## Surface and subsurface

## dynamics of two vortex patches

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ДИНАМИКА
ВИХХРЕВЫХ СТРУКТУР В ЄЈРАТИФИЦИРОВАННОЙ

ВРАЩАЮЩЕЙСЯ


2


Mkhal A. Sokolovskly
Jacques Verron

## Dynamics of Vortex Structures in a Stratified Rotating Fluid

## Dedicated

to the blessed memory of
Professor Vadim Fedorovich Kozlov,
the Founder of Far Eastern School
of Geophysical Fluid Dynamics,
Vladivostok, Russia
(1933-2005)
and
to Emil Hopfinger,
Directeur de Recherche CNRS,
Ex-Director of Laboratoire des Ecoulements
Géophysiques et Industriels,
Grenoble, France

## BOOK REVIEW

Dynamics of Vortex Structures in a Stratified Rotating Fluid, by
Mikhail A. Sokolovskiy and JacquesVerron, Atmospheric and Oceanographic Sciences Library, Springer International Publishing, Switzerland, 2014, XII + 382 pp., $£ \$ 179.00$ (hardback), ISBN 978-3-319-00788-5, (eBook) ISBN 978-3-319-00789-2.

## Appendix A

## E.J. Hopfinger. Experimental Study of Hetons

This Appendix is a summary of the experiments with hetors, conducted by Griffiths and Hopfinger [309,310), wimed at illustrating heson generation in the laboratory. These experiments were motivated by the wark of Hogg and Stommel [350, 351] aho considered discrete, baroclinic geostrophic vortices that have the capacity
of trasporting heat. Real vortices have a finite core sime and it was of interest of trassporting heat. Real vortices have a finite core sire and it was of interest to compare experiments with the theorstical, idealized point vorter solutions. Numerical solutions of, finite core sime geostrophic vortices have followed and are resented in this book.
Oo lyyer stratified fluid in a tank 100 cm in diameter and 45 cm doep wit otating, avo lyyer stratified fluid in a tank 100 cm in diameter and 45 cm deep with the fluid layers being of equal deph, $H=20 \mathrm{~cm}$. The rocating tank was firs filled with fresh through a tube placed near the sidewall at the bottom of the tank. When the lower, denser layer reaxhod 20 cm the system was left to spin-up to solid body rotation. It needs to be mentioned that the two liyer sytem never reached complete solid body rotation. A weak wimuthal drift, generally $<3 \%$ of the tark rotation, remained that swichod from being axisymmetric to non-axisymmetric and reverse. Griffitse and Linden [312] interpeted this dritt as being the result of a meridional Eddingtossegar solution inslead of salt solution reduced this diffusion, hence the importance of this circulation. Lower diffusion resulted also in a thinner interface thicksess, ahich was close to 2 cm at the time when the heton experiments were starad. The tank rotation was $\Omega-1.0 \mathrm{rad} s^{-1}\left(f-2 s^{-1}\right)$ and the reduced gravity $g^{\prime}-g \Delta \rho / \rho$, wis chasen such that the itternal Rossby nadius of deformation $\lambda^{\prime}-\left(\rho^{\prime} H\right)^{1 / 2 / f} / f$ $-5,10$ and 15 cm . Note that the Roosby rafux $\lambda$ used in this book
$\left.\lambda-\mid g^{\prime} h_{1} h_{2} /\left(h_{1}+h_{2}\right)\right)^{1 / 2} / f$, hence, when $h_{1}-h_{2}-H, \lambda-\lambda^{\prime} / \sqrt{2}$.
$\lambda-\lg / h_{1} h_{2} /\left.\left(h_{1}+h_{3}\right)\right|^{1 / 2} / f$, hesce, when $h_{1}-h_{2}-H, \lambda-\lambda^{\prime} / \sqrt{2}$.
The vortices in this two layer stratified, rotating system were generated by The vortices in this two layer stratified, rotating system were generalod by
ources (anticyclones) and sinks (oyclones) placed at the froe surface or the bottom These wotices have a core radias $k^{\prime}$ of shout 4 cm at which the arimuthal velocity is maximum. The vortex strench is a function of the flow rte. With sinks strong



## Classical problem: interaction of two identical vortices

Annu. Rev. Fluid Mech. 1993. Vol. 25, pp. 241-249

## VORTICES IN ROTATING FLUIDS

## E.J. Hopfinger

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$$
\mathrm{R}_{\mathrm{cr}} \approx 1.6
$$

# Geophysical problem: interaction of two identical vortices of upper layer in two-layer rotating fluid 

J. Fluid Mech. (1987), vol. 178, pp . 73-97

## Coalescing of geostrophic vortices

By R. W. GRIFFITHS
Research School of Earth Sciences, Australian National University, GPO Box 4, Canberra, A.C.T. 2001. Australia

## and E. J. HOPFINGER

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Diagrams in the rectangular domain of plane ( $\gamma, \mathbf{R}$ ) of possible states of two circular vortices of upper layer with the specified values of the upper-layer thickness: M (merger) - a state, when after the merging of the vortex patches and a short intermediate stage of the thin vortex-line formation, there remains one compact vortex of larger scale. MT (merger/triplet) - merging of vortex patches, and the subsequent formation of a triplet composed of a large central vortex and two peripheral smaller like signed vortices. DMT (double merger/triplet) - the first stage of the evolution is similar to that of MT but later, a new vortex merger occurs leading to the birth of a new triplet. MNS (merger/non-symmetric separation) - a merging, followed by separation into unequal vortex patches. MSS (merger/symmetric separation) - the division is symmetrical, i.e. after a temporary merging, two identical vortex patches are formed again. NM (no merger) - vortex patches, pulsating, rotate around a common center of vorticity without merging.

$$
\begin{gathered}
\mathrm{h}_{1}=0.02 \\
\gamma=3.5, \mathrm{R}=1.43 \\
\text { (NM-type) }
\end{gathered}
$$

$$
00
$$

$$
t=24
$$


$\mathrm{t}=48$


$$
\mathrm{t}=72
$$



$$
8
$$

## $\mathrm{t}=120$



$$
0_{0}
$$

$$
0_{0}
$$

$$
\mathrm{t}=192
$$



$$
\infty
$$

$$
0^{\circ}
$$

$$
0
$$

## $\mathrm{t}=288$



## $\mathrm{t}=312$



$$
t=336
$$



## $t=360$



$$
\mathrm{t}=384
$$



## $\mathrm{t}=408$




Diagrams in the rectangular domain of plane ( $\gamma, \mathbf{R}$ ) of possible states of two circular vortices of upper layer with the specified values of the upper-layer thickness: M (merger) - a state, when after the merging of the vortex patches and a short intermediate stage of the thin vortex-line formation, there remains one compact vortex of larger scale. MT (merger/triplet) - merging of vortex patches, and the subsequent formation of a triplet composed of a large central vortex and two peripheral smaller like signed vortices. DMT (double merger/triplet) - the first stage of the evolution is similar to that of MT but later, a new vortex merger occurs leading to the birth of a new triplet. MNS (merger/non-symmetric separation) - a merging, followed by separation into unequal vortex patches. MSS (merger/symmetric separation) - the division is symmetrical, i.e. after a temporary merging, two identical vortex patches are formed again. NM (no merger) - vortex patches, pulsating, rotate around a common center of vorticity without merging.

$$
\begin{gathered}
\mathrm{h}_{1}=0.02 \\
\gamma=0.4, \mathrm{R}=1.65 \\
\text { (MSS-type) }
\end{gathered}
$$

$$
00
$$

$$
0^{0}
$$

$$
8
$$

$$
\mathrm{t}=5
$$



$$
t=6
$$



$$
0^{\circ}
$$

$$
0
$$

## $\mathrm{t}=10$



$$
\mathrm{t}=11
$$



$$
t=12
$$



$$
0^{0}
$$

$$
0
$$

## $t=16$



$$
t=17
$$



## $\mathrm{t}=18$



$$
0^{0}
$$

## $t=20$


$\mathrm{t}=22$


$$
\mathrm{t}=23
$$




Diagrams in the rectangular domain of plane ( $\gamma, \mathbf{R}$ ) of possible states of two circular vortices of upper layer with the specified values of the upper-layer thickness: M (merger) - a state, when after the merging of the vortex patches and a short intermediate stage of the thin vortex-line formation, there remains one compact vortex of larger scale. MT (merger/triplet) - merging of vortex patches, and the subsequent formation of a triplet composed of a large central vortex and two peripheral smaller like signed vortices. DMT (double merger/triplet) - the first stage of the evolution is similar to that of MT but later, a new vortex merger occurs leading to the birth of a new triplet. MNS (merger/non-symmetric separation) - a merging, followed by separation into unequal vortex patches. MSS (merger/symmetric separation) - the division is symmetrical, i.e. after a temporary merging, two identical vortex patches are formed again. NM (no merger) - vortex patches, pulsating, rotate around a common center of vorticity without merging.

$$
\begin{gathered}
h_{1}=0.02 \\
\gamma=0.4, R=1.60 \\
\text { (MNS-type) }
\end{gathered}
$$

$00$

$$
\infty
$$

g

## $\mathrm{t}=3$



$$
t=4
$$



$$
t=5
$$


(0)
©
${ }^{0}$


## $\mathrm{t}=10$

## $\mathrm{t}=11$



## $\mathrm{t}=12$



$$
t=13
$$



$$
t=14
$$



$$
t=15
$$



## $\mathrm{t}=16$



$$
t=17
$$



## $\mathrm{t}=18$



## $\mathrm{t}=19$



## $t=20$



## $\mathrm{t}=21$



## $\mathrm{t}=22$



## $\mathrm{t}=23$




Diagrams in the rectangular domain of plane ( $\gamma, \mathbf{R}$ ) of possible states of two circular vortices of upper layer with the specified values of the upper-layer thickness: M (merger) - a state, when after the merging of the vortex patches and a short intermediate stage of the thin vortex-line formation, there remains one compact vortex of larger scale. MT (merger/triplet) - merging of vortex patches, and the subsequent formation of a triplet composed of a large central vortex and two peripheral smaller like signed vortices. DMT (double merger/triplet) - the first stage of the evolution is similar to that of MT but later, a new vortex merger occurs leading to the birth of a new triplet. MNS (merger/non-symmetric separation) - a merging, followed by separation into unequal vortex patches. MSS (merger/symmetric separation) - the division is symmetrical, i.e. after a temporary merging, two identical vortex patches are formed again. NM (no merger) - vortex patches, pulsating, rotate around a common center of vorticity without merging.

$$
\begin{gathered}
h_{1}=0.02, R=1.12, \gamma=2.5 \\
\text { (M-type) }
\end{gathered}
$$

$$
\infty
$$

## $\mathrm{t}=1$



## $\mathrm{t}=2$



## $\mathrm{t}=3$



$$
t=4
$$



$$
t=5
$$



$$
\theta^{\prime}
$$

$$
0
$$

$\Omega$

B
B

$$
t=15
$$



## $\mathrm{t}=16$



## $\mathrm{t}=\mathbf{1 7}$



## $\mathrm{t}=18$



$$
t=19
$$



## $t=20$



## $\mathrm{t}=21$



## $\mathrm{t}=22$



## $\mathrm{t}=23$



## $\mathrm{t}=24$



$$
\infty
$$

## $t=26$



## $\mathrm{t}=27$


$\mathrm{t}=\mathbf{2 8}$


## $\mathrm{t}=29$



$$
0
$$

$$
0
$$

## $\mathrm{t}=32$



$$
8
$$

$\Omega$
$8$

$$
\mathfrak{B}
$$

## $\mathrm{t}=39$




Diagrams in the rectangular domain of plane ( $\gamma, \mathbf{R}$ ) of possible states of two circular vortices of upper layer with the specified values of the upper-layer thickness: M (merger) - a state, when after the merging of the vortex patches and a short intermediate stage of the thin vortex-line formation, there remains one compact vortex of larger scale. MT (merger/triplet) - merging of vortex patches, and the subsequent formation of a triplet composed of a large central vortex and two peripheral smaller like signed vortices. DMT (double merger/triplet) - the first stage of the evolution is similar to that of MT but later, a new vortex merger occurs leading to the birth of a new triplet. MNS (merger/non-symmetric separation) - a merging, followed by separation into unequal vortex patches. MSS (merger/symmetric separation) - the division is symmetrical, i.e. after a temporary merging, two identical vortex patches are formed again. NM (no merger) - vortex patches, pulsating, rotate around a common center of vorticity without merging.

$$
\begin{gathered}
\mathrm{h}_{1}=0.02, \mathrm{R}=1.22: \\
\text { (a) } \gamma=2.4, \text { (b) } \gamma=2.5 \text {, (c) } \gamma=2.6 \\
\text { (MT/DMT/MT-types) }
\end{gathered}
$$

$\mathrm{t}=\mathbf{0}$


$$
\gamma=2.4
$$


$\gamma=2.5$

$\gamma=2.6$
$\gamma=2.4,2.5,2.6$


$$
\gamma=2.4
$$


$\gamma=2.6$
$\gamma=2.4,2.5,2.6$

## $t=2$



## $\mathrm{t}=3$



$\gamma=2.6$
$\gamma=2.4,2.5,2.6$






0

$\gamma=2.6$
$\gamma=2.4,2.5,2.6$
$t=10$

$\mathrm{t}=11$

$\mathrm{t}=12$

$\gamma=2.4$
$\gamma=2.6$
$\gamma=2.4,2.5,2.6$
$\mathrm{t}=13$

$\gamma=2.4$


0

$\gamma=2.5$
$\gamma=2.6$
$\gamma=2.4,2.5,2.6$
$\mathrm{t}=14$

$\mathrm{t}=15$

$t=16$


## $\mathrm{t}=\mathbf{1 7}$


$\mathrm{t}=18$


## $\mathrm{t}=19$



## $t=20$



## $\mathrm{t}=21$



## $t=22$



## $\mathrm{t}=23$


$\mathrm{t}=24$

$\mathrm{t}=25$

$\mathrm{t}=26$


## $\mathrm{t}=27$


$\mathrm{t}=\mathbf{2 8}$


## $t=29$



## $\mathrm{t}=\mathbf{3 0}$



## $\mathrm{t}=\mathbf{3 1}$



## $\mathrm{t}=32$



## $\mathrm{t}=33$


$\mathrm{t}=34$

0
電


$\gamma=2.6$
$\gamma=2.4,2.5,2.6$
$t=35$
$0<0$

OO
$\gamma=2.4$



0
$\gamma=2.6$
$\gamma=2.4,2.5,2.6$
$\mathrm{t}=36$

$\gamma=2.4$


0

$\gamma=2.6$
$\gamma=2.4,2.5,2.6$

## $\mathrm{t}=\mathbf{3 7}$


$\mathrm{t}=38$
$0 \bigcirc 0$

$\bigcirc$
$\gamma=2.4$

0


$$
\gamma=2.6
$$

$\gamma=2.4,2.5,2.6$

## $\mathrm{t}=39$



$\gamma=2.6$
$\gamma=2.4,2.5,2.6$
$\mathrm{t}=\mathbf{4 1}$

$\underbrace{\gamma=2.4}_{\gamma=2.5}$



0
$\gamma=2.6$
$\gamma=2.4,2.5,2.6$
$\mathrm{t}=42$

$\gamma=2.4$

0


0
$\gamma=2.5$

$\gamma=2.6$
$\gamma=2.4,2.5,2.6$
$\mathrm{t}=43$


0



$\gamma=2.6$
$\gamma=2.4,2.5,2.6$


Diagrams in the rectangular domain of plane ( $\gamma, R$ ) of possible states of two circular vortices of upper layer with the specified values of the upper-layer thickness: M (merger) - a state, when after the merging of the vortex patches and a short intermediate stage of the thin vortex-line formation, there remains one compact vortex of larger scale. MT (merger/triplet) - merging of vortex patches, and the subsequent formation of a triplet composed of a large central vortex and two peripheral smaller like signed vortices. DMT (double merger/triplet) - the first stage of the evolution is similar to that of MT but later, a new vortex merger occurs leading to the birth of a new triplet. MNS (merger/non-symmetric separation) - a merging, followed by separation into unequal vortex patches. MSS (merger/symmetric separation) - the division is symmetrical, i.e. after a temporary merging, two identical vortex patches are formed again. NM (no merger) - vortex patches, pulsating, rotate around a common center of vorticity without merging.

$$
\begin{gathered}
\gamma=4.2, h_{1}=0.1: \\
\text { (a) } \mathrm{R}=1.46 \text {, (b) } \mathrm{R}=1.34
\end{gathered}
$$

## Case (a): <br> evolution of vortex patches $S_{2}=0.257, S_{3}=0.325$

## $\mathrm{t}=2$



$$
t=4
$$



## $\mathrm{t}=12$



## $\mathrm{t}=14$



## $\mathrm{t}=16$



## $\mathrm{t}=18$



0

## $\mathrm{t}=20$

○

$\bigcirc$

## $t=22$

$\bigcirc$

$\bigcirc$

## $\mathrm{t}=24$



$$
t=26
$$



$$
\mathrm{t}=28
$$



## $\mathrm{t}=30$



## $\mathrm{t}=32$



## $\mathrm{t}=34$


$t=36$

$\mathrm{t}=38$

$\mathrm{t}=42$


$$
t=44
$$



$$
t=46
$$


$\mathrm{t}=48$


$$
t=50
$$

O


## $t=52$



$$
t=54
$$



$$
t=56
$$



$$
t=58
$$



$$
t=62
$$



$$
t=64
$$



## $t=66$



$$
t=68
$$

## $\mathrm{t}=70$



$$
\mathrm{t}=72
$$



$$
t=76
$$



## $\mathrm{t}=78$



## $\mathrm{t}=80$



## $\mathrm{t}=82$



$$
t=84
$$

$$
t=86
$$

O


$$
t=88
$$



$$
t=94
$$



## $t=96$



$$
t=98
$$



$$
t=100
$$

## $\mathrm{t}=102$



$$
t=106
$$



## $\mathrm{t}=108$



## $\mathrm{t}=110$



## $\mathrm{t}=112$



## $\mathrm{t}=114$



$$
t=116
$$

## $\mathrm{t}=118$



## $\mathrm{t}=120$



## $t=122$



## $\mathrm{t}=124$


$\bigcirc$

## $t=126$



## $\mathrm{t}=128$



## Case (b): evolution of vortex patches $\mathrm{S}_{2}=0.346, \mathrm{~S}_{3}=0.172$

## $\mathrm{t}=2$



## $\mathrm{t}=4$



## $\mathrm{t}=10$



## $\mathrm{t}=12$



$$
t=14
$$



## $\mathrm{t}=16$



## $\mathrm{t}=18$



## $t=20$

$\bigcirc$


0

## $\mathrm{t}=22$

0


0

## $\mathrm{t}=24$



## $t=26$


$\mathrm{t}=28$


## $\mathrm{t}=30$


$\mathrm{t}=32$


## $t=36$



## $\mathrm{t}=38$



$$
t=40
$$



## $\mathrm{t}=42$



## $\mathrm{t}=44$



## $t=46$



$$
t=48
$$



$$
t=50
$$



## $t=52$



O

$$
t=54
$$

0


0

$$
t=56
$$

O


## $\mathrm{t}=58$



## $t=60$



## $\mathrm{t}=62$



$$
t=64
$$



$$
t=66
$$



$$
t=68
$$



## $\mathrm{t}=72$



## $\mathrm{t}=74$



## $\mathrm{t}=76$


$\mathrm{t}=78$

$t=80$


$$
t=82
$$



$$
t=88
$$



## $\mathrm{t}=90$

$\bigcirc$

$\mathrm{t}=92$


0
$\mathrm{t}=94$

O

$\bigcirc$

## $t=96$



$$
\mathrm{t}=98
$$



$$
t=100
$$



$$
\mathrm{t}=102
$$



## $\mathrm{t}=104$



$$
t=106
$$

0


## $\mathrm{t}=108$



## $\mathrm{t}=110$



## $\mathrm{t}=112$



## $\mathrm{t}=114$



## $\mathrm{t}=116$



$$
\mathrm{t}=118
$$

0


## $\mathrm{t}=120$

$\bigcirc$


## $t=122$



## $\mathrm{t}=124$

## o <br>  <br> $\bigcirc$

## $\mathrm{t}=126$

0


O

## $\mathrm{t}=128$



## $\mathrm{t}=130$



From above: the trajectories of centers of one of the peripheral vortices belonging to the triplet of the upper layer. Bellow: distances $r_{c}$ as functions of time. Panels (a) and (b) correspond to cases 1 and 2, respectively. The red lines represent the trajectories of these centers in the time interval before re-merging, and the green lines are their trajectories after the second separation. Everywhere, a circle marker corresponds to beginning point of the motion, and square marker to its end point.


## Summary

1. The boundary between the domains of stability and coalescing of two identical top-layer circular vortices is a non-monotonous function of the stratification parameter.
2. Reduction the upper layer thickness contributes to the expansion of the stability domain (there is allowed an abnormally close relative position of the vortex-patch centers when they behave as point vortices).
3. The coalescing area has a complex structure. The general trend is as follows: when the distance between the vortex centers being decreasing, there takes place the transition between states merger/symmetric separation --merger/non-symmetric separation -- merger/triplet - merger takes place.
4. The process of the vortex-patch merging creates an intensive mixing of water masses located inside the vortices.
5. A sub-domain of parameters is found within the area merger/triplet which stimulate a new movement subtype: a double merger/triplet, when the triplet of the first evolution phase is unstable, and an intermediate phase of vortexpatch triplet merging takes place; then there follows the secondary separation into three parts (double merger/triplet). The latter structure represents a quasi-periodic steady state.

## PS

The question to pioneers* of this problem:

> Is possible to obtain experimentally the stable triplet vortex structures as a result of interaction of two identical circular vortex columns of upper layer in two-layer rotating fluid?

## Thank you very much!



