Temperature fluctuations induced by turbulent dissipation

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Abstract

IN practically every fluid flow, kinetic energy is converted into heat through viscous friction. In a turbulent flow this heat is generated in an inhomogeneous matter and the temperature distribution in the fluid will subsequently not be uniform. We investigate these temperature fluctuations in isotropic turbulence. It is shown by numerical simulations and theory how these fluctuations interact with the turbulent flow that generated them. The intermittent nature of the dissipation rate fluctuations is shown to play a fundamental role in the physics of viscous heating.

1. How much heat generates a turbulent flow?

Using Taylor's zeroth order law to estimate the dissipation:

$$\langle \epsilon \rangle \sim U^3/L$$

In a large wind-tunnel:
$L = 1m$, $U = 1m/s \rightarrow \langle \epsilon \rangle \approx 1m^2/s^3$.
In a closed system this will heat the fluid,

$$\frac{d\langle \theta \rangle}{dt} = \frac{\langle \epsilon \rangle}{c_p}$$

with $c_p = 10^3 J/kg/K$ the specific heat.
After one hour: $\Delta \langle \theta \rangle = 3600 * 1/10^3 = 3.6K$.

For large wind-tunnel turbulence experiments, see for instance: Anselmet, Gagne, Hopfinger and Antonia (J. Fluid Mech. 1984)

2. How large are the fluctuations? What is their size?

The average heat is easily estimated, but the fluctuations? Only few relevant studies (De Marinis et al. JFM 2013, Cadot & Plaza APS 2005).

Introduce: $\theta = \langle \theta \rangle + \theta'$, $\epsilon(\boldsymbol{x}, t) = \langle \epsilon \rangle + \epsilon'$

$$\frac{\partial \theta'}{\partial t} + \boldsymbol{u} \cdot \nabla \theta' = D \Delta \theta' + \frac{\epsilon'}{c_p}$$

so that

$$\frac{d \frac{1}{2} \langle \theta'^2 \rangle}{dt} = \frac{\langle \epsilon' \theta' \rangle}{c_p} - D \left\langle (\nabla \theta')^2 \right\rangle.$$

Unclosed equation $\rightarrow$ we need Simulation/Theory/Experiment.

3. EDQNM

Introducing

$$\int E(k) dk = \frac{3}{2} \left\langle u^2 \right\rangle, \qquad \int E_\theta(k) dk = \frac{1}{2} \left\langle \theta^2 \right\rangle$$

EDQNM expression for the viscous heat production:

$$\frac{\langle \epsilon' \theta' \rangle}{c_p} = 16\pi \int k^2 \left(\frac{\nu}{c_p} \right)^2 \iint \delta(\boldsymbol{k} - \boldsymbol{p} - \boldsymbol{q}) \int_0^t G_\theta(k, t, s) \left[(p_m q_m)^2 \Phi_{ij}(\boldsymbol{p}, t, s) \Phi_{ij}(\boldsymbol{q}, t, s) + 2 p_m q_m p_i q_j \Phi_{aj}(\boldsymbol{p}, t, s) \Phi_{ia}(\boldsymbol{q}, t, s) + p_i p_j q_m q_n \Phi_{mn}(\boldsymbol{p}, t, s) \Phi_{ij}(\boldsymbol{q}, t, s) \right] ds\, d\boldsymbol{p}\, d\boldsymbol{q}\, dk$$

Numerical integration of the equations:

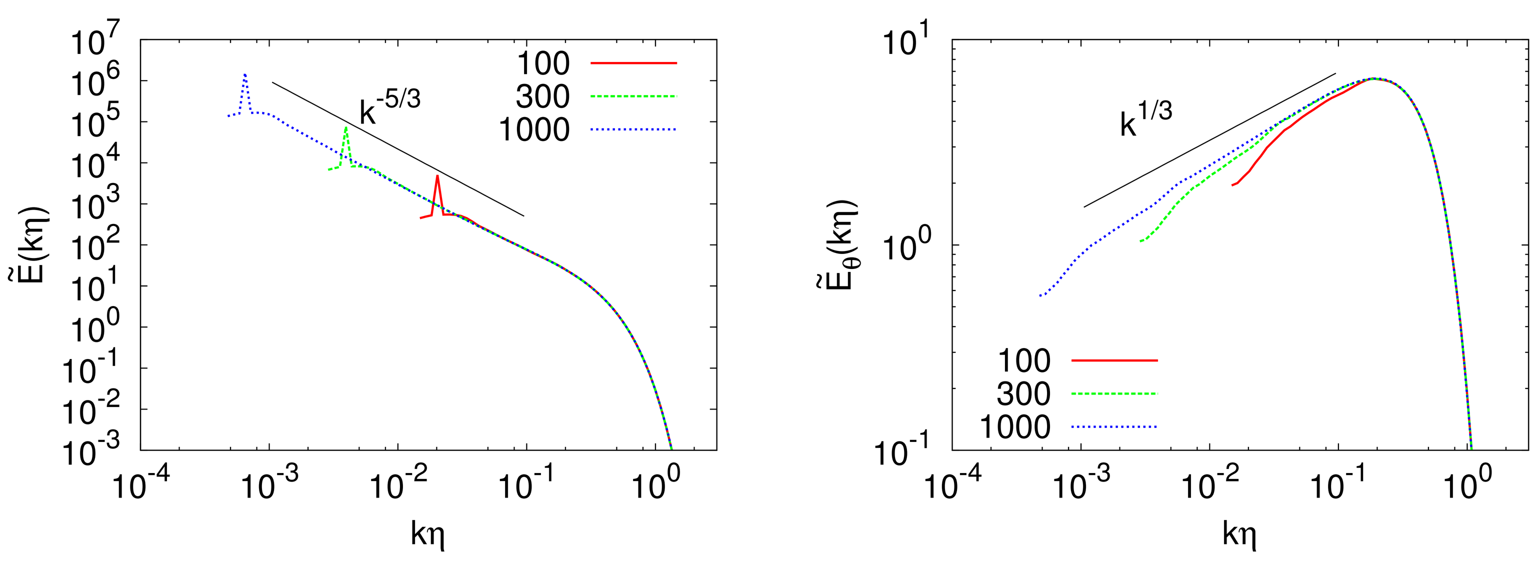


Fig. 1: *Left: Energy spectrum, normalized by Kolmogorov variables, for three different Reynolds numbers. Right: corresponding temperature fluctuation spectrum, generated by frictional heating ($Pr = 1$).*

4. How large are the heat fluctuations?

$$\left\langle \theta^2 \right\rangle = \int_0^{k_\eta} E_\theta(k) dk = \frac{\langle \epsilon \rangle \nu}{c_p^2}$$

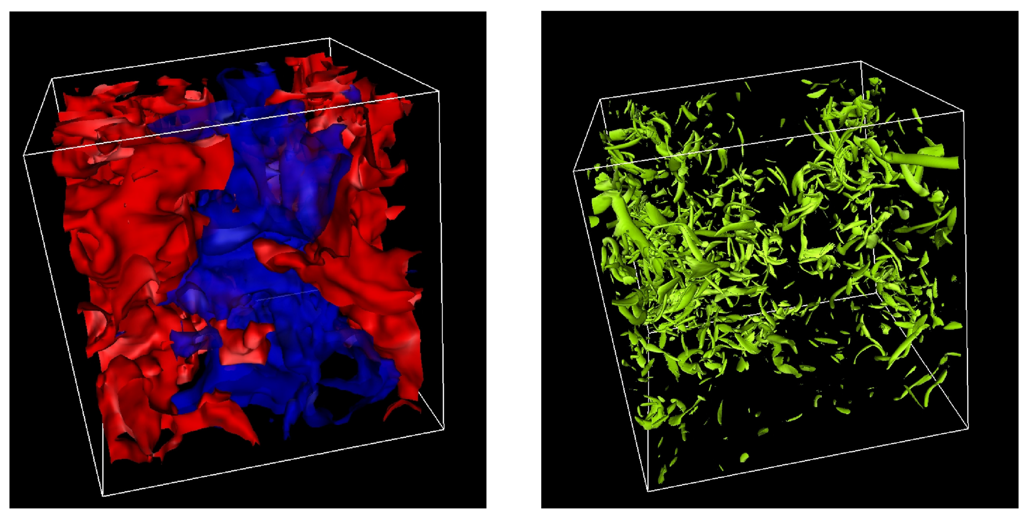
In the air experiment $\nu = 10^{-5} m^2 s^{-1}$, $\langle \epsilon \rangle = 1 m^2 s^{-3}$, $c_p = 10^3 J kg^{-1} K^{-1}$,

$$\left\langle \theta^2 \right\rangle = \int_0^{k_\eta} E_\theta(k) dk \sim 10^{-11} K^2$$

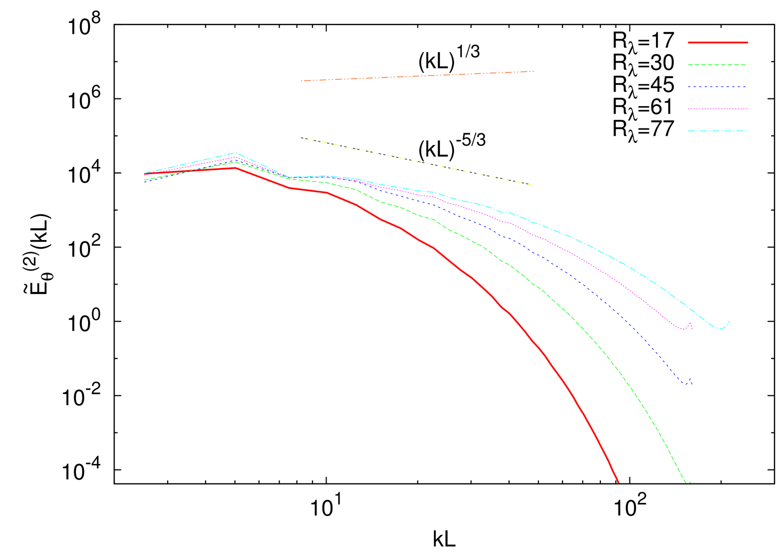
That is not very large since $\langle \theta^2 \rangle \sim Re^{-1}$ (or $\sim R_\lambda^{-2}$).

Bad news for the experimentalist: $\theta_{rms} \approx 3 \mu K$

5. Let's check this by DNS



- Left: Temperature fluctuations are correlated at large scales,
- Right: isovorticity. Positive temperature fluctuations exist in the zones where many worms are clustered.



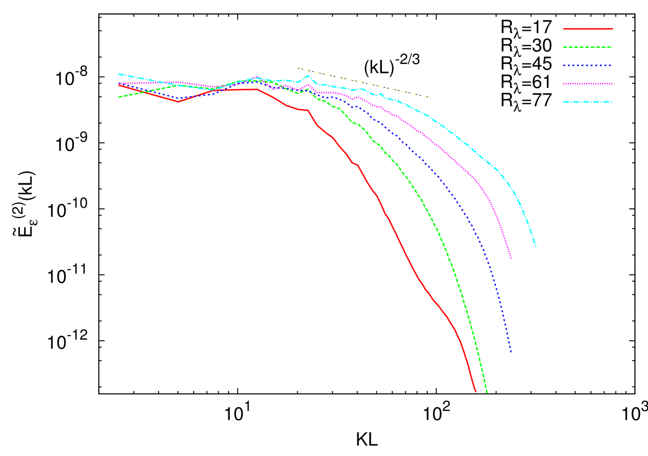
This is in complete disagreement with the previous results !!!

6. Why? Dissipation rate fluctuations are very intermittent

$$\frac{\langle \epsilon'(\boldsymbol{x}, t) \theta(\boldsymbol{x} + \boldsymbol{r}, t) \rangle}{c_p} = \frac{1}{c_p^2} \int_0^t \int \langle g_\theta(\boldsymbol{x} + \boldsymbol{r}, t | \boldsymbol{y}, s) \epsilon'(\boldsymbol{x}, t) \epsilon'(\boldsymbol{y}, s) \rangle\, d\boldsymbol{y}\, ds,$$

$$\frac{\langle \epsilon' \theta' \rangle}{c_p} \approx \int \frac{\tau(k)}{c_p^2} E_\epsilon(k) dk. \quad \text{with} \quad \int E_\epsilon(k) dk = \frac{1}{2} \left\langle \epsilon'^2 \right\rangle \qquad (1)$$

Dissipation rate spectra at $R_\lambda \leq 77$:



Yaglom-Novikov estimate (See Monin & Yaglom, New Testament pp. 605-608):

$$E_\epsilon(k) \sim \langle \epsilon \rangle^2 L (kL)^{-1+\mu}$$

with $0.3 < \mu < 0.5$.

See also Castaing, Gagne and Hopfinger, Physica D 1990.

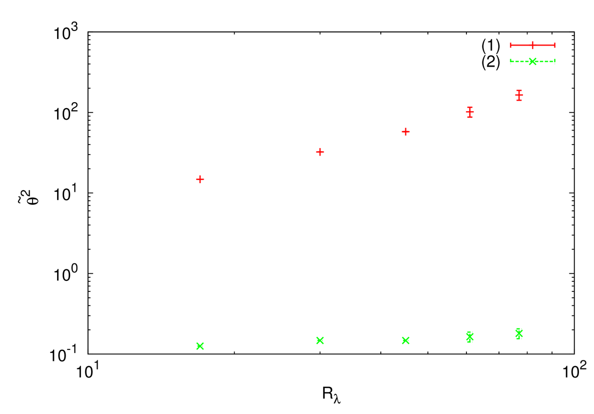
7. Implications for the temperature variance

Using Yaglom's expression in equation (1), with $\tau(k) \sim \epsilon^{-1/3} k^{-2/3}$ we have

$$E_\theta(k) \sim \frac{\langle \epsilon \rangle^{4/3} L^{2/3} k^{-5/3}}{c_p^2}$$

$\langle \theta^2 \rangle = \int E_\theta(k) dk \sim \frac{(\langle \epsilon \rangle L)^{4/3}}{c_p^2}$

- EDQNM: $\overline{\theta^2}^{(1)} \sim C^{(1)} \frac{\langle \epsilon \rangle \nu}{c_p^2}$
- With intermittency: $\overline{\theta^2}^{(2)} \sim C^{(2)} \frac{(\langle \epsilon \rangle L)^{4/3}}{c_p^2}$



Implications for the value

$$\theta_{rms} \sim \frac{\langle \epsilon \rangle^{1/2} L^{2/3}}{c_p} \sim 10^{-3} K$$

Message to the experimentalist: $\theta_{rms} \approx 1mK$ $\rightarrow$ small but measurable!

References

- W.J.T. Bos. The temperature spectrum generated by frictional heating in isotropic turbulence. *J. Fluid Mech.* (2014).
- W.J.T. Bos, R. Chahine, A.V. Pushkarev. On the scaling of temperature fluctuations induced by frictional heating. *Phys. Fluids* (2015).