Energetic budget of Direct Numerical Simulations in a turbulent stratified flow.

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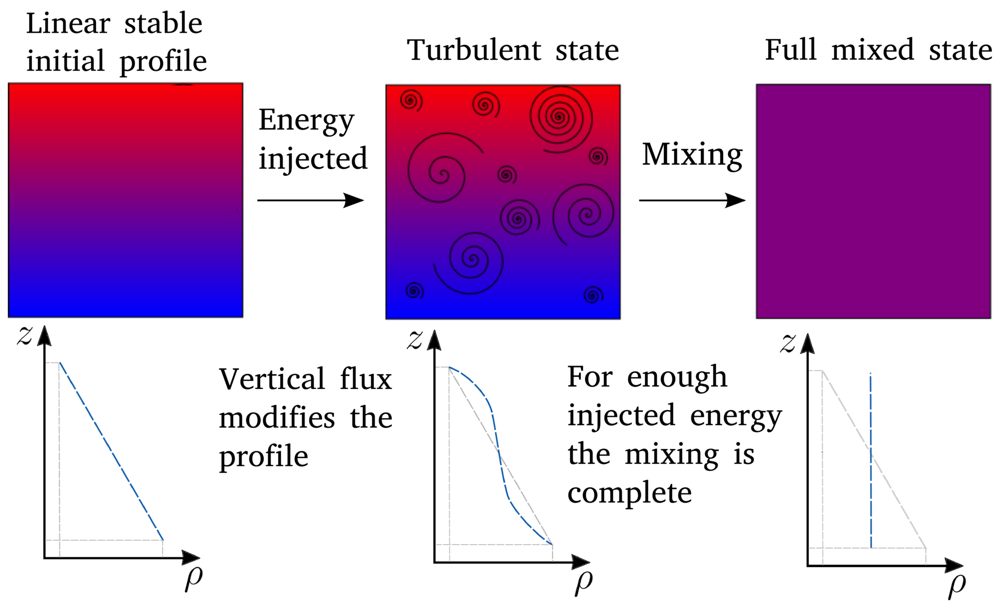
Stably stratified environment

The vertical profile under a turbulent mixing process:



How does the vertical density profile evolves under turbulent mixing?

Global balance: advection and diffusion

For a stationary density profile the local vertical advection and diffusion must be globally conserved

$$w\partial_z\rho = \partial_z(K\partial_z\rho)$$

w : vertical upwelling
K : turbulent diffusivity
ρ : density

Osborn (1980)

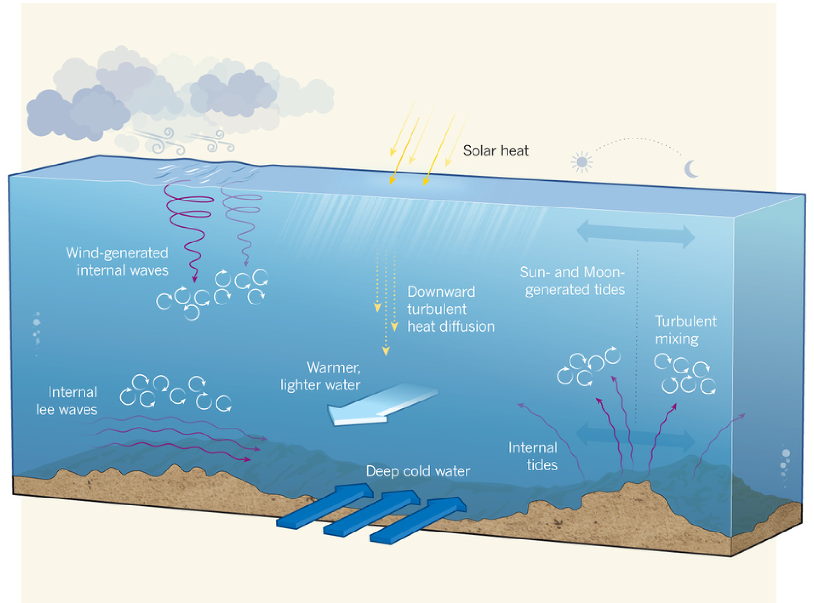
$$\varepsilon_p = \eta \cdot \varepsilon = K \cdot N^2 \quad \text{and} \quad \eta = 1/6$$

ε_p : fraction of the dissipation available to mix the fluid
$N^2 = -\frac{g}{\rho}\partial_z\rho$: buoyancy frequency
η : mixing efficiency
ε : viscous dissipation

In the limit of a homogeneous fluid, mixing cannot drive a buoyancy flux

$$K \cdot N^2 = \eta \cdot \varepsilon \underset{N^2 \to 0}{\to} 0$$

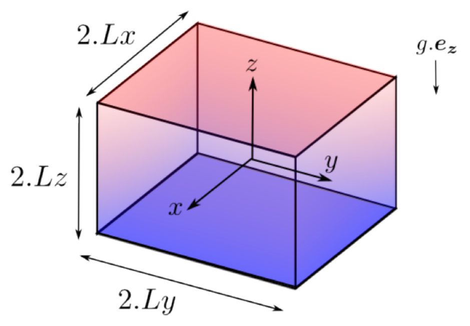
Motivation: the Ocean



MacKinnon 2013. Nature.

- Many different processes generate mixing in the ocean interior.
- Mixing drives downward transport of heat and dissolved greenhouse gases, and upward transport of biological nutrients.
- Drives a global network of currents (MOC).
- Induces meridional heat transport.
- Important effect on climate.

Method: Direct Numerical Simulations



- Navier-Stokes under Boussinesq approximation.
- Standard pseudospectral method (Fourier).
- Velocity field imposed for initial conditions.
- Penalisation region in the boundaries.
- Background density can evolve.

Equations: Dimensionless Navier-Stokes

$$\frac{\partial \mathbf{u}}{\partial t} - \omega \times \mathbf{u} = -\nabla p - \theta \cdot Ri \cdot z + \frac{1}{Re} \cdot \nabla^2 \mathbf{u}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla\right)\theta = \frac{1}{Re \cdot Sc}\nabla^2\theta$$

$$\nabla \cdot \mathbf{u} = 0$$

Dimensionless numbers:
$Ri = \frac{\theta_0 \cdot g \cdot L_0}{U_0^2}$, $Re = \frac{L_0 \cdot U_0}{\nu}$, $Sc = \frac{\nu}{\kappa}$

Reduced buoyancy
$\theta = \frac{\rho - \rho_0}{\rho_0}$

$\theta_0 = \frac{\rho^{max} - \rho^{min}}{\rho_0}$

Turbulent integral scales
L_0, U_0

DNS parameters:
$N_{grid} = 1024, Re = 1000, Sc = 5$

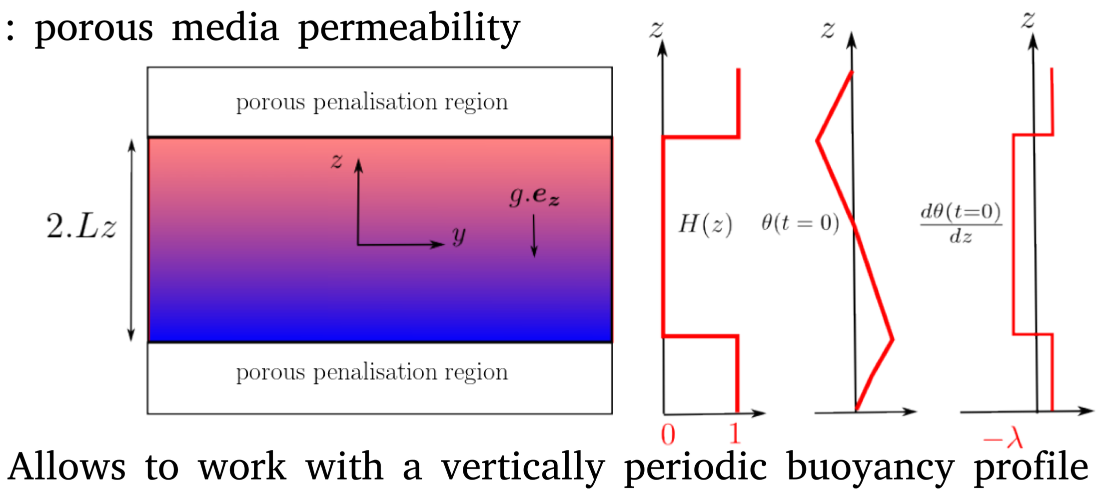
run	1	2	3	4	5	6	7
Ri	2	10	44	180	740	3000	11000

In the penalisation region: Inspired by Kadosh et al. 2012

$$\frac{\partial \mathbf{u}}{\partial t} - \omega \times \mathbf{u} = \nabla p - \theta \cdot Ri \cdot (z) \cdot (1 - H) + \frac{1}{Re} \cdot \nabla^2 - \frac{1}{\eta_p} \cdot H \cdot \mathbf{u}$$

$$\frac{\partial \theta}{\partial t} = \nabla \cdot [(1 - H) \cdot \mathbf{u} \cdot \theta] + \nabla \cdot \left\{\left[\frac{1}{Re \cdot Sc}(1 - H) + \eta_p \cdot H\right] \cdot \nabla\theta\right\}$$

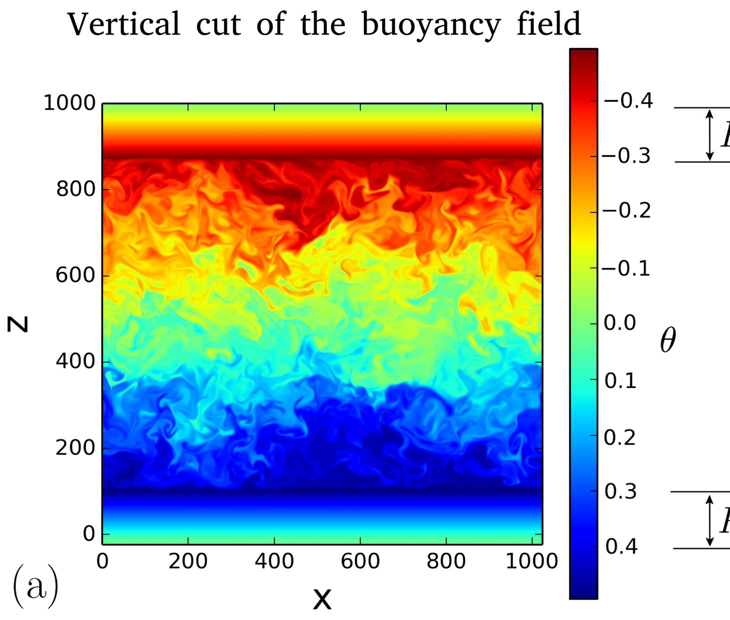
η_p: porous media permeability



Allows to work with a vertically periodic buoyancy profile

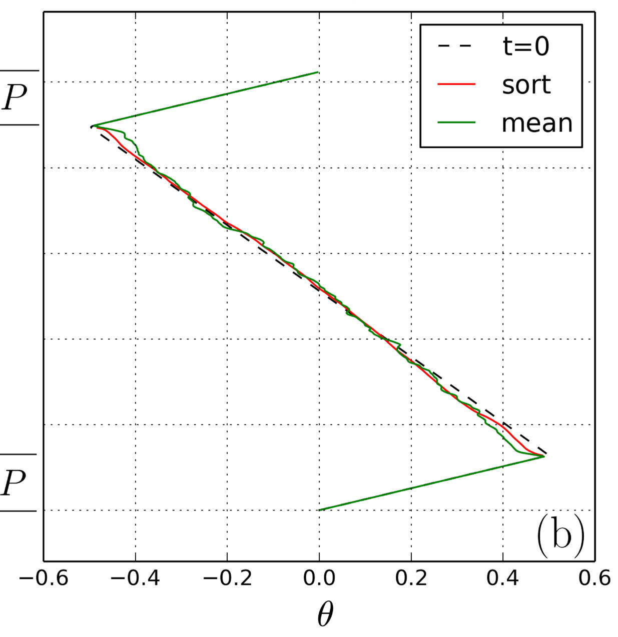
Results: buoyancy field

Vertical cut of the buoyancy field



The penalisation region is indicated by the letter P

Buoyancy vertical profile



Buoyancy fields:

- Horizontal average buoyancy field
  $< \theta(x, y, z, t) >_{x,y}$
- Horizontal average sorted buoyancy field
  $< \theta_S(x, y, z, t) >_{x,y}$

The sorted buoyancy field is obtained by sorting every element of fluid of the buoyancy field. It allows to measure instantaneously the irreversible mixing.

- Initially, the buoyancy field is sorted
  $\theta(x, y, z, t = 0) = \theta_S(x, y, z, t = 0)$

Potential energies

Potential energy:

$E_p = g \cdot \rho_0 \int_V \theta \cdot z \cdot dV$ — Given by the instantaneous center of mass of the fluid

Background potential energy:

$E_b = g \cdot \rho_0 \int_V \theta_S \cdot z \cdot dV$ — Indicates the irreversible mixing done in the fluid

Available potential energy:

$E_{ape} = g \cdot \rho_0 \int_V (\theta - \theta_S) \cdot z \cdot dV$ — Represents the instantaneous potential energy that has not yet been transformed in irreversible mixing

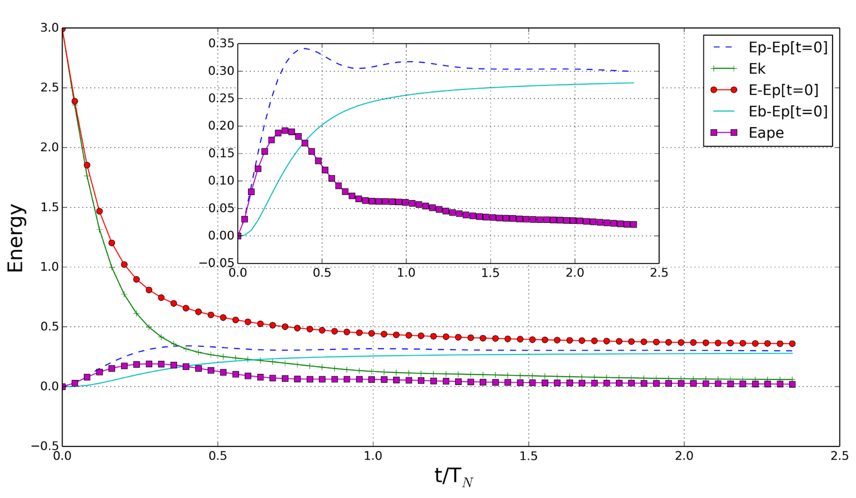
Kinetic energy:

$E_k = \frac{\rho_0}{2} \int_V \mathbf{u}^2 \cdot dV$

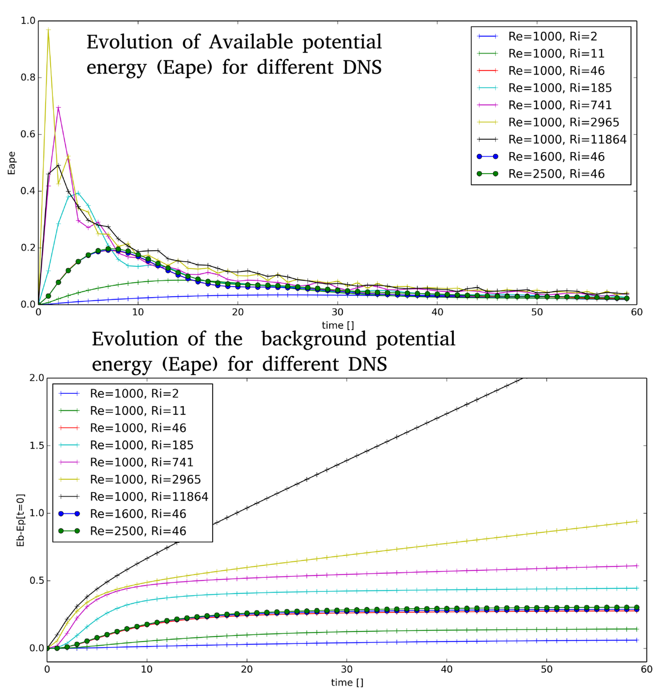
Winters 1995:

Energies: Evolution

Evolution of potential and kinetic energies:



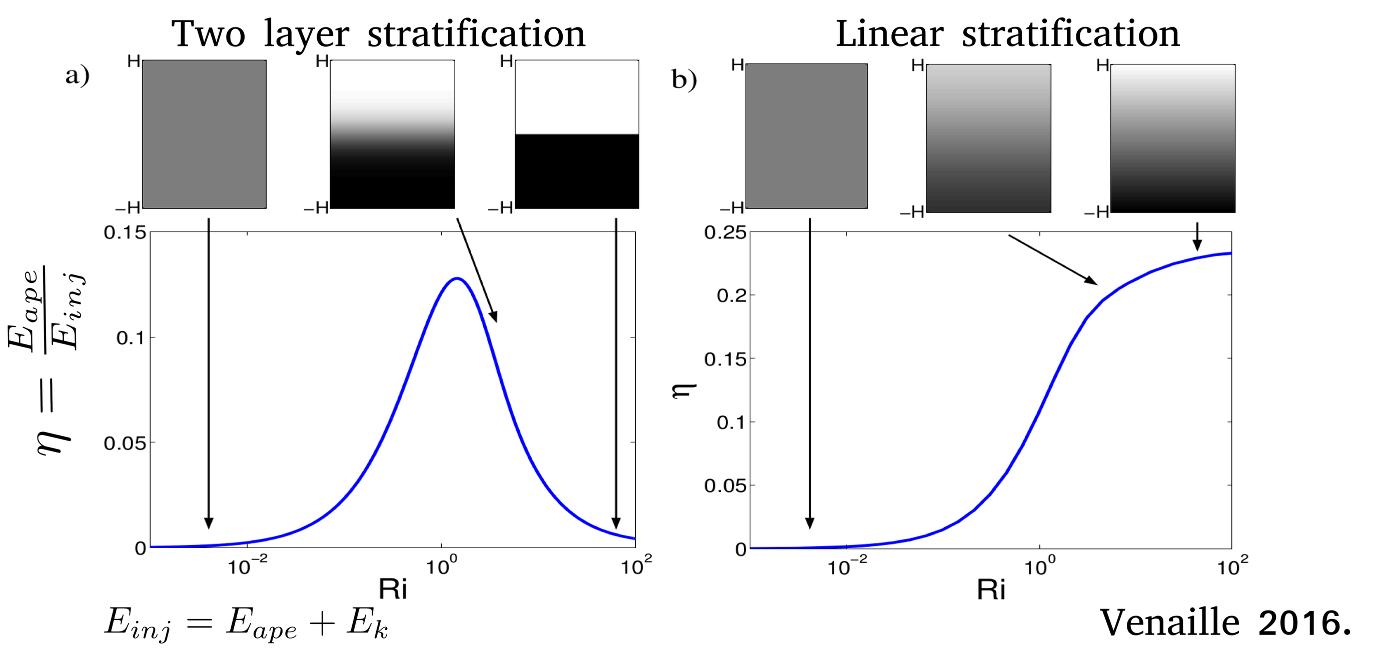
Re=1000, Ri=46



Three phases of the flow:

- Initially the energy is injected through kinetic energy
- Once the turbulence is developed, the kinetic energy decays and, therefore the rate of irreversible mixing decreases.
- Finally, turbulence is weaker and the flow transitions towards laminarization.

Perspectives: mixing efficiency through a statistical theory



$\eta = \frac{E_{ape}}{E_{inj}}$

$E_{inj} = E_{ape} + E_k$

Venaille 2016.

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