From stratified gravity flow to a practical hydraulics problem

By Gilles Corcos,
Emeritus Professor, University of California, Berkeley

Abstract

As an aside (section 3) in his JFM 1968 article on gravity currents, Brook Benjamin had established the properties of a liquid emptying from a horizontal tube and of the upstream propagation with respect to that liquid of a semi infinite pocket or bubble of a lighter liquid or gas. It turns out that in the hydraulics of pipe flow under gravity, a phenomenon is frequently present which is closely related to Benjamin's idealized model. The latter also informs the mechanics of siphons. Useful but little known practical consequences of this fact have now been exploited.

Introduction.
Designers of water conduction lines utilizing gravity between a source of water at atmospheric pressure and a lower reservoir are well aware of the mischiefs that are frequently caused by the presence or ingestion of air in the piping: It frequently acts as a block to the flow of water. Since for an obvious practical reason the conduction line is forced to follow the relief of the ground, it most often encounters local high points near which the air may accumulate, and therefore some lengths of horizontal conduits. The usual practice is to provide automatic air relief valves at each of the high points encountered. This blind practice has serious disadvantages, but very little literature has provided alternatives by analysing the behaviour of these pockets of air which when stationary are in fact long bubbles.

Having become late in my career one of these designers I chose to look into the matter. In a UC Berkeley lab I set up a scaled-down model of such a conduction line allowing both the introduction of air upstream in the pipe and a variable water flow rate and using transparent plastic pipes to visualize the phenomenon.

I had also remembered a lecture given on the Berkeley campus by Brook Benjamin on the subject of stratified gravity flow in which an idealized model of such flows was given a remarkably simple representation, that of a horizontal two-dimensional semi infinite channel sealed on one end, initially full of water and discharging under gravity at the other end.

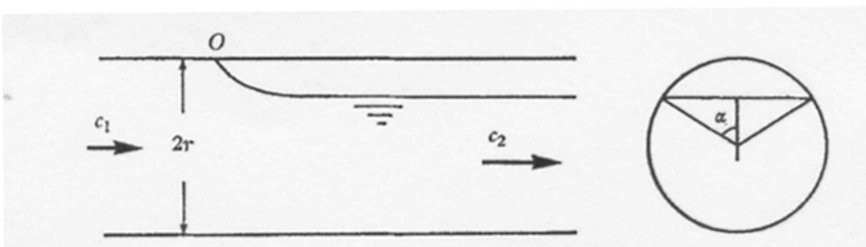

Figure 1.Brook Benjamin's (1968) inviscid model of a water-filled pipe emptying at one end

As an aside, in a published version of his study he extended his model to the discharge of a horizontal tube of water, closed at one end and discharging to the atmosphere at the other.

The Brook Benjamin model .

In that model the coordinates are chosen so that the nose of the bubble is
stationary, there is a cross-wise uniform flow at upstream infinity and a stagnation point at the nose of the bubble on the upper part of the pipe wall and there is again a uniform flow under the bubble sufficiently far downstream of the bubble nose.
The fluid is assumed inviscid.
Use of the Bernoulli equation, as well as the integral momentum and continuity equations lead to an algebraic equation in terms of trigonometric functions of the cross-sectional geometry of the flow far downstream and this equation yields a unique real solution in which the far downstream velocity with respect to the nose of the bubble is supercritical; its Froude number is greater than 1. This is important since it implies that perturbations of that flow downstream of the bubble nose cannot affect the flow around it.

While I had overlooked the existence of extensive and relevant laboratory studies by Zukoski (2), I had simulated the conditions of Brook Benjamin's model somewhat differently, i.e. more in keeping with the practical situation I had encountered in the field, where the fluid has some viscosity and the oncoming flow is not crosswise uniform

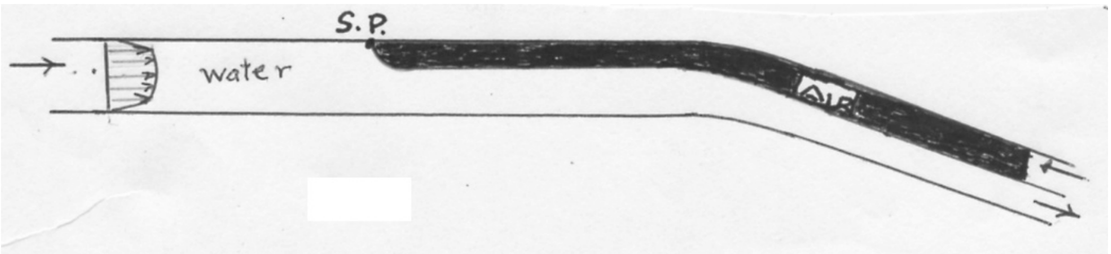

Fig 2. The essential part of my experimental set up

The pipe diameter was typical of those used in our field conduction lines.. The flow rate was measured when, as it was gradually decreased the bubble just stopped receding downstream and advanced in the horizontal part of the configuration shown. When the bubble nose was made stationary there, the flow rate was taken to correspond to the velocity of propagation of the bubble nose.
The downstream Froude number was deduced to be 1.38 from the flow rate by using continuity and a visual (and therefore approximate) measure of the interface height, (about half way up). In view of the small value of the non-dimensional surface tension parameter, it was assumed that it and the Reynolds number were, for our practical use, of small import so that the basic parameter influencing the critical flow rate (the flow rate with respect to which the propagation velocity of the bubble is stationary) was of the form :

$$Q_{cr}=A(d^{5/2}g^{1/2}); \quad (1)$$

where A is assumed a constant. For the horizontal pipe A was determine to be approximately

$$A=0.38$$

Zukoski 's experiments inform us to what extent this assumption is justified.

In summary my very limited experimental results are close to Zukoski's and to the theoretical results of Brook Benjamin, this, in spite of the differences between the three flows. This extends to the measurements for tubes inclined to the horizontal. There, while in general no theoretical results have apparently been reported, Zukoski and I both find that for comparable values of the parameters, the propagation velocity of the bubble with respect to fluid at rest first increases as the slope of the pipe increases from the horizontal and reaches a maximum around 35 to 40 degrees and then decreases to a value inferior to that for a horizontal tube.

Consequences for the flow with air bubbles in the vicinity of local high points in gravity systems.

First the fact that the flow below the bubble becomes supercritical in the sense of shallow water theory implies as noted above that downstream perturbations cannot affect the behaviour of the flow near the bubble nose. That is the case in particular when the flow is subject downstream to something similar to a hydraulic jump as the water fills the pipe again. So we get the interesting result that finite bubbles can and often do propagate with respect to fluid within horizontal pipes with a finite velocity closely dependent only on the pipe diameter.

Second in the downstream, lowering side of the high point for our field conditions the bubble will not be chased downstream until the flow rate has reached a value larger than the critical flow rate for the horizontal pipe. The value of this flow rate depends on the slope of the pipe.
To be conservative in our designs the criterion used as sufficient though seldom necessary to expel a bubble downstream in $Q= Ad^{5/2}g^{1/2}$ has been chosen as:

$$A>0.5$$

Third, as in the simplified example of Brook Benjamin, the velocity of the incoming stream with respect to the bubble nose is lower than the critical velocity $(gr)^{1/2)}$, (r being the pipe radius and g the gravity constant) ; consequently according to shallow water theory the transition to supercritical flow can only occur on a horizontal sill so that the nose of the bubble will always be found right at the high point of the pipeline when the flow rate generates a supercritical condition downstream of the bubble nose. Additionally it is clear that if there is flow the bubble cannot originate ahead of (i.e. upstream lower than) the high point since in that case the velocity needs to increase around the bubble, while the potential head also needs to increase and the pressure remains constant as soon as the bubble head is reached.
As a result: for lower flow rates, the head of the bubble is found at the high point and its end is found on the downstream side of that local maximum. There lies the reason for head losses due to air pockets: This head loss is equal to the level difference

between the head and the tail of the bubble. The mass of the air bubble remains fixed but its volume for a given mass depends on the pressure within the bubble.

On the other hand if the flow rate exceeds that for which the bubble can propagate on the downstream inclined segment of the pipe, the bubble will be swept downstream along with the water.
To summarize: for velocities inferior or equal to that required for a semi infinite bubble to be stationary in a horizontal pipe the bubble nose is found at the high point while its tail is found downstream and lower.

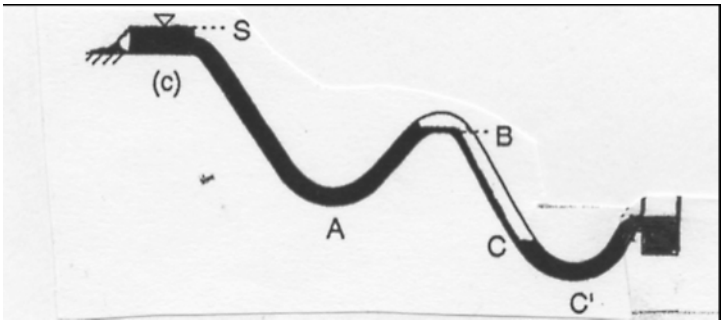

Figure 3. The geometry of a stationary bubble near a local high point (case of a restricted flow)

This is the case also when the incoming velocity while higher is still insufficient to chase the bubble down the inclined section downstream of the high point.
Finally when the flow rate exceeds that needed to maintain the bubble stationary within the inclined section, the bubble is expelled downstream and the transient bubble occasions no average head loss.

Several practical consequences can be drawn from these results:

1. If, as needed when the flow rate is sufficiently small, an automatic air relief valve is used to expel the air of the bubble, it imperatively needs to be located somewhat <u>downstream</u> of the high point. To minimize the head loss due to the bubble the location of the valve will be as close as possible to that high point.

2. When the total head available allows the extra loss due to the choice of a locally smaller diameter it is possible by so doing to chase the bubble downstream. Alternatively it is always possible to make it stationary by increasing the local value of the pipe diameter, (see equation 2). Theses manipulation turns out to be of great utility. When it is advantageous or necessary to keep the bubble in place and get rid of it with an air release valve, the velocity of the local incoming flow should be lower than required to chase the bubble downstream. The reason is that if it is not, incoming air from upstream will transit through the valve and activate it off-and-on ceaselessly, therefore shortening its life. This condition can always be accommodated. When either the velocity of the water in the pipe is naturally high enough or can be made so by decreasing the local pipe diameter the bubble is expelled downstream and the loss of head corresponding to the level difference between head and tail of the bubble is essentially totally avoided.

The foregoing and related considerations have been systematically exploited in many dozens of gravity conduction lines whose average slope was as small as 0.002 but whose local elevation excursions were very large, (i.e. for which the potential for head losses from air bubbles was far greater than the head available). And these applications have turned out to be uniformly successful.
A practical manual (Air in Pipes, second edition) to exploit the use of the technique is available on line from the website www.aplv.org, A computer program due to C. Huizenga and based on these considerations and automatizing the optimum design of conduction lines is available from the same source.

The sketchy discussion presented above invites further studies. This is true in particular of the unsteady flows that results from the possibly steady accretion of air upstream of the high points. The occurrence of these transients spans very large times and they can be very violent.

References

1. Brook Benjamin , JFM, 1968, vol. 31, part 2, pp. 209-248
2. Zukoski, E.E. JFM, 1966, vol. 35, part3, pp. 821-837.
3. Gilles Corcos, Air in Pipes, second edition ; can be downloaded from www.aplv.org.