Isotropy recovery in rotating-stratified turbulence: the role of Ozmidov and Hopfinger scales.

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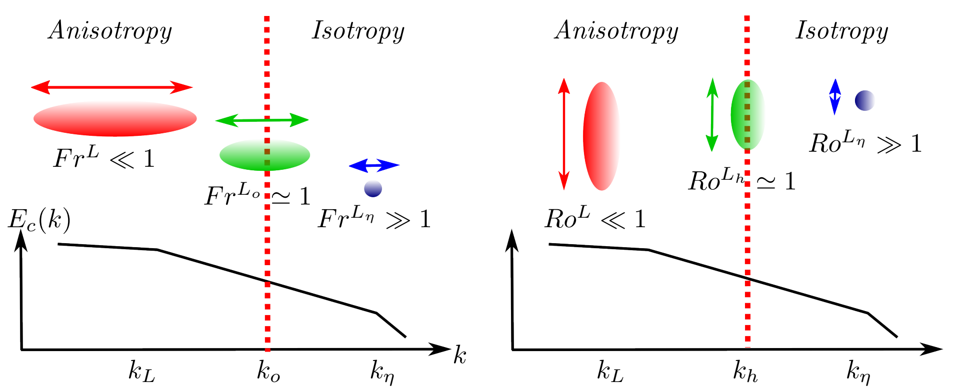
Background

Transition to isotropy at certain scale :

Stratified turbulence
Ozmidov's scale L_o [1] (1965)
Eddy of size l : $Fr^l = \frac{Inertia}{Gravity} = \frac{v}{N.l}$
Transition $Fr^{L_o} \simeq 1 \Rightarrow L_o = \sqrt{\frac{\epsilon}{N^3}}$
In fourier space : $k_o = \sqrt{\frac{N^3}{\epsilon}}$

Rotating turbulence
Hopfinger's scale L_h [2, 3] (1985)
Eddy of size l : $Ro^l = \frac{Inertia}{Coriolis} = \frac{v}{2.\Omega.l}$
Transition $Ro^{L_h} \simeq 1 \Rightarrow L_h = \sqrt{\frac{\epsilon}{(2\Omega)^3}}$
In fourier space : $k_h = \sqrt{\frac{(2\Omega)^3}{\epsilon}}$



Goal of this study :

- Proof directly the Ozmidov and Hopfinger phenomenology.
- Characterize the structure and the non linear transfer in terms of scale.

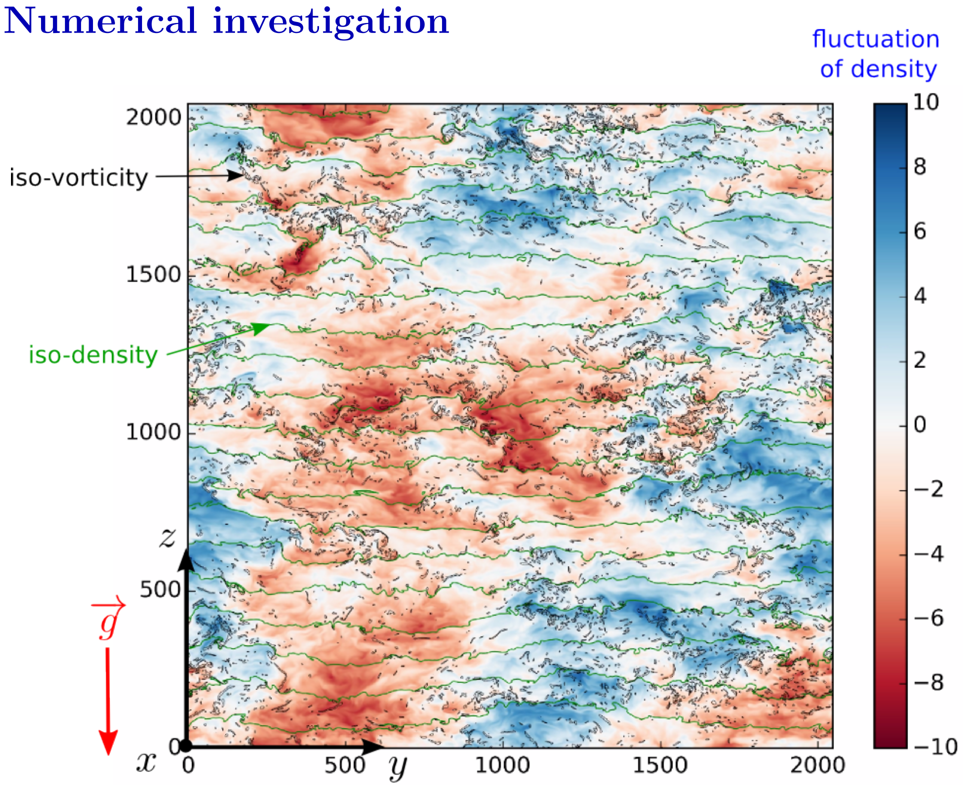
Interest :

- Better estimation of the dissipation at small scale and of the turbulent mixing
- Useful for turbulent models (LES ...)
- The Osmidov scale is widely used for analyzing data (oceanography ...)

Difficulty :

- Large separation of scale (from large to small scale)
- Characterize the anisotropy / isotropy in terms of scale and direction.

Numerical investigation



Exemple of numerical simulation : a vertical cut of pure stratified turbulence

- **Navier-Stokes & Boussinesq** for perturbation around state $\rho_0(z)$:

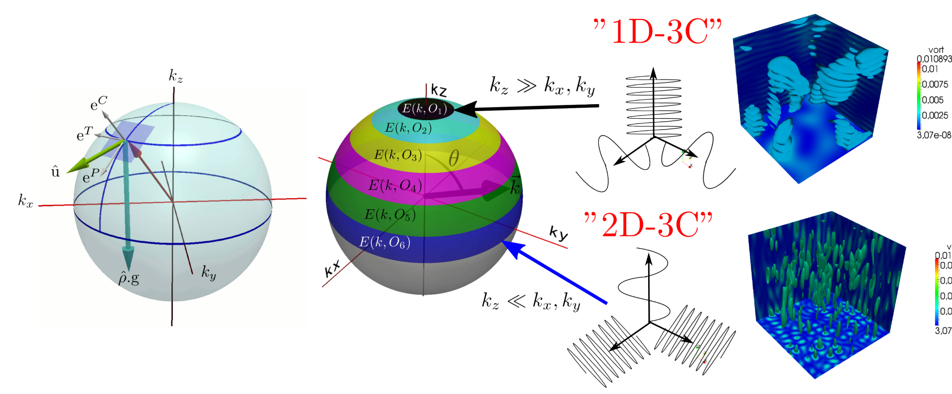
$$\left(\frac{\partial}{\partial t} - \nu\Delta\right) \boldsymbol{u}(\boldsymbol{x},t) = -\boldsymbol{\nabla} p + \boldsymbol{\omega} \times \boldsymbol{u} - (2\boldsymbol{\Omega} + \boldsymbol{\omega}) \times \boldsymbol{u}$$
$$\left(\frac{\partial}{\partial t} - \nu_\rho\Delta + \boldsymbol{u}(\boldsymbol{x},t).\nabla\right) \rho(\boldsymbol{x},t) = N^2.u_z$$

with $\boldsymbol{\omega} = \nabla \times \boldsymbol{u}$, $\boldsymbol{u}$ perturbation of velocity and ρ fluctuation of density and $N = \frac{d\rho_0(z)}{dz}$ the Brunt-Väisälä frequency, Ω the rate of rotation.

- **High resolution** (2048^3 points and $k_{max}.\eta \sim 3$) with pseudospectral method.
- **Turbulence freely decreasing** : analyse after initial eddy turnover time.

A scale by scale analysis with anisotropy

From physical $(\boldsymbol{u}, \rho)$ to Fourier space $(\hat{\boldsymbol{u}}, \hat{\rho})$



- **Craya and velocity's components** ($div\ \boldsymbol{u} = 0 \Rightarrow i.\boldsymbol{k} \cdot \hat{\boldsymbol{u}} = 0 \Rightarrow \boldsymbol{k} \perp \hat{\boldsymbol{u}}$) :

$$\hat{\boldsymbol{u}}(\boldsymbol{k},t) = \underbrace{u^{(T)}(\boldsymbol{k},t)\boldsymbol{e}^{(T)}(\boldsymbol{k})}_{Toroidal} + \underbrace{u^{(P)}(\boldsymbol{k},t)\boldsymbol{e}^{(P)}(\boldsymbol{k})}_{Poloidal}$$

$u^{(T)} =$ pure horizontal velocity, $u^{(P)} =$ vertical/horizontal velocity, linear limit of **stratified turbulence** [4] $u^{(P)} =$ **internal gravity wave** and $u^{(T)} =$ **vortex**

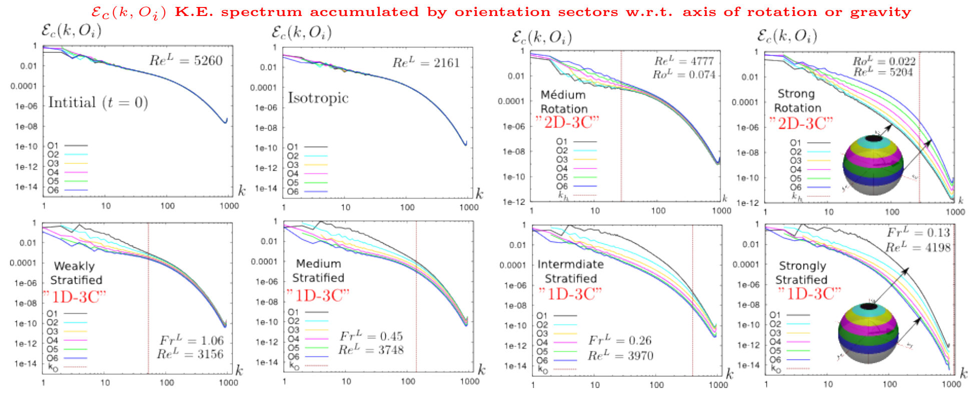
- **Average on each ring** O_i for potential $\mathcal{E}_{pot}$ and kinetic $\mathcal{E}_c$ energy :

$$\mathcal{E}_c(k,O_i) = \underbrace{\frac{1}{m_k^i}.\sum_{\boldsymbol{k}\in O_i} |u^{(T)}(\boldsymbol{k},t)|^2}_{\mathcal{E}^T(k,O_i)} + \underbrace{\frac{1}{m_k^i}.\sum_{\boldsymbol{k}\in O_i} |u^{(P)}(\boldsymbol{k},t)|^2}_{\mathcal{E}^P(k,O_i)}$$

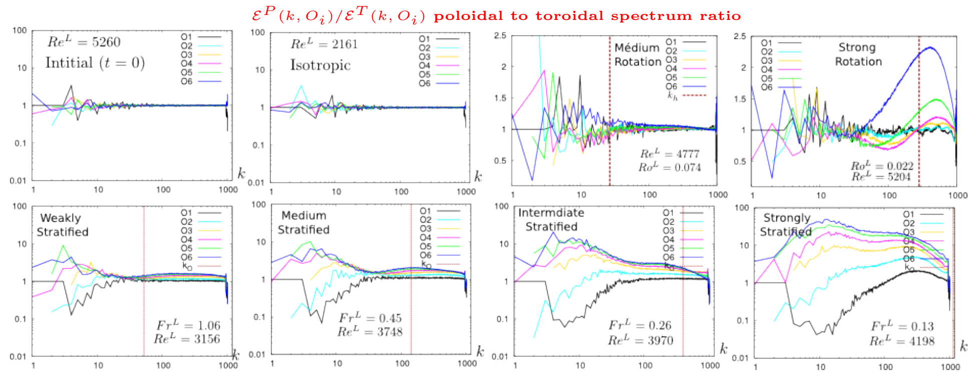
$\Rightarrow$ anisotopy of distribution of energy (from "1D-3C" to "2D-3C" state)

Results

$\mathcal{E}_c(k,O_i)$ **K.E. spectrum accumulated by orientation sectors w.r.t. axis of rotation or gravity**



$\mathcal{E}^P(k,O_i)/\mathcal{E}^T(k,O_i)$ **poloidal to toroidal spectrum ratio**



Conclusion

- After Ozmidov's scale and Hopfinger's scale [5], the isotropy seems to be restored ($\mathcal{E}^T \sim \mathcal{E}^P$)
- Stratfied turbulence : the "1D-3C" state is promoted with an important role of vortex mode in dynamics at **large scale** ($\mathcal{E}^T \gg \mathcal{E}^P \sim \mathcal{E}^{pot}$) whereas at **small scale**, the energy of "1D-3C" state tends to an equilibrium between vortex/gravity wave ($\mathcal{E}^T \sim \mathcal{E}^P \sim \mathcal{E}^{pot}$).
- Rotating turbulence : **near Hopfinger's scale** k_h, the "2D-3C" state is promoted with "jetal" eddies (high vertical velocity i.e $\mathcal{E}^P > \mathcal{E}^T$)

Bibliography

[1] R.V. Ozmidov, Izvestia Acad. Sci. USSR, Atmos. and Ocean Phys., 8 (1965)
[2] Mory, M. & Hopfinger, Macroscopic Modelling of Turbulent Flows, (1984)
[3] O. Zeman, Phys. Fluids 6, 3221 (1994).
[4] Riley, J. J., Metcalfe, R. W., & Weissman, M. A. , AIP Publishing (1981).
[5] Delache, A., Cambon, C., & Godeferd, F. , Phys. Fluids, 26(2), (2014).