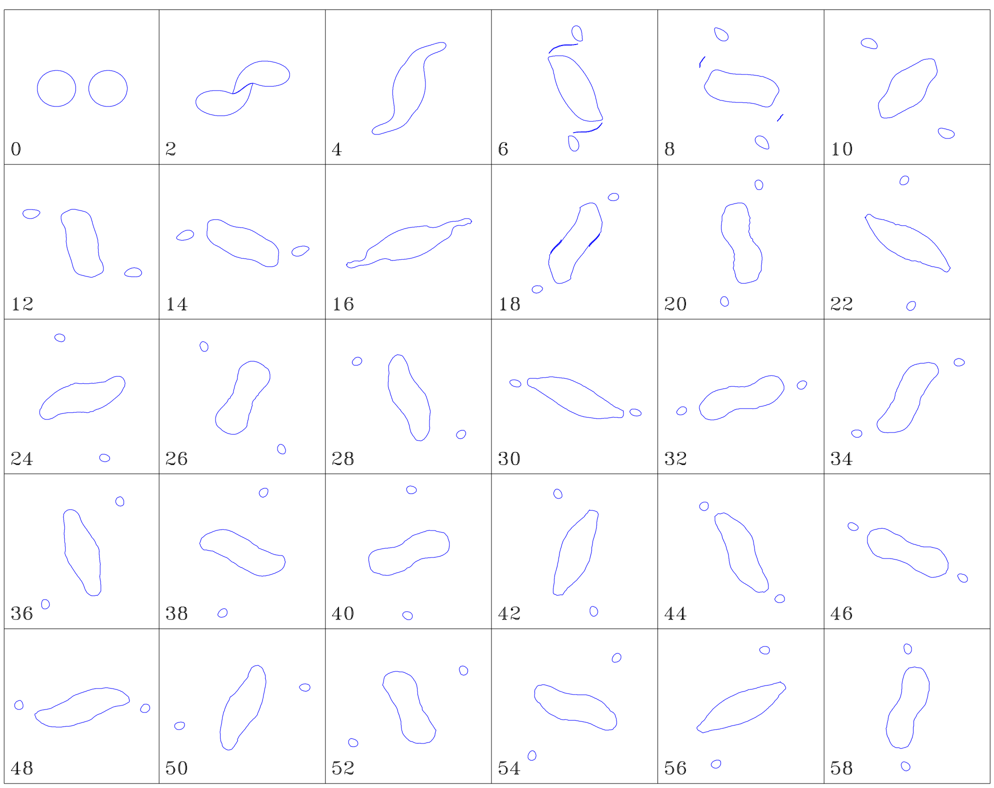

Figure 4: The same as in Figure 3, but with $R = 1.34$.

The following possible states of the vortex states have been found:

- **M** (merger) – a state, when after the merging of the vortex patches and a short intermediate stage of the thin vortex-line formation, there remains one compact vortex of larger scale.
- **MT** (merger/triplet) – merging of vortex patches, and the subsequent formation of a triplet composed of a large central vortex and two peripheral smaller like signed vortices.
- **DMT** (double merger/triplet) – the first stage of the evolution is similar to that of **MT** but later, a new vortex merger occurs leading to the birth of a new triplet. Below, this new class of movements will be discussed in details.
- **MNS** (merger/non-symmetric separation) – a merging, followed by separation into unequal vortex patches. This the process can be single or multiple.
- **MSS** (merger/symmetric separation) – in contrast to the previous case, the division is symmetrical, i. e. after a temporary merging, two identical vortex patches are formed again.
- **NM** (no merger) – vortex patches, pulsating, rotate around a common center of vorticity without merging.

Note, that, according to the Figure 2, the shape of the domains of each of the motion types depends strongly on the degree of stratification, what is typical for two-layer quasi-geostrophic vortex interactions [4].

As an example let us consider the features of the most interesting motion type, such as **DMT** with a fixed value of γ and two different values of R.

Figure 3 illustrates the evolution of two vortex patches with a half distance between their centers $R = 1.46$ (the upper marker in Figuure 2), and Figure 4 – the same with $R = 1.34$ (the lower marker in Figure 2). The difference of scenarios consists in the fact, that in Figure 3, in course of the first triplet formation, the areas of each of the peripheral vortices are smaller than their areas during the subsequent triplet formation (the ratio of the areas is of $1/1.26$), and in Figure 4 the first peripheral areas are larger than the subsequent, and this ratio is $1/0.49$. In both cases, the last motion stage can be considered to be quite stable.

Figure 5 represents the motion of one of the vortex centers movement of one of the peripheral vortices for the example given in Figure 3 (panel 5a) and for the example in Figure 4 (panel 5b). In the upper part of the figure, the red lines represent the trajectories of these centers in the time interval before re-merging, and the green lines are their trajectories after the second division; in the lower part of the figure the same colors correspond to curves $r_c(t)$, where rc is the radius vector of the vorticity center of one of the peripheral vortices. Here, the calculation is made for a somewhat larger time interval than in Figsures 3 and 4. The figure shows that $r_c(t)$ has a quasi-periodic character, and this function reaches the minimum (maximum) values in those times when the satellites are located along the major (minor) axes of the central quasi-elliptic vortex patch.

In the plane of given variables, the domains of existence of triplets are significant both for the shallow and deep surface layers (note that this behavior is similar to that for surface temperature vortices, see Figure 6 in [9]), the **DMT** structures are more specific of the second case.

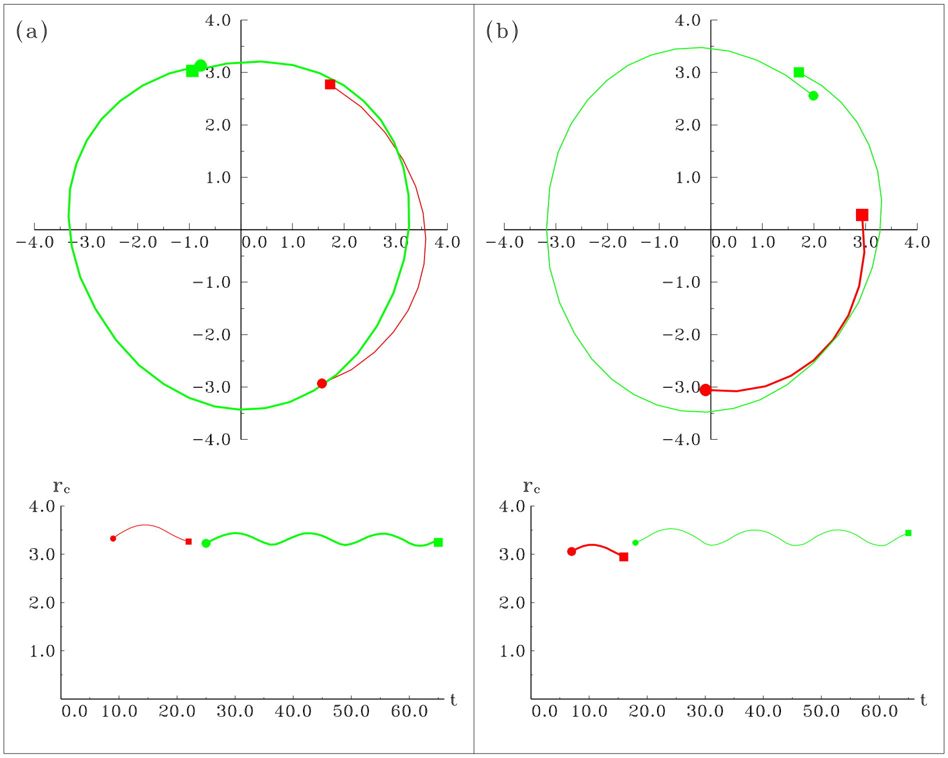

Figure 5: From above: the trajectories of centers of one of the peripheral vortices belonging to the triplet of the upper layer. Bellow: distances r_c as functions of time. Panels (a) and (b) correspond to Figures 3 and 4, respectively. Everywhere, a circle marker correspods to beginning point of the motion, and square marker to its end point.

Conclusion

We briefly reviewed some of the details of interaction of surface and subsurface vortices, but the study of the peculiarities of their dynamics is of great practical interest for the general problem of vortex processes in geophysical hydrodynamics.

Acknowledgements

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