Flow through a rotating, tilted rectangular box

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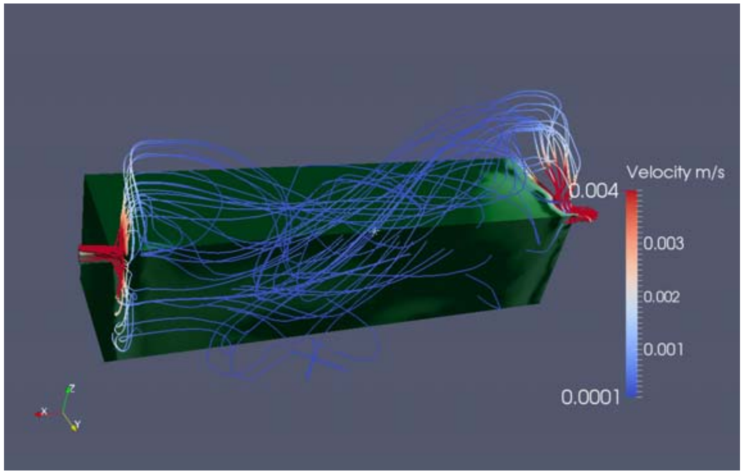
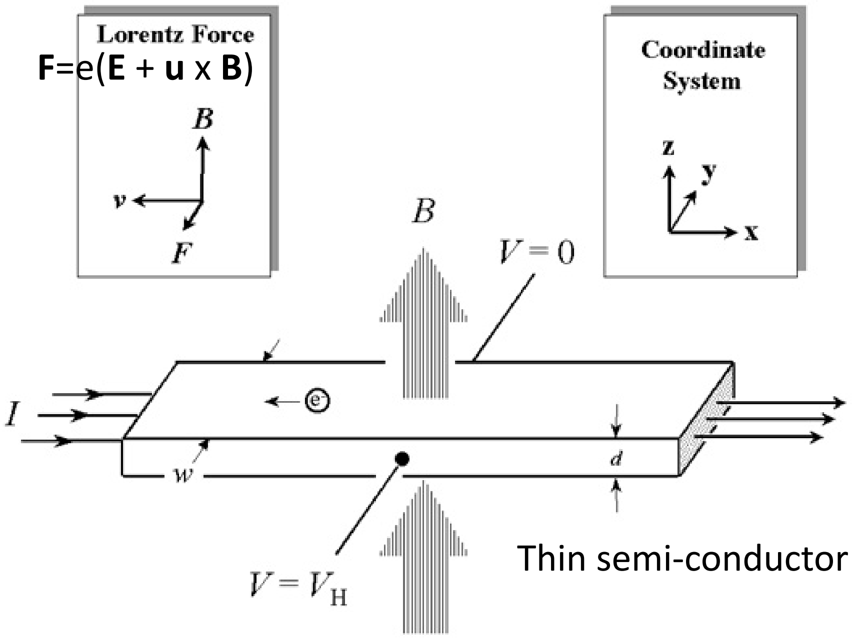
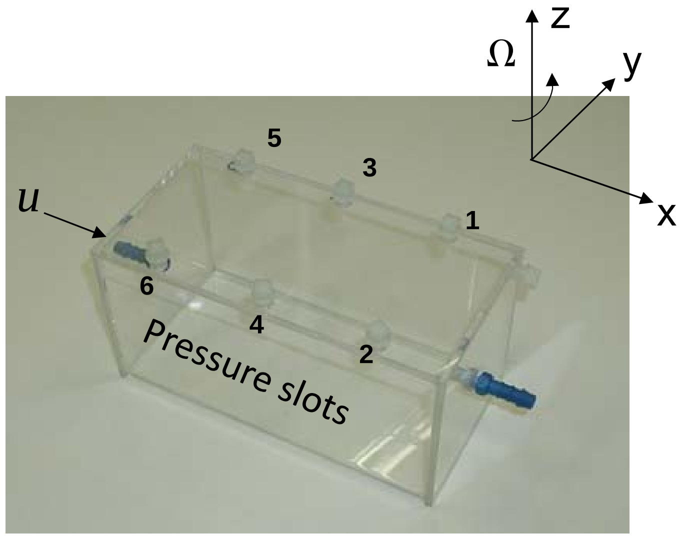
Costanza Rodda, Matias Duran-Matute



Experimental support by undergrads:
Niels Smit, Maurits Kruijt, John Wright,
Marlies van der Lugt, Marcus Therkildsen

Analogy of Lorentz force to Coriolis force





Charge separation creates lateral ΔV, analogous to geostrophy in rotating fluid

$$\frac{dV}{dy} = -Bu$$

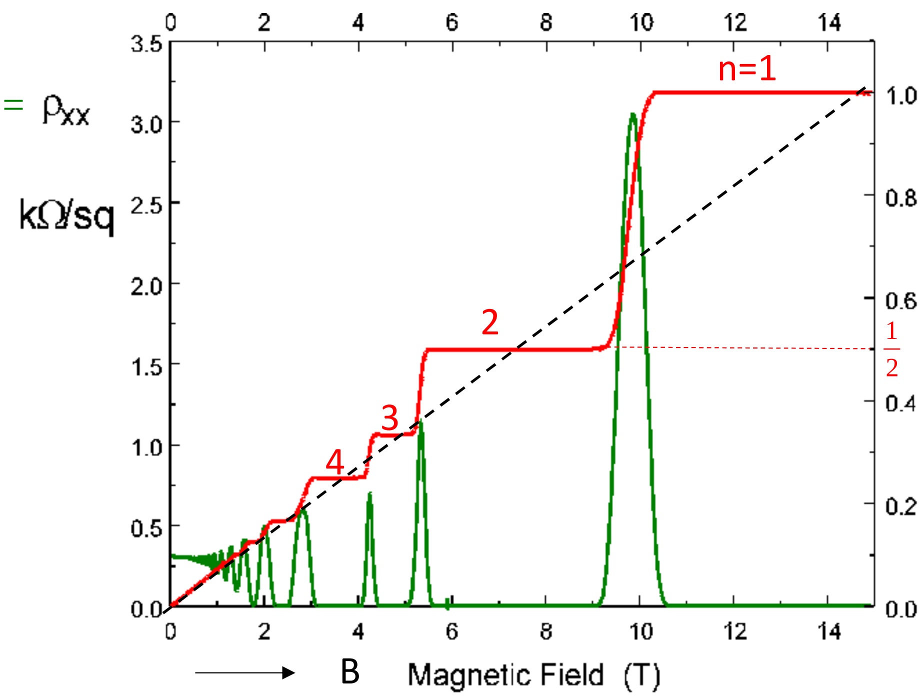
$$\frac{dp}{dy} = -2\Omega u$$

Hall effect (Hall, 1879)   Resistance $R_H = \frac{V_H}{I} = \frac{uBW}{uWNe} = \frac{B}{Ne}$

Integer (*n*) quantum Hall effect (von Klitzing et al 1980)

30 mK: 'electron fluid'

Longitud. resistance = ρ_{xx}



Hall resistance $\rho_{xy} = R_H$

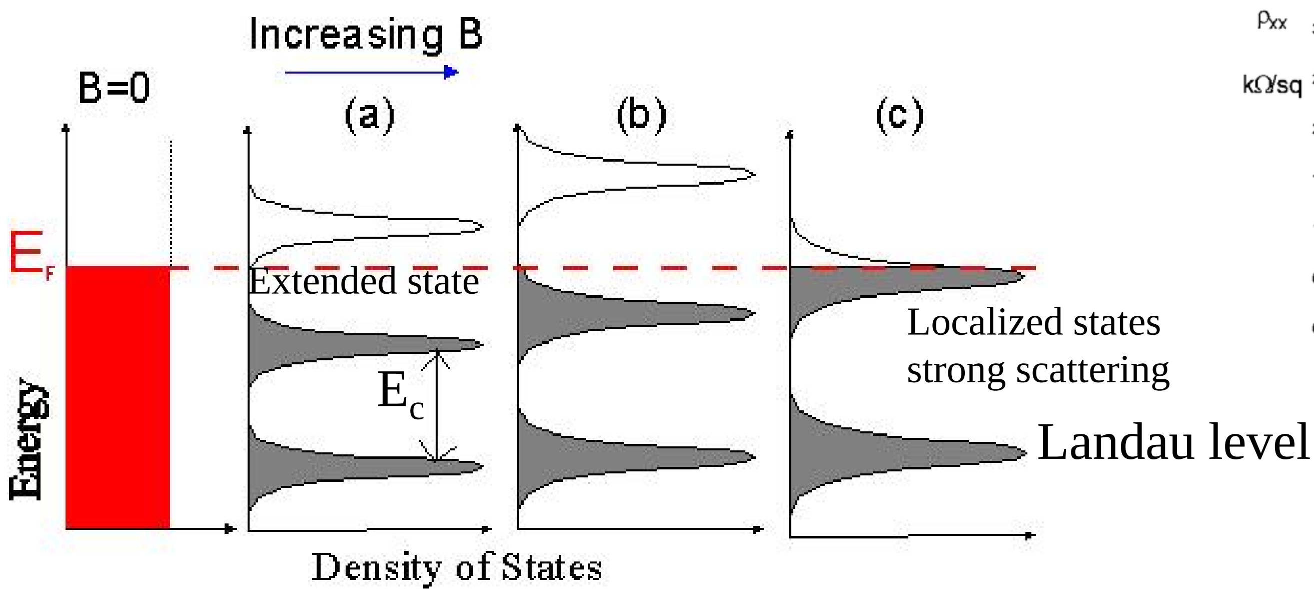
h/e^2 = 1 'Klitzing'

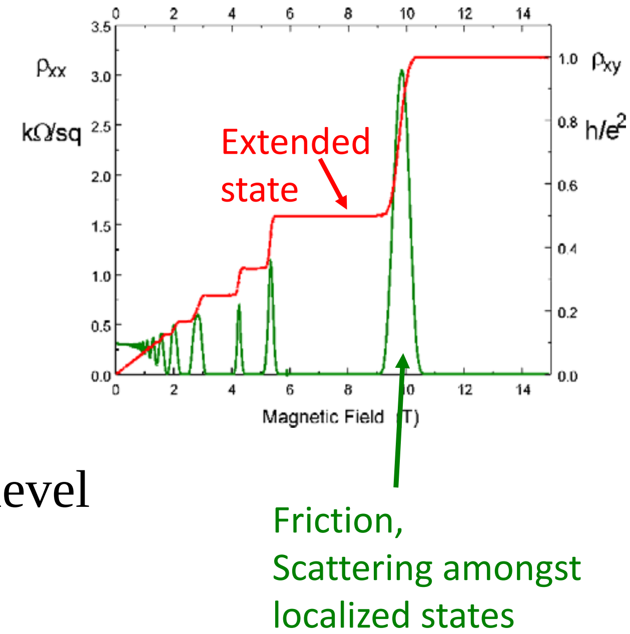
Plateaus: $\dfrac{h}{n\,e^2}$ $\quad n \in \mathbb{N}$

Constant at experimental accuracy of levels $<10^{-8}$!
Measure microscopic values from macroscopic data!

IQHE: 'Single-electron' states discretised by clumping of available states (Landau levels)

Explanation: a different view on ‘friction’





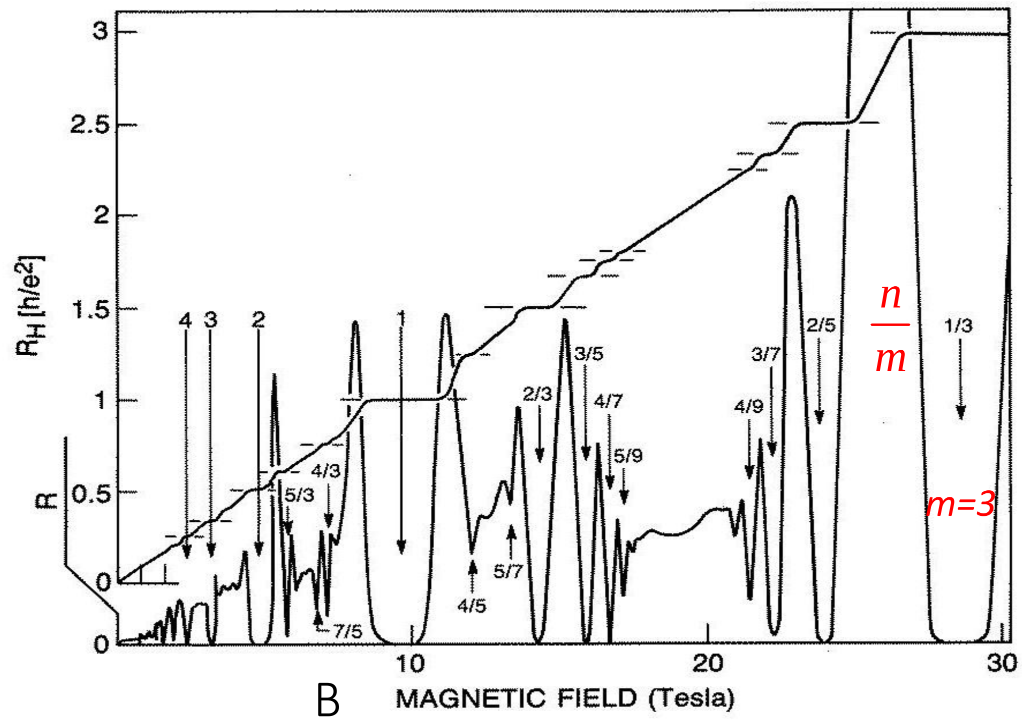
Cyclotron energy $E_c = h\,(eB/mc)$

Fixed E_f = Fermi energy : fixed carrier density. If E_f between Landau levels, no scattering: “extended state”

Fractional quantum Hall effect (Tsui, Störmer, Gossard '82)

Less *impurities*
Stronger B
Fractional charge…
(quasi-particles)
'Explanation' :
new, collective
multi-electron state
(Laughlin, 1983)

Note: also edge states
'Chiral boson states',
or Lutinger liquid
(like Kelvin waves)!



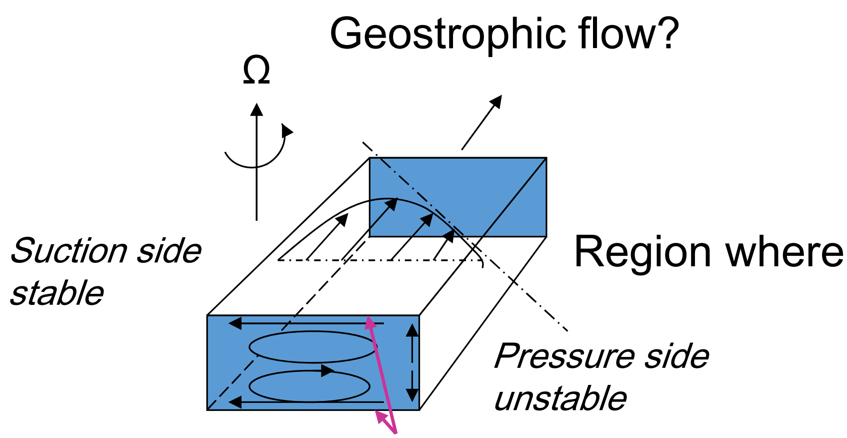
$\frac{mh}{ne^2}$, $m,n \in \mathbb{N}$

'Odd-denominator' phenomenon (odd m)

Eisenstein & Störmer '90

Plateaus (quantization) of geostrophy?

Partial compensation of absolute vorticity, 2Ω, by (mean) relative vorticity, $\hat{\boldsymbol{k}} \cdot \nabla \times \bar{\boldsymbol{u}}$



$2\Omega - \bar{u}_y = 0$

Ekman layers lead to two-cell secondary flow

Odd-denominator, reminiscent of odd number of reflections of wave attractors

But, do ***wave*** attractors facilitate **mean** throughflow?

$\Delta p_{||}$: energy fund → turbulence → waves → mixing, or focusing & throughflow

Geometric interpretation of FQHE as wave attractor regimes?

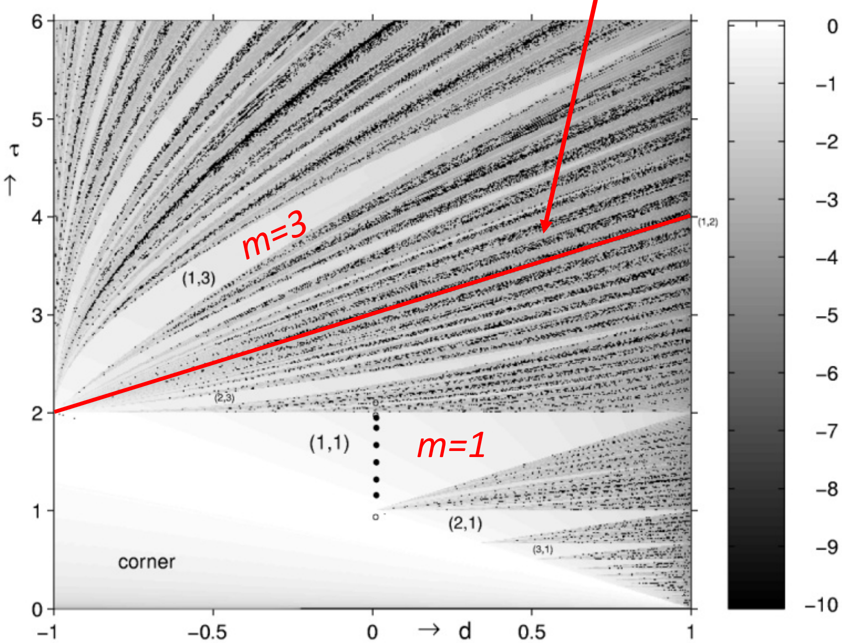
Lyapunov exponent:

Arnol'd tongues

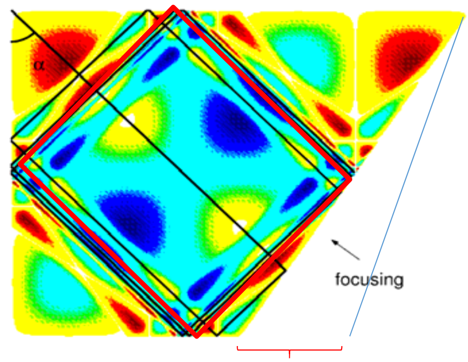
(n,m) attractors : odd m

$$\tau = \frac{H}{W}\sqrt{\frac{1}{\frac{4\Omega^2}{\sigma^2}-1}}$$

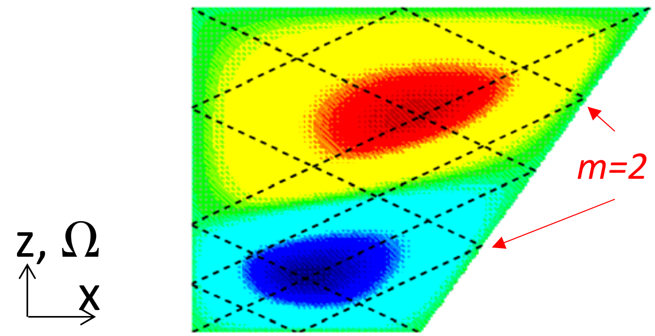
(1,2) regular mode

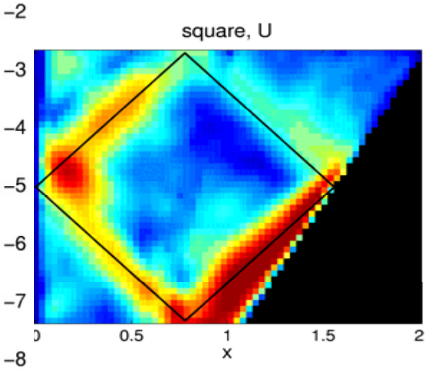


(1,1)-attractor



(1,2)-regular mode





Old Coriolis tank

wave attractors *do* affect **mean flow**

M., Benielli, Sommeria & Lam 1997, Maas 2001 and, …

Do we find geostrophy in enclosed box?

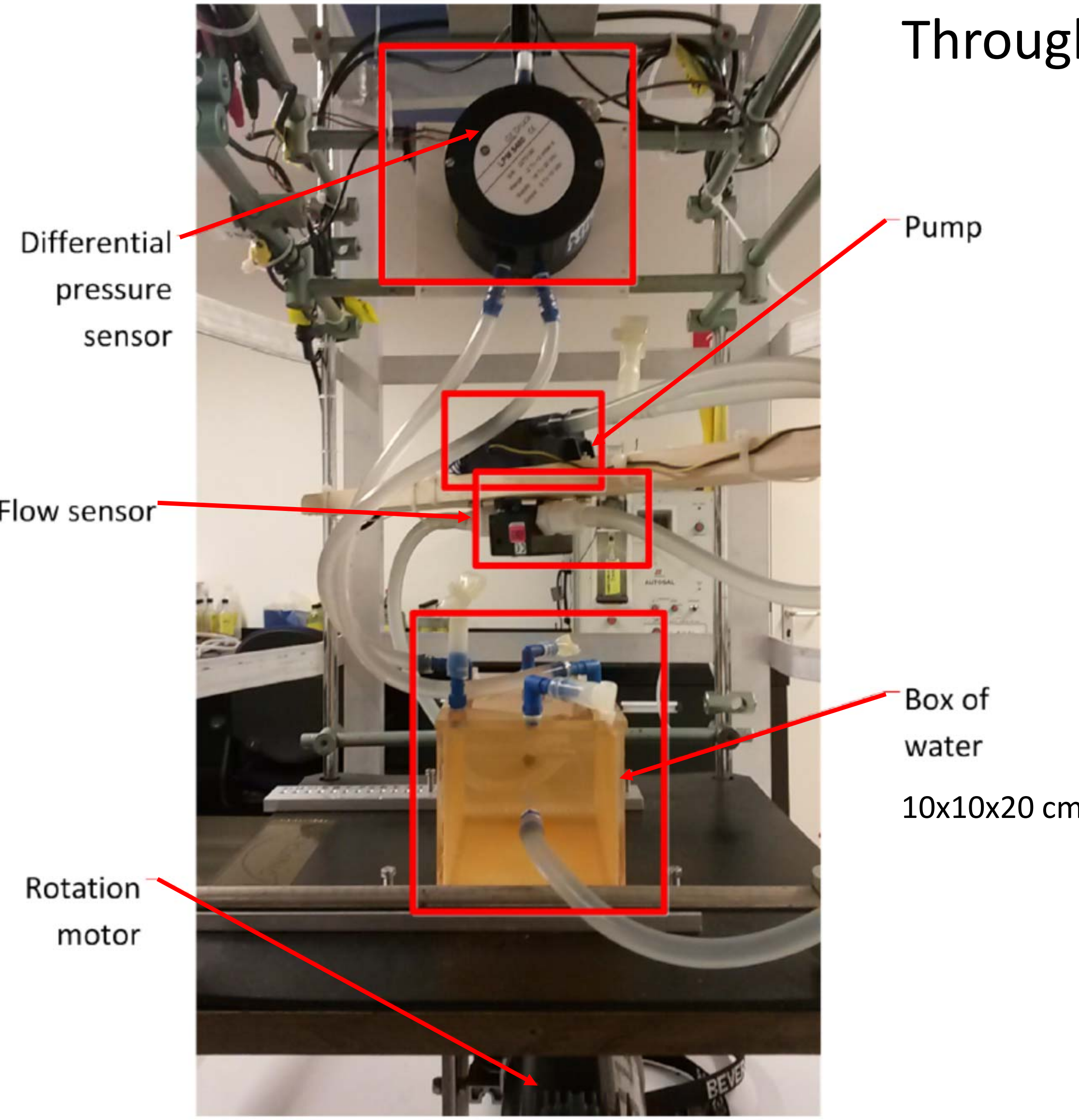
Is through-flow stronger, or weaker upon rotation?

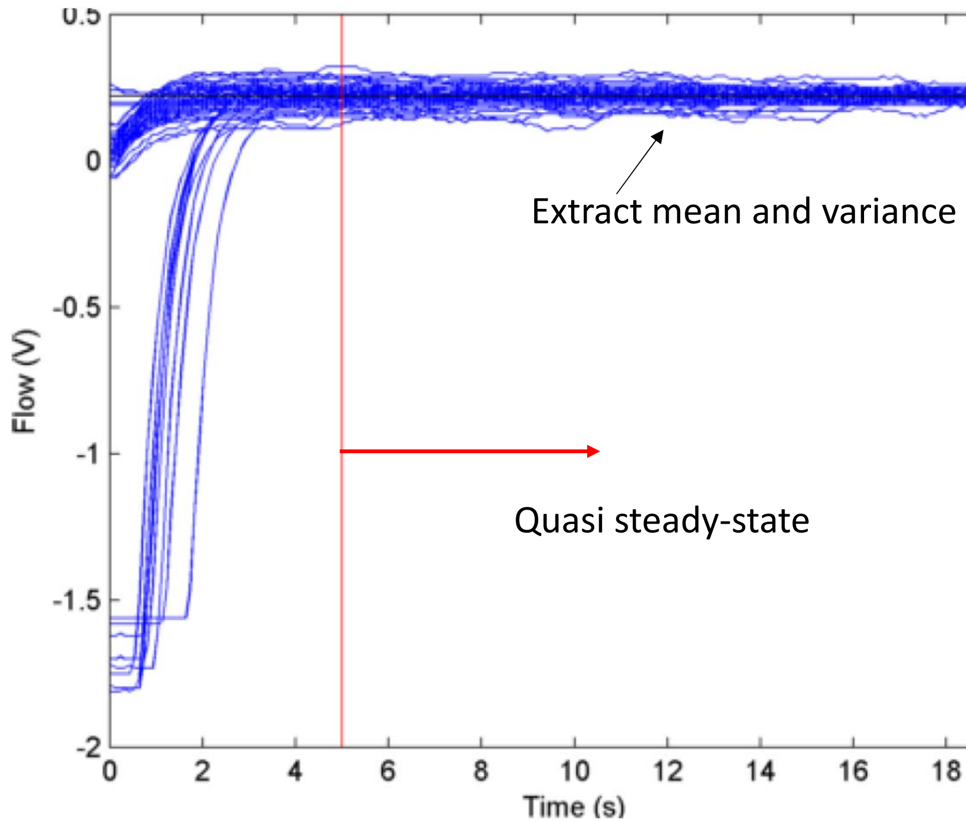
What are the ***impurities*** of a rotating box?

Do rotating tanks also possess a *non-uniform* **Density Of States** (DOS)?

Equivalent of *low temperature*, or of micro-properties?

Throughflow adjustment when changing Ω

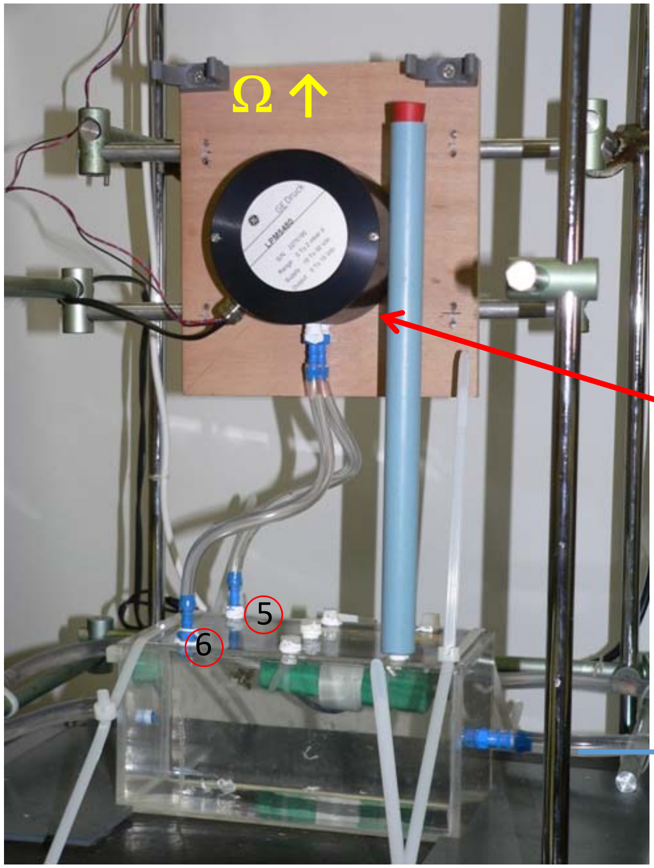




Rotating flows – geostrophy *without* gravity

$$2\Omega u = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

Apply $\Delta p_{\parallel} \propto u$



Labview:
Control: Ω, u ($\Delta p_{\parallel}$)

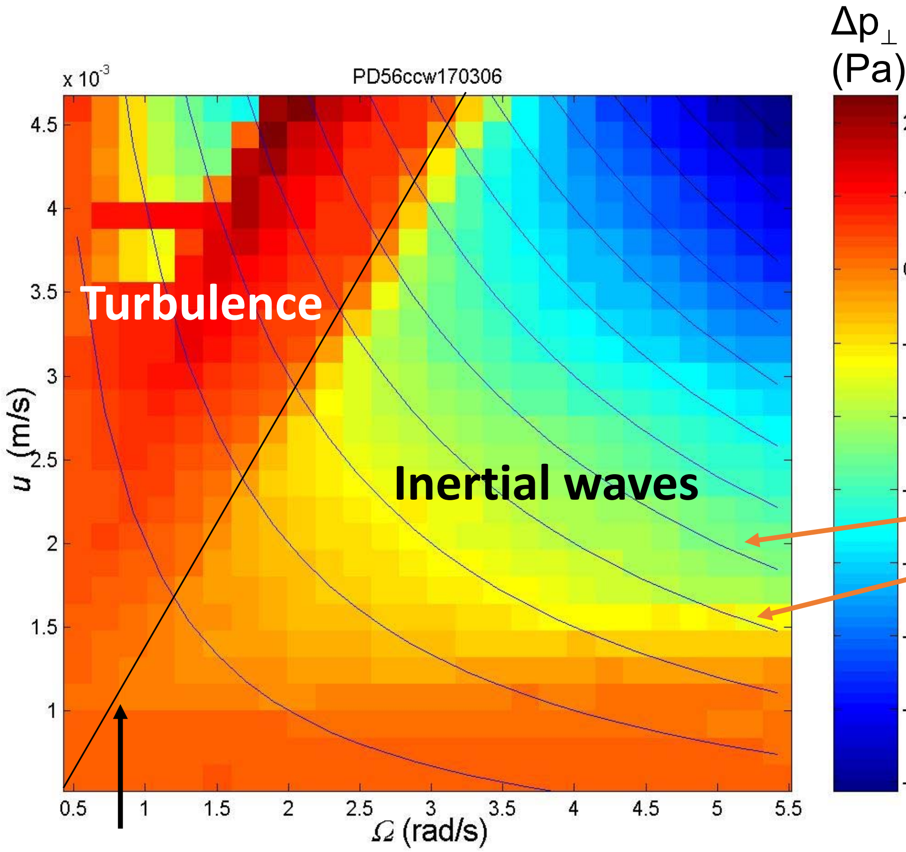
Measure: differential pressure $\Delta p_{\perp}$
& flow rate **q**

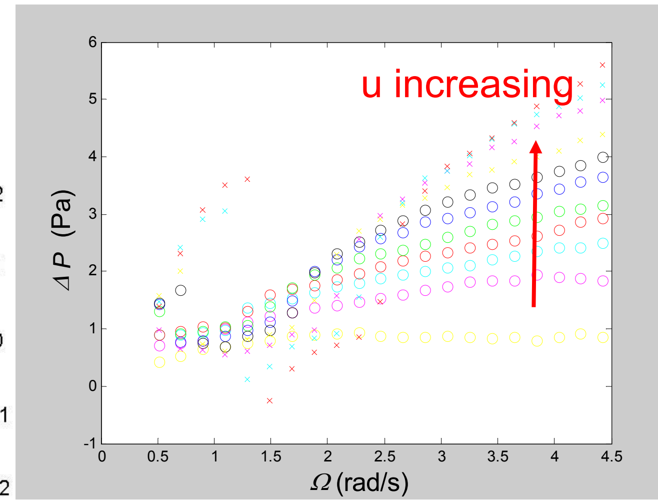
$\Delta p_{\perp}$

q

Can waves, excited at ω=u/L, modify throughflow under tilt?

Geostrophy? Expect isobars along hyperbola, u $\propto \Omega^{-1}$





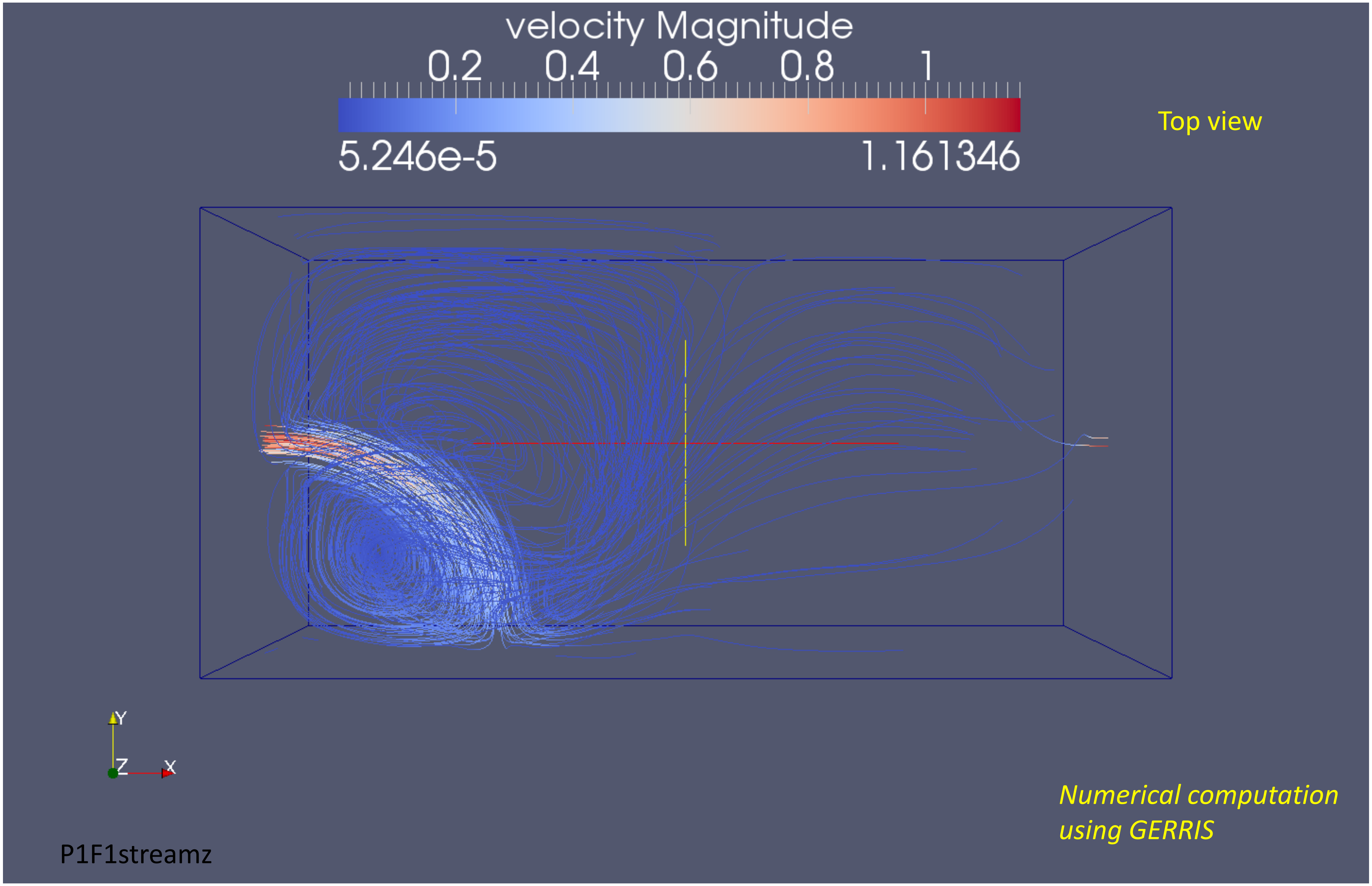
Geostrophy:

$$2\Omega u = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

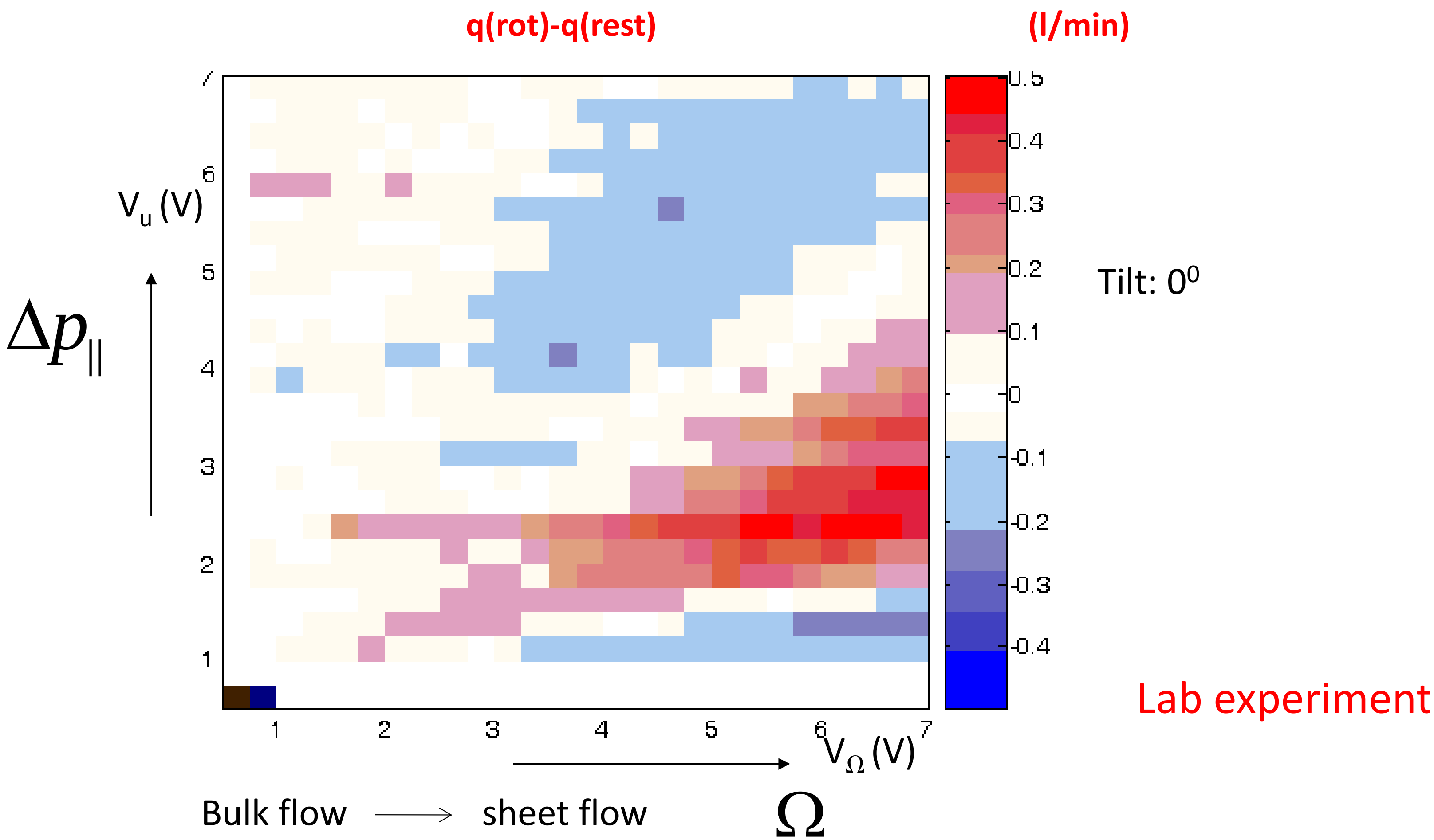
Ro = U/2ΩL = const If L = boundary layer scale → Ro = 1

M. 2007

Reminiscent of transition in grid-generated turbulence, Hopfinger et al 1982


velocity Magnitude
0.2
0.4
0.6
0.8
1
5.246e-5
1.161346
Top view
Y
Z
X
Numerical computation
using GERRIS
P1F1streamz

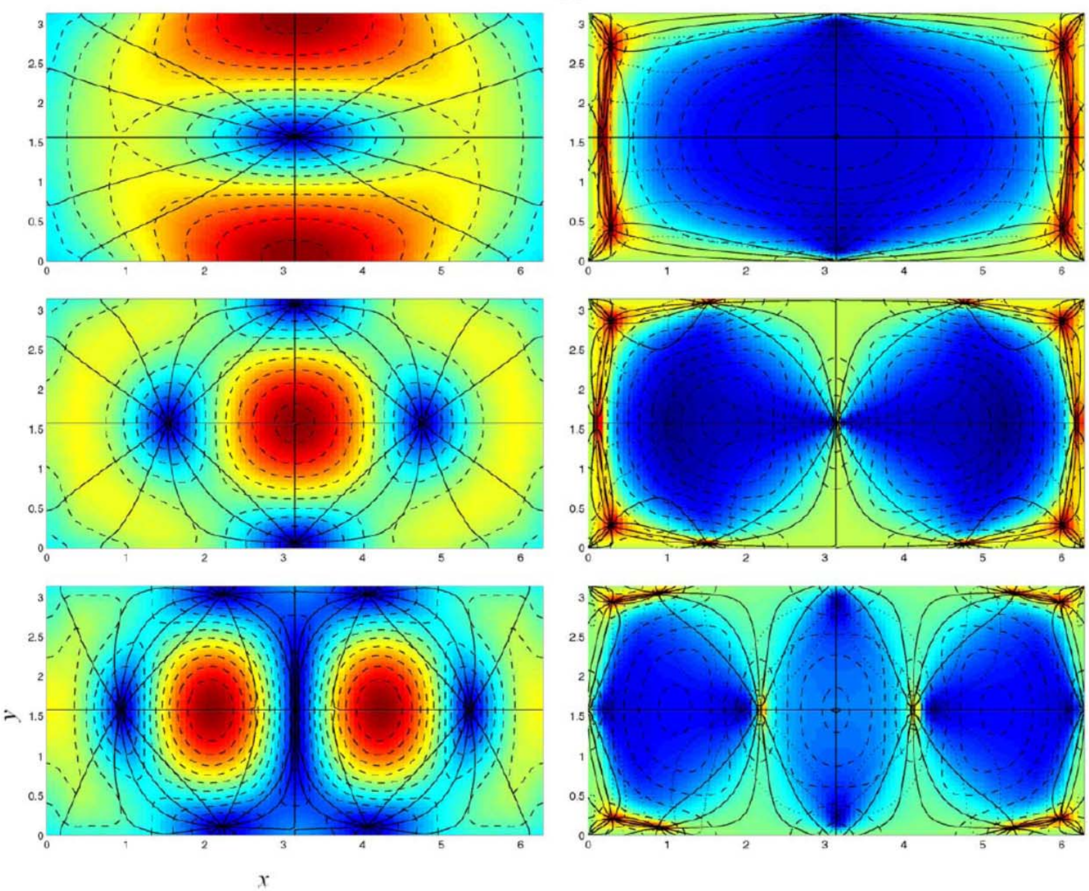
Throughflow, q, affected during rotation?



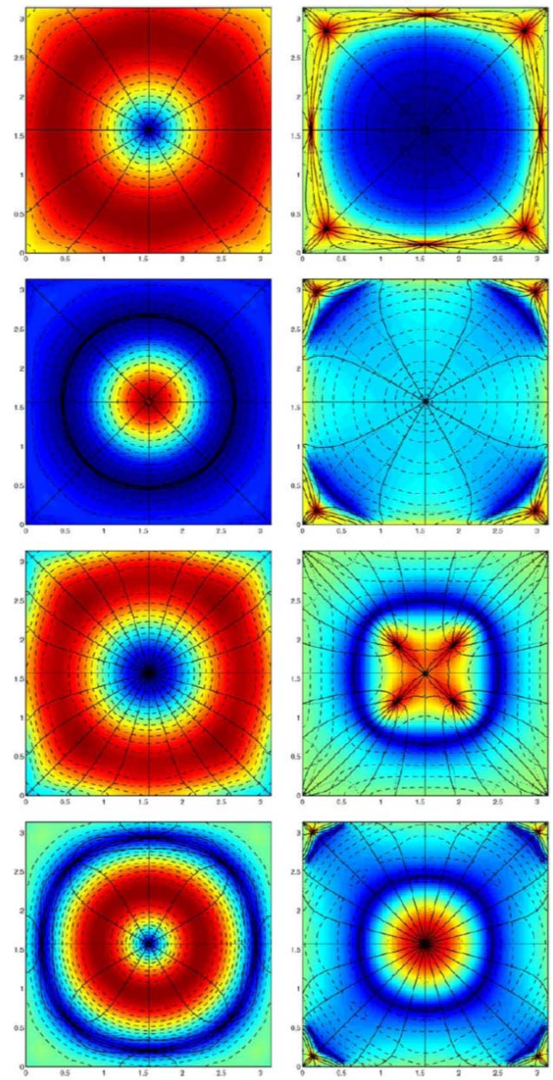
Non-uniform **Density of States**?

$\zeta, \quad w = \frac{d\zeta}{dt}$ $\qquad$ Ω $\qquad$ u, v

Eigenmodes
rotating box

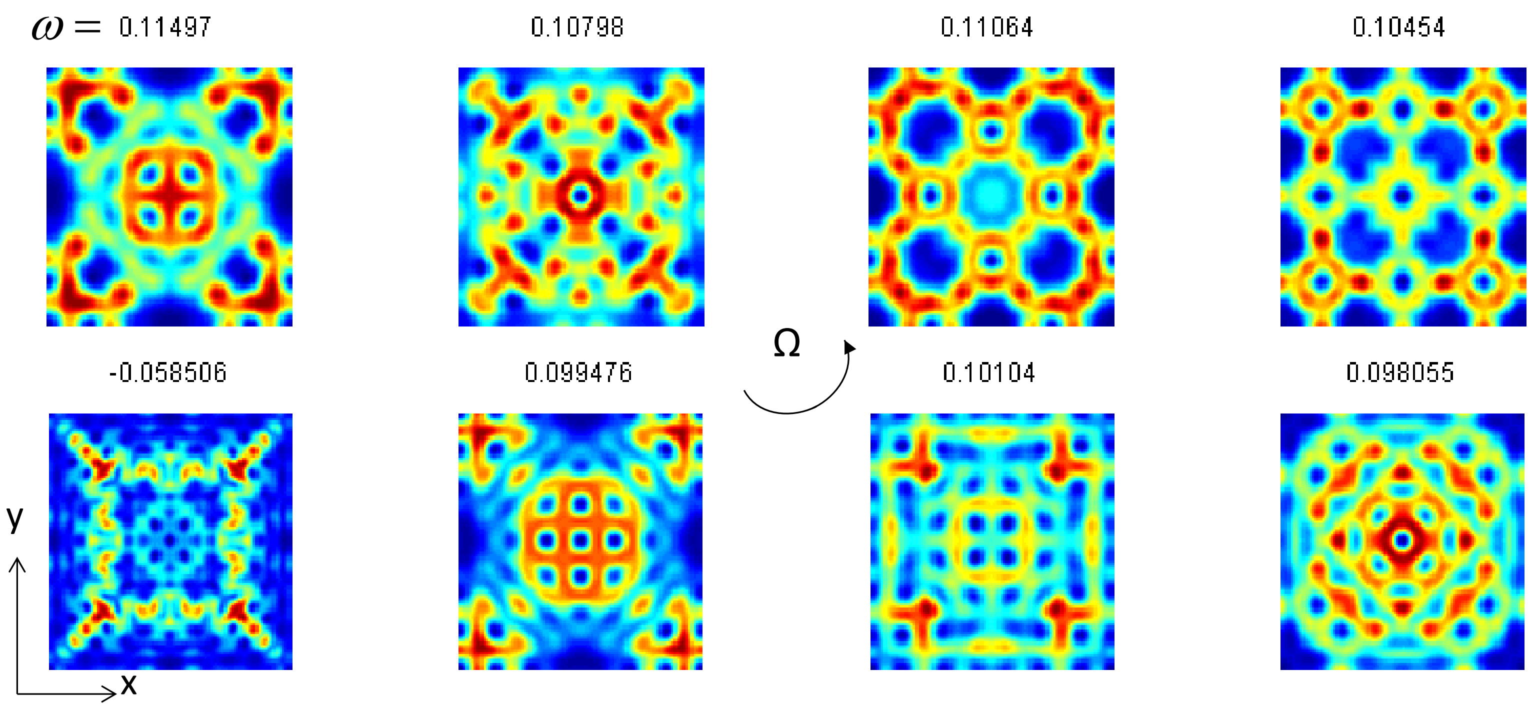


ζ $\qquad$ u, v



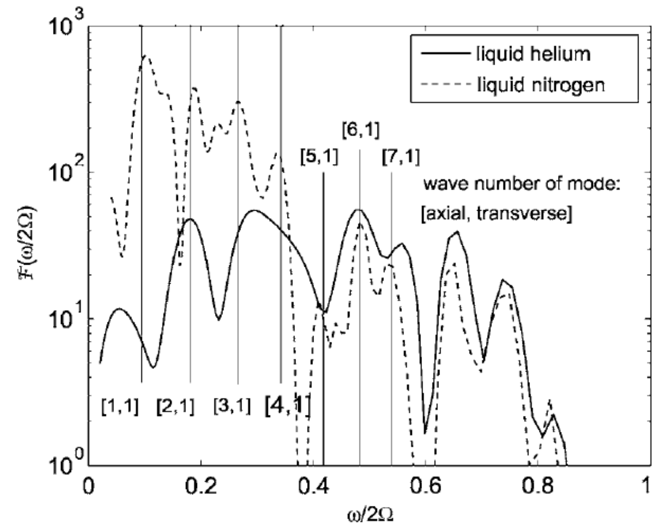
M. 2003

Energy distribution low-frequency modes rotating cube



Generalized eigenvalue problem - but, every (?!) $\omega \in (0,1)$ is eigenfrequency
Each 'physical mode': infinite Fourier series
Competition circular-rotational, versus geometric-rectangular symmetry

Experimental confirmation

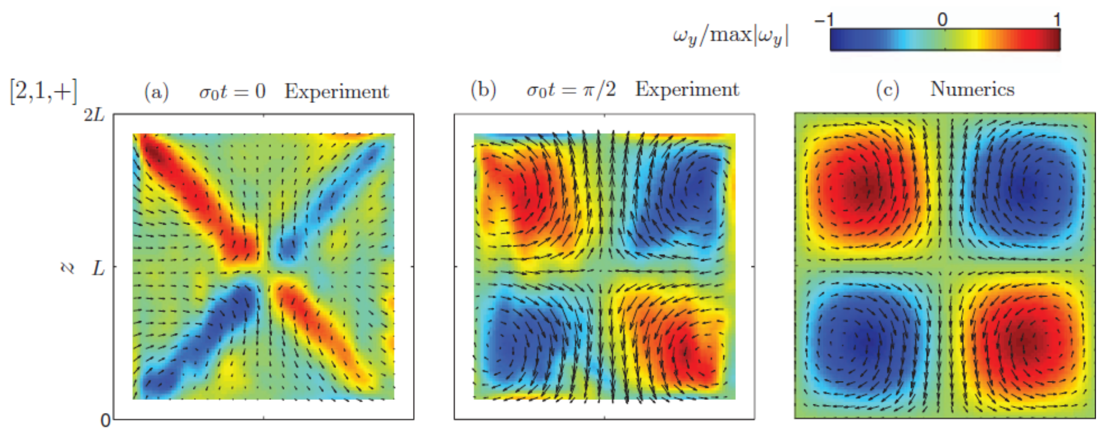


Eigenfrequencies:

Decaying, rotating grid 'turbulence'

Bewley, Lathrop, M., Sreenivasan 2007

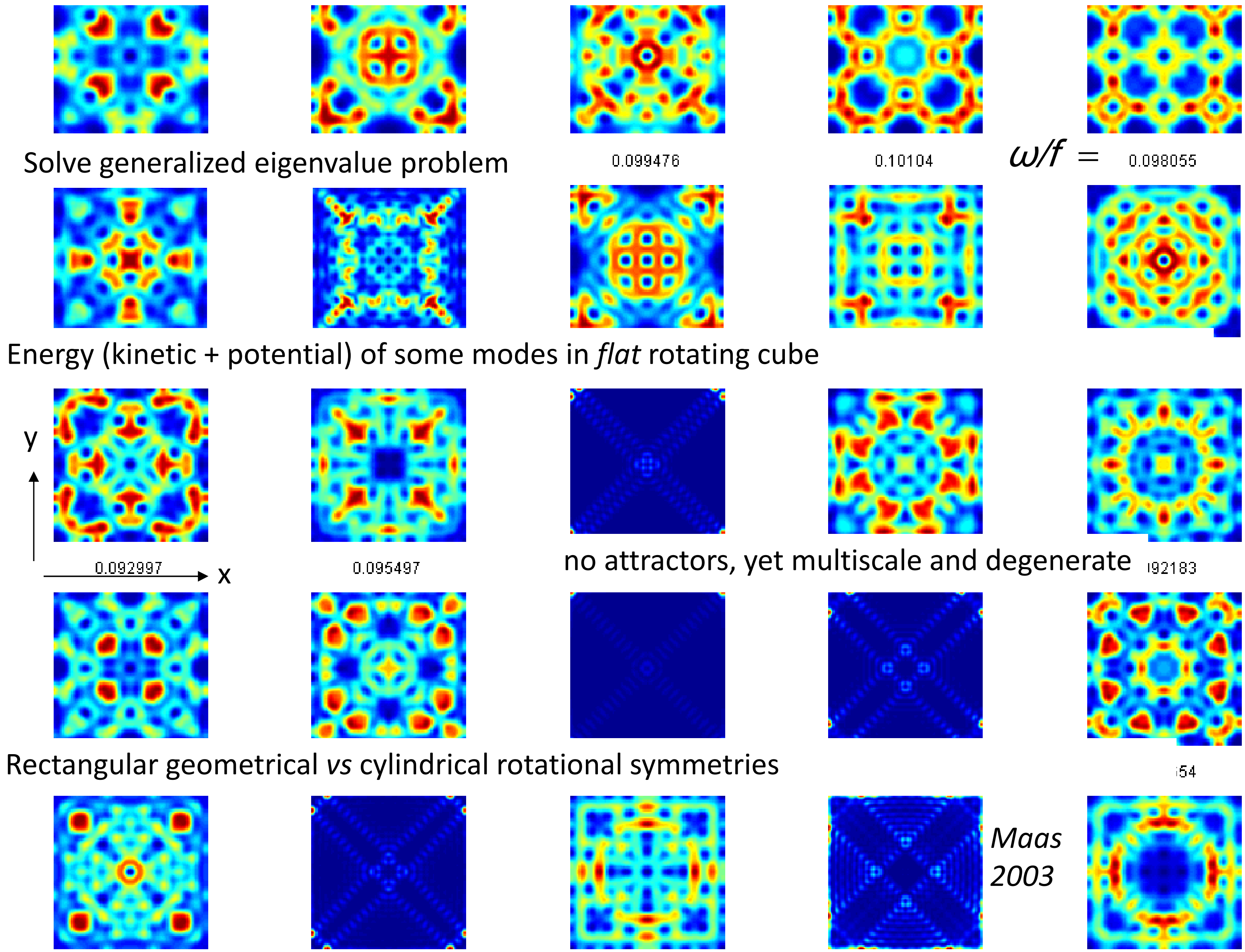
Lamriben, Cortet, Moisy & M. 2011



Eigenmode:

Libration of rotating cube

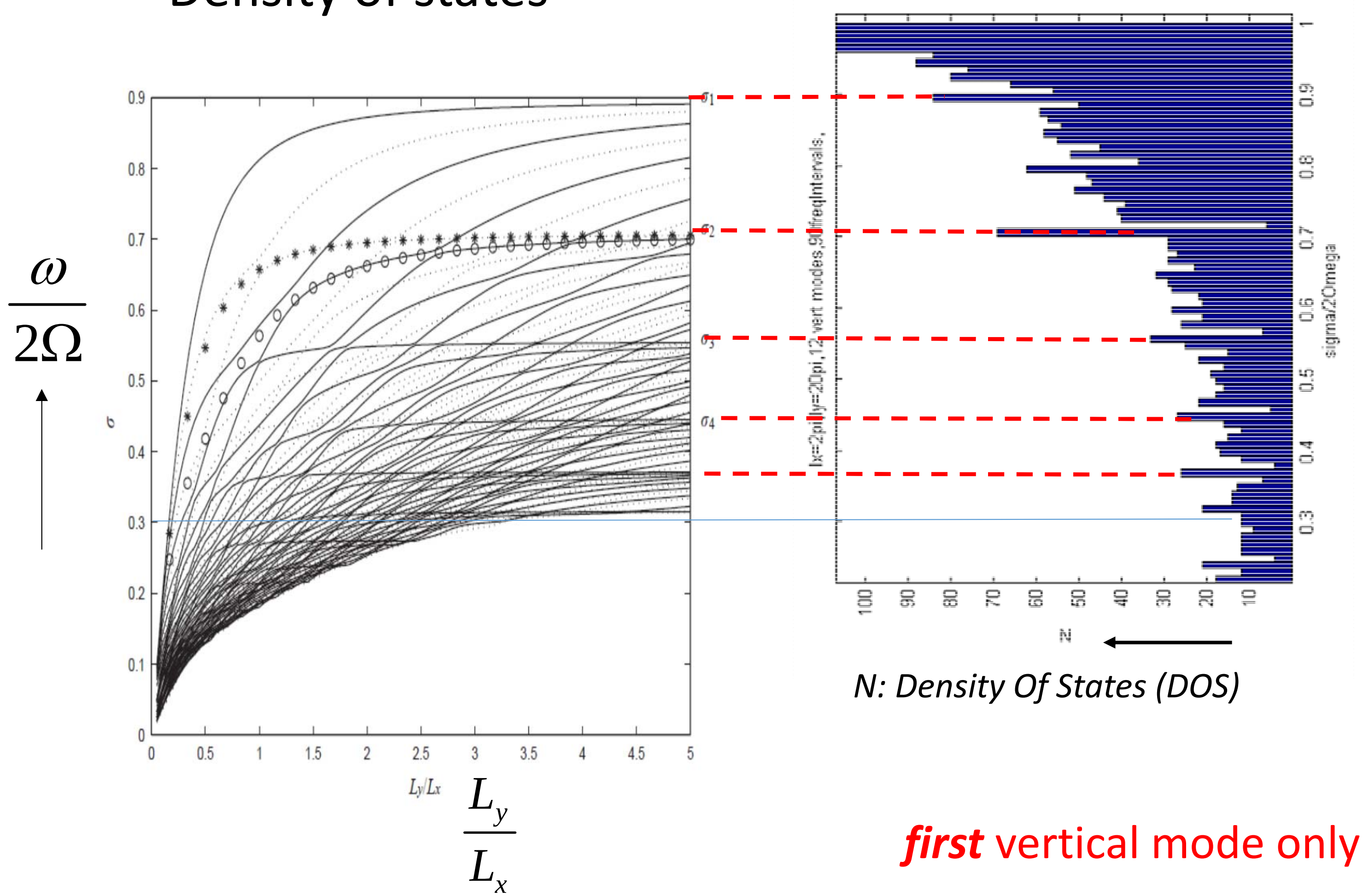
Boisson, Lamriben, M., Cortet & Moisy 2012



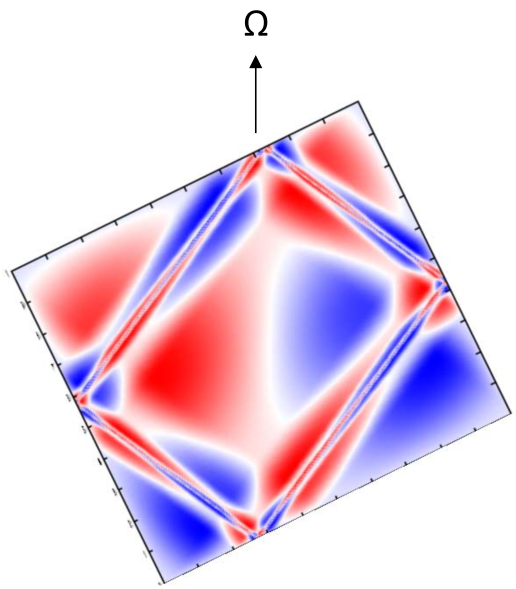
Solve generalized eigenvalue problem
0.099476
0.10104
ω/f =
0.098055
Energy (kinetic + potential) of some modes in *flat* rotating cube
y
x
0.092997
0.095497
no attractors, yet multiscale and degenerate
Rectangular geometrical *vs* cylindrical rotational symmetries
Maas 2003

Density of states

Eigenfrequencies first 1200 modes of a rotating 1X1X10 rectangular box

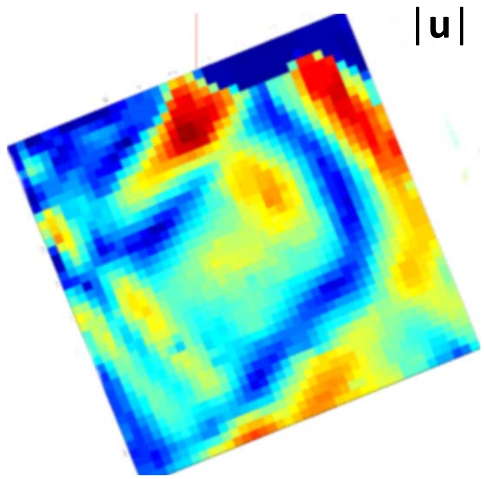


‘Impurity’ rotating box: *Tilt*, leading to focusing on wave attractor

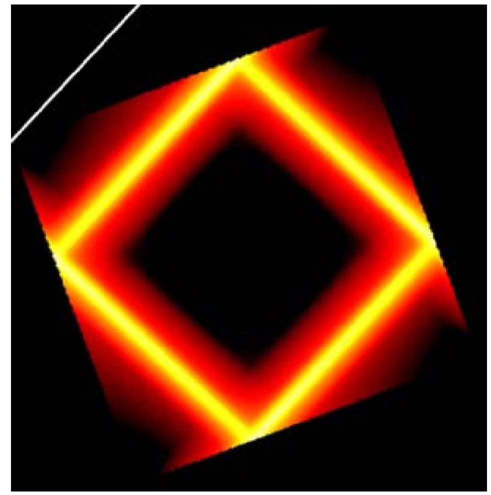


In tilted rectangle, inertial waves focus onto attractor - *Kopecz, 2006*

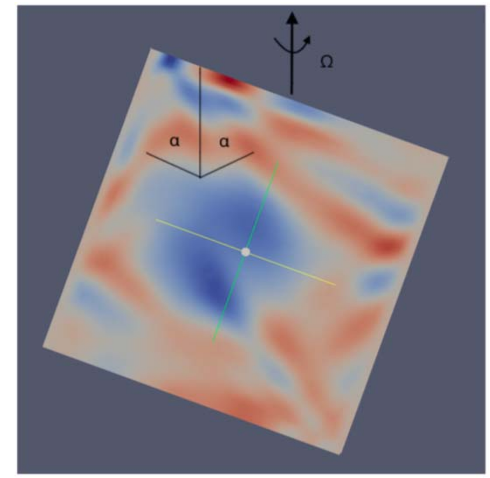
2D Stream function



Lab. experiment modulating rotation - *Manders & M. (unpubl.)*



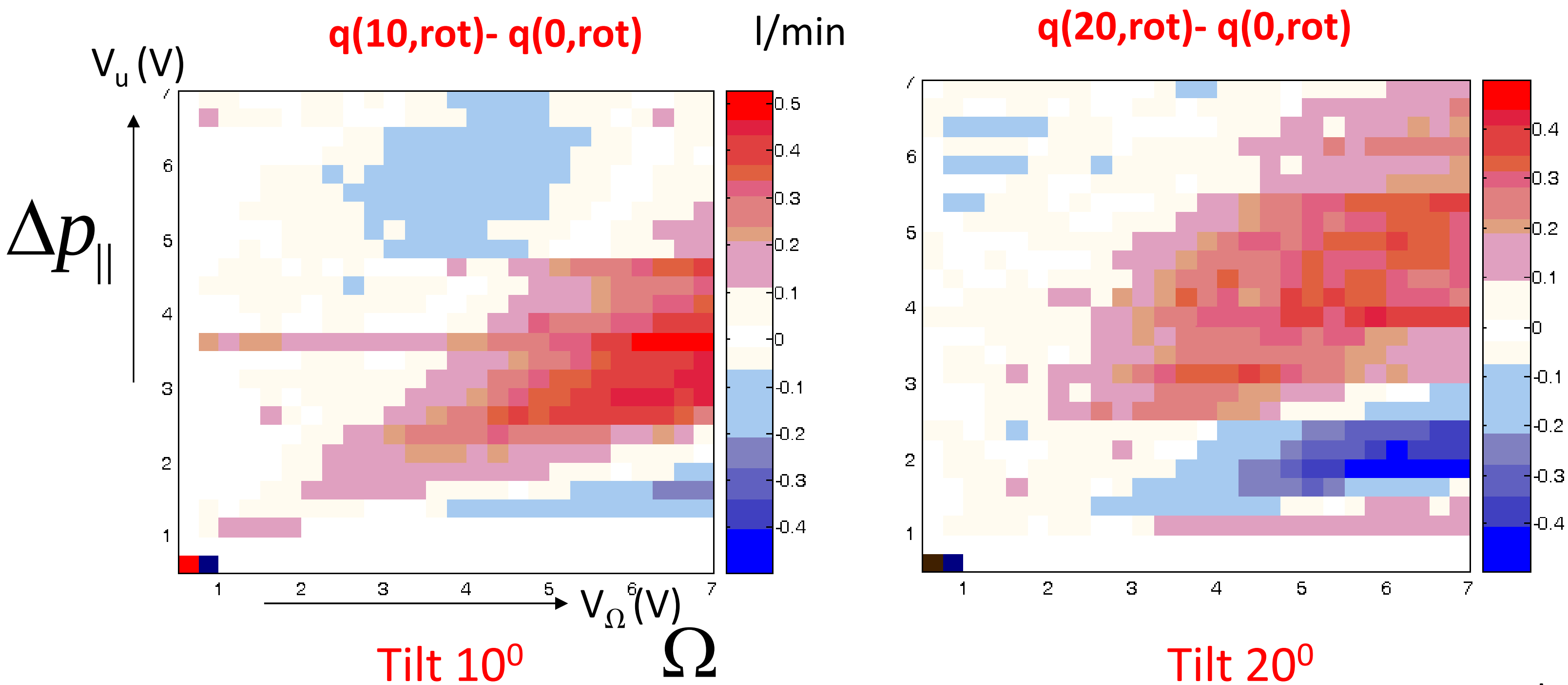
Approximate numerical soln. periodically forced viscous eqn. - *Ogilvie 2005*



GERRIS solution midplane

Velocity magnitude

Extra flux (q) in rotating channel due to tilt

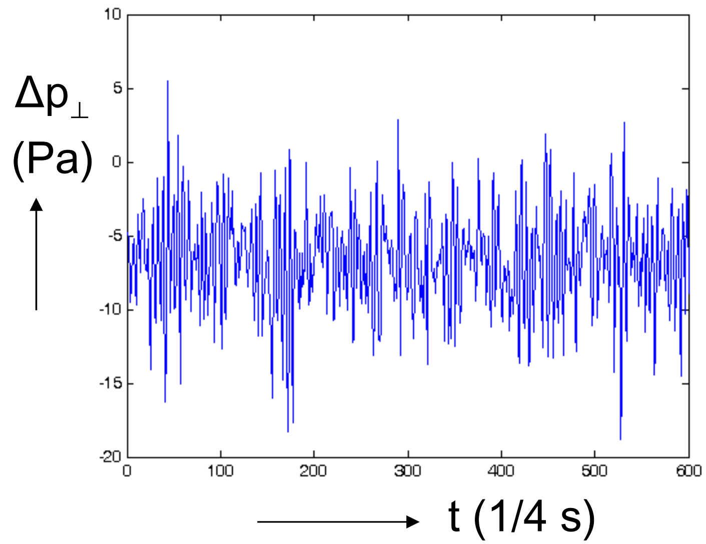


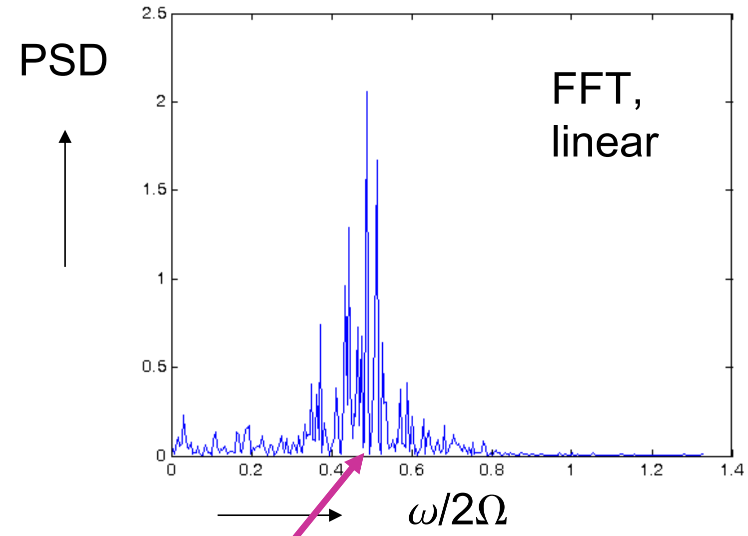
Lab experiment

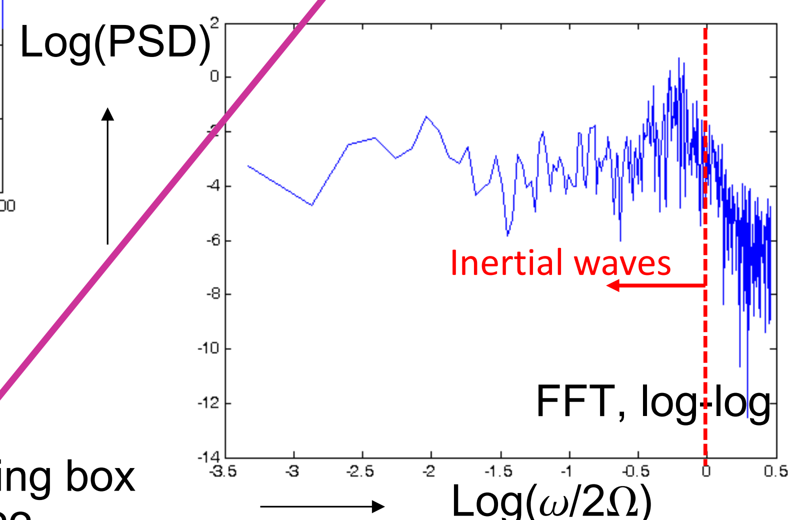
Channel tilt amplifies throughflow when inertial waves dominate

M. 2007

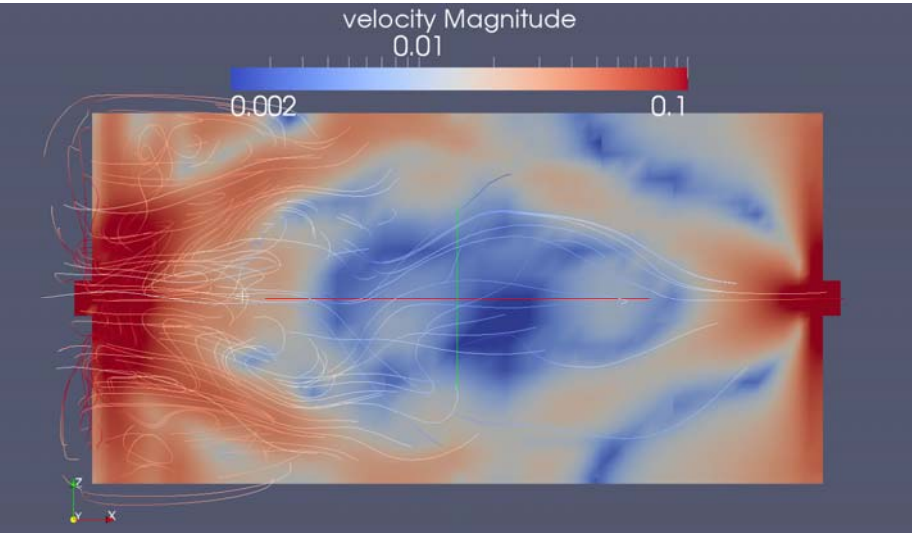
Single 150 s experiment
with 10^o tilt
V_Ω=5V, V_u=4V





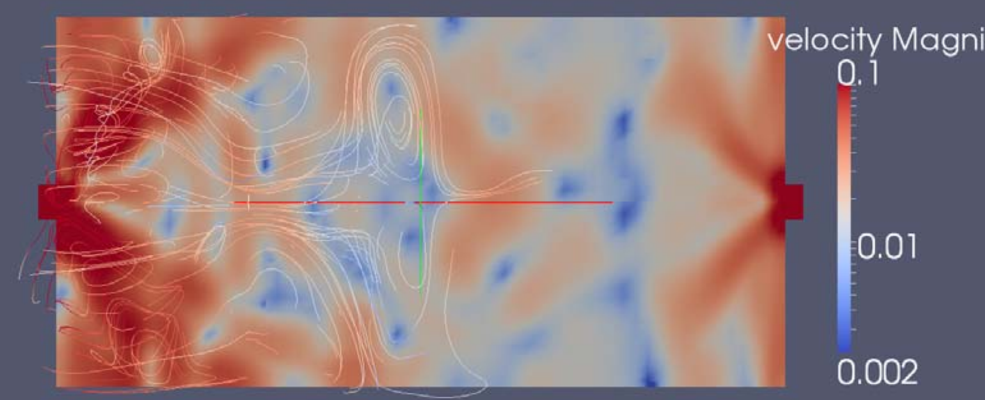


Eigenmodes rotating box
Attractor regimes??



Side views

Higher rotation rate

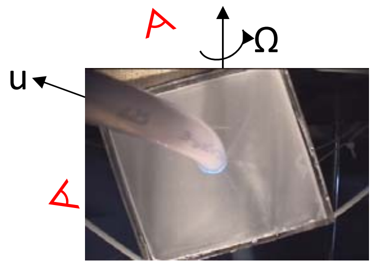


Indications inertial waves generated by turbulence

GERRIS

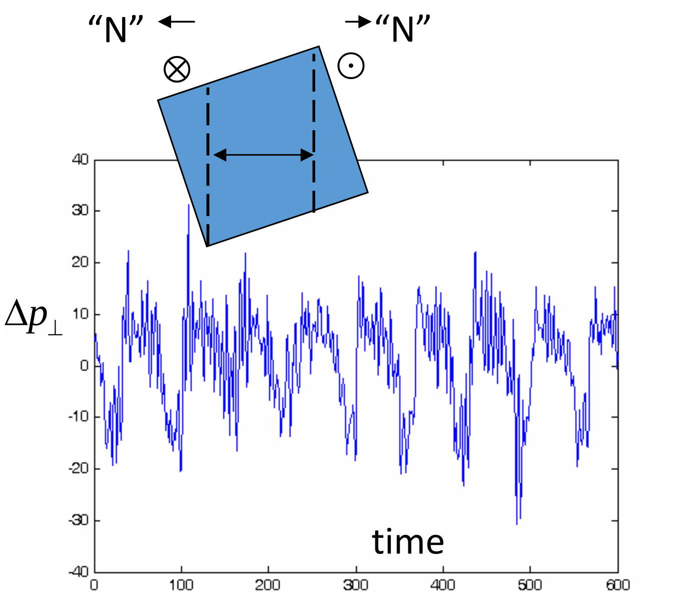
Experiment on topographic Rossby wave

Tilted rectangular channel

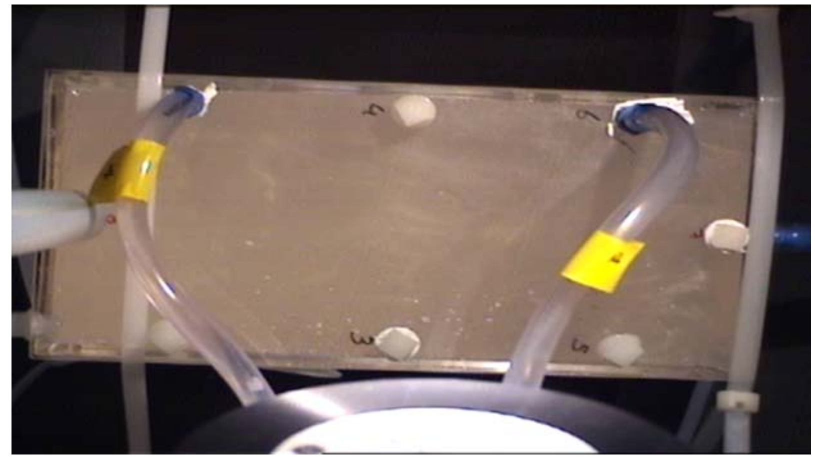


Vortex tube
Compression

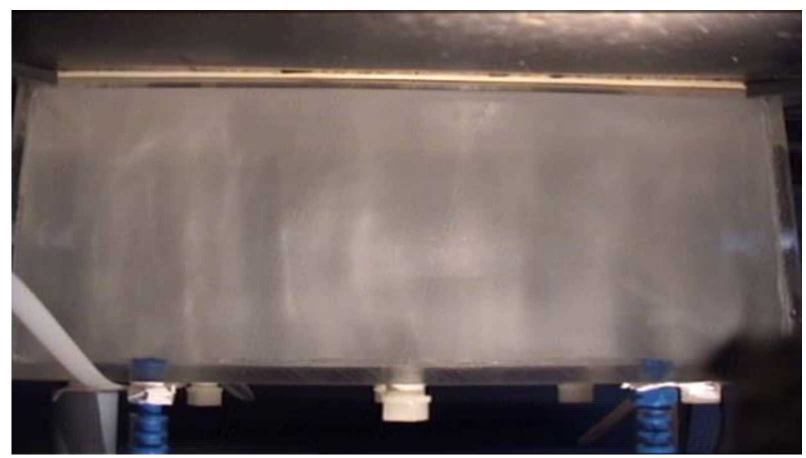
End view



Top view



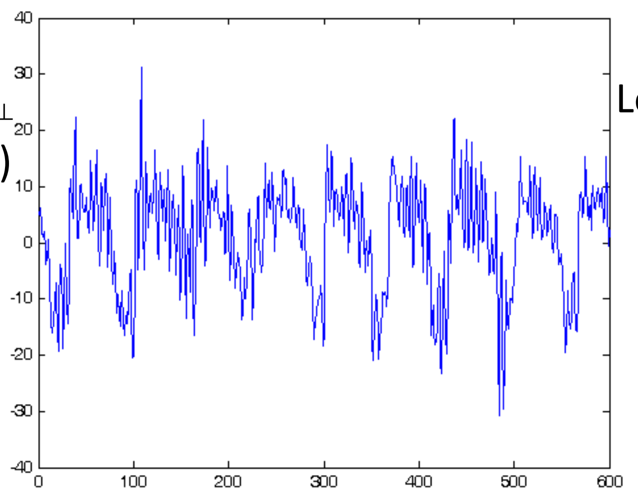
Side view



Transverse pressure in rotating tilted box

Rossby (R) and inertial (I) waves

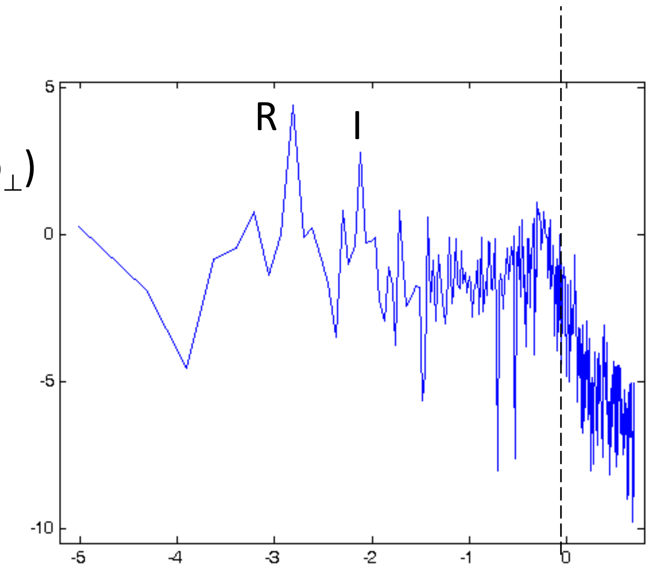
$\Delta p_\perp$ (Pa)



time (1/4 s)

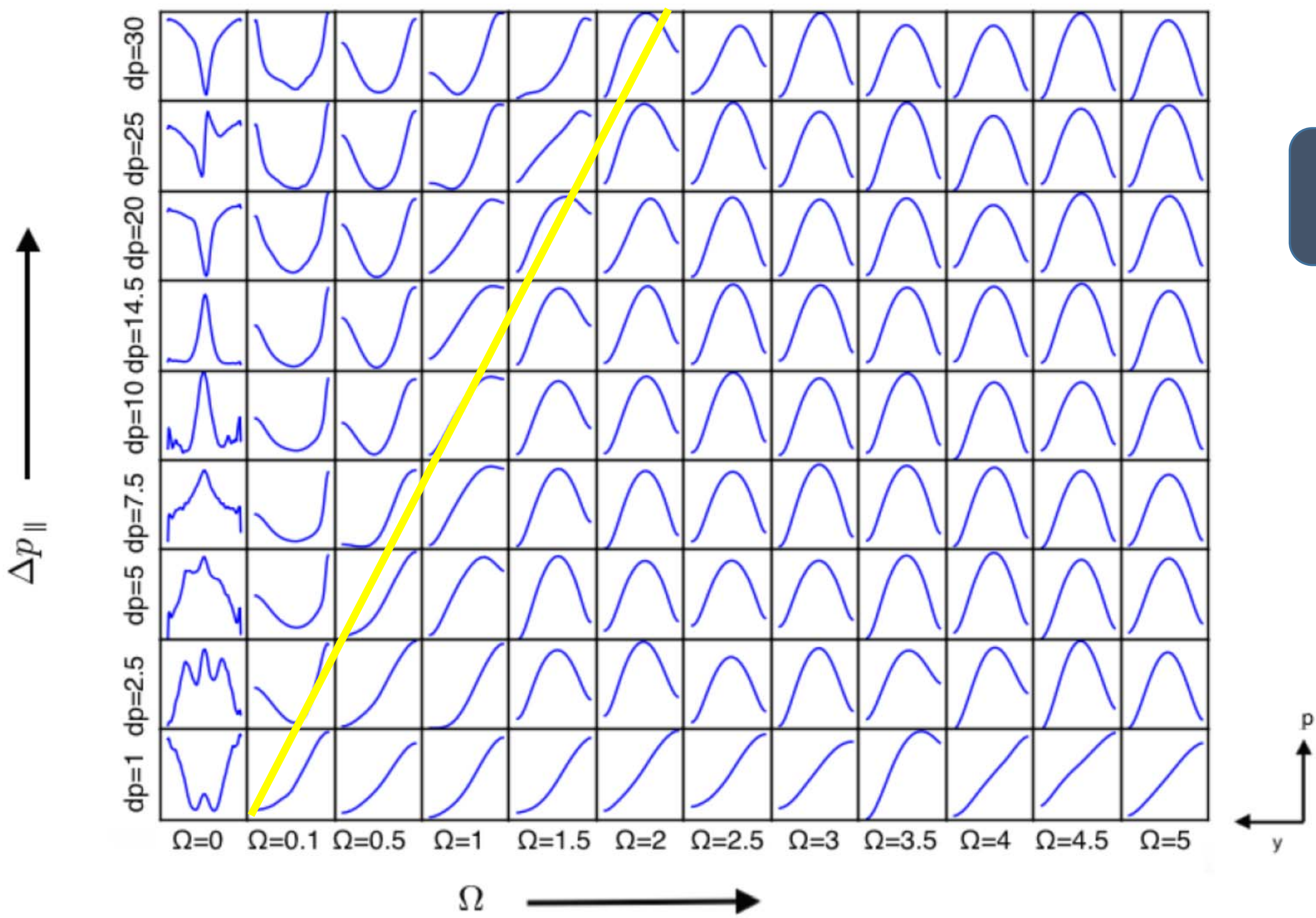
Large pump & rotation rates

$\text{Log}(\Delta p_\perp)$

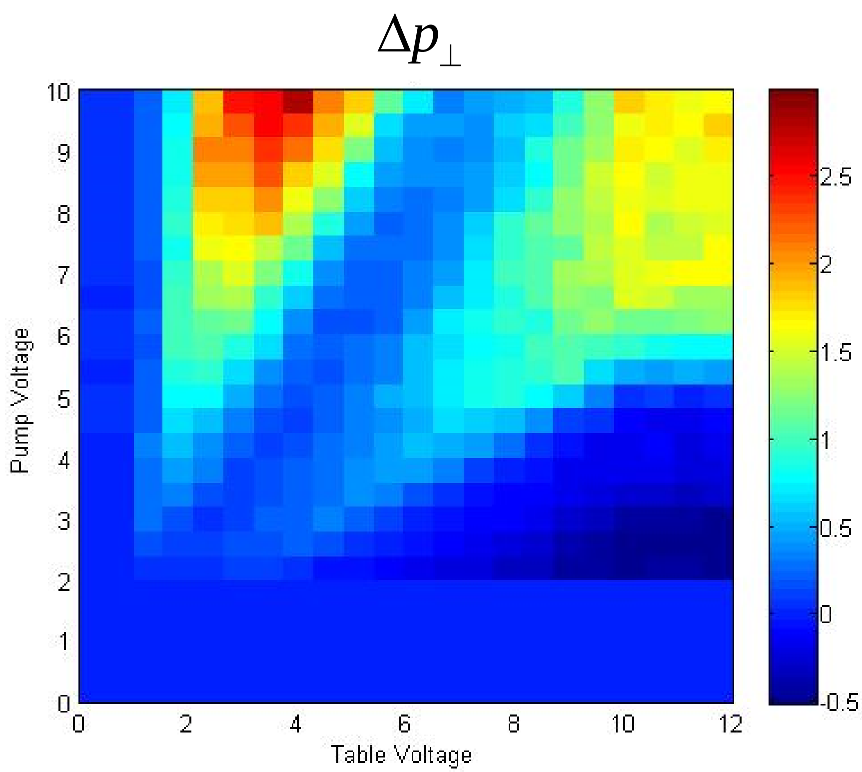


$\text{Log}(\omega/2\Omega)$

Vertically averaged pressure in transverse plane p(y)



GERRIS



20° tilt, clockwise

Tilt leads to *multiple* regions of strong lateral pressure differences

Needs scaling by *observed* flow..

Conclusions

Flow through box may be enhanced (LAB) or reduced (LAB,NUM) due to rotation

NUM: mass conservation problems, strong forcing, numerical dissipation

Flow through rotating box $\rightarrow$ participation eigenmodes $\rightarrow$ flow change (LAB & NUM)

Response in box dominated by inertial waves: extended, or localized?

In tilted box: focusing & attractors and Rossby waves

Plateau formation Geostrophic flow: unclear.