Energy cascade in internal wave attractors

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in collaboration with
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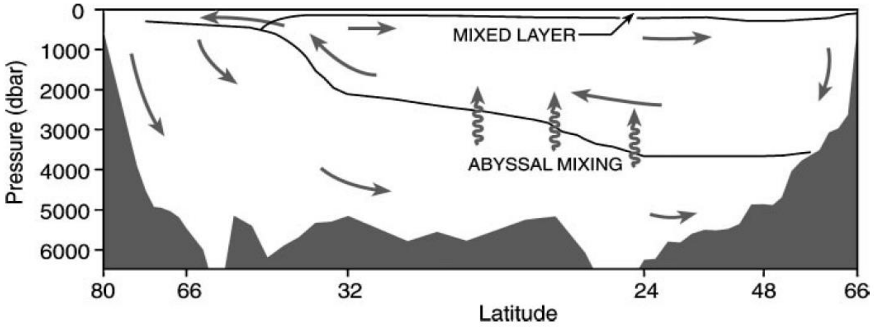
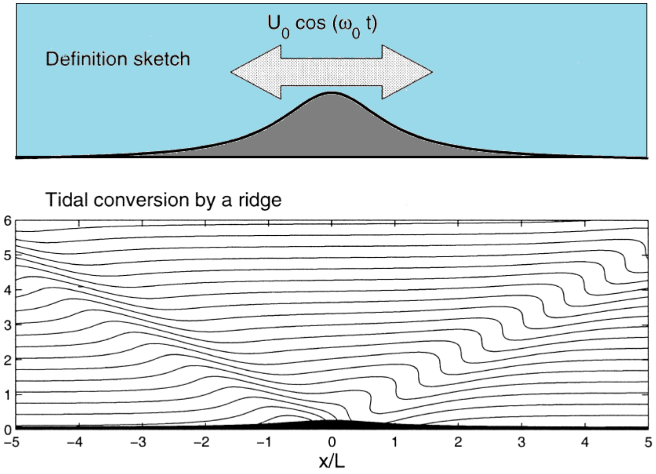
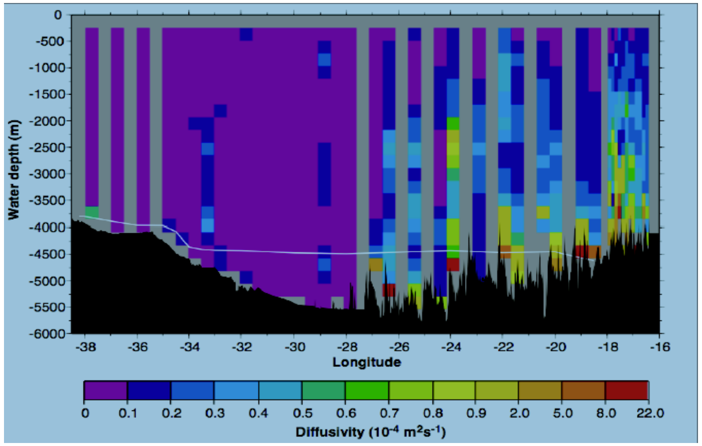
Motivation



Meridional circulation in the ocean:
Wunsch & Ferrari (Annu. Rev. Fluid Mech. 2004)





Tidal conversion at a ridge:
Llewellyn Smith & Young (JPO 2002), Bell (JFM 1975)

Diapycnal diffusivity in Brazil Basin:
Polzin *et al*. (Science 1997)

Dispersion relation

$$N(z) = \left[-(g/\rho)\, d\rho/dz\right]^{1/2}$$

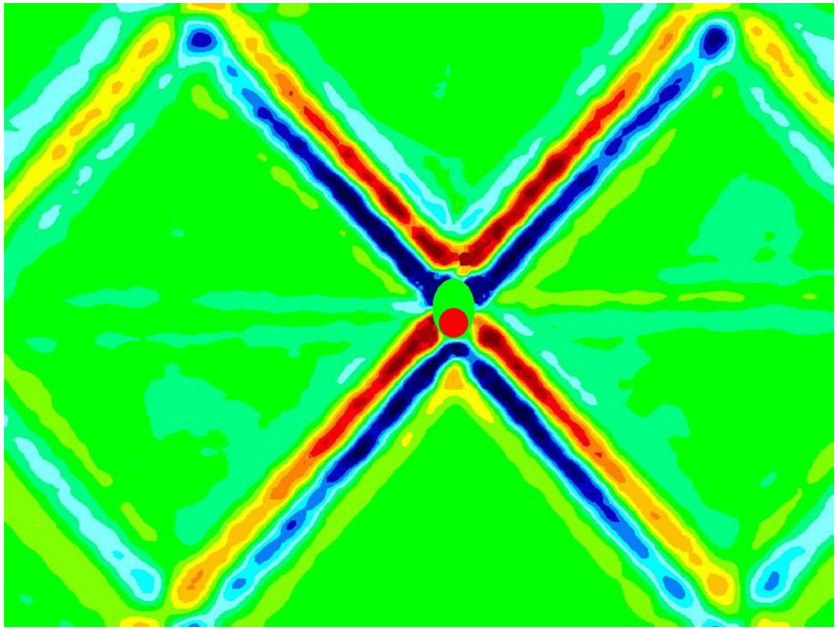
Buoyancy frequency

$$\Omega = \omega / N$$

Forcing frequency

$$\Omega = \frac{\omega}{N} = \pm \sin\theta$$

Dispersion relation



Internal waves emitted by oscillations of a circular cylinder
(color shows density gradient perturbations)

Geometric focusing

$$N(z) = \left[-(g/\rho)\, d\rho/dz\right]^{1/2}$$

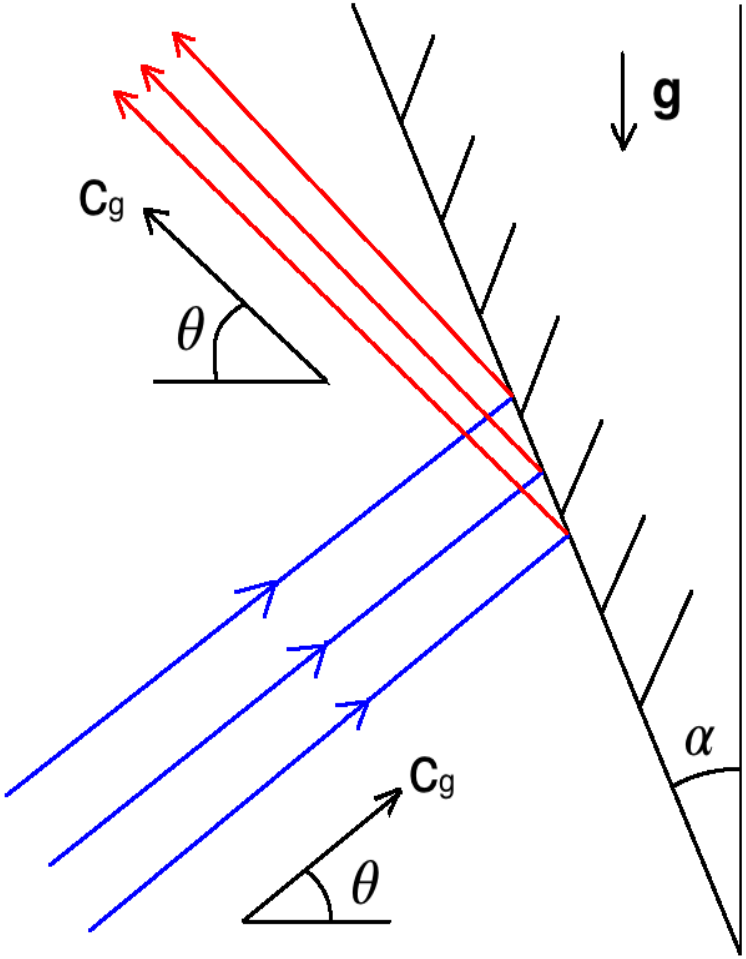
Buoyancy frequency

$$\Omega = \omega / N$$

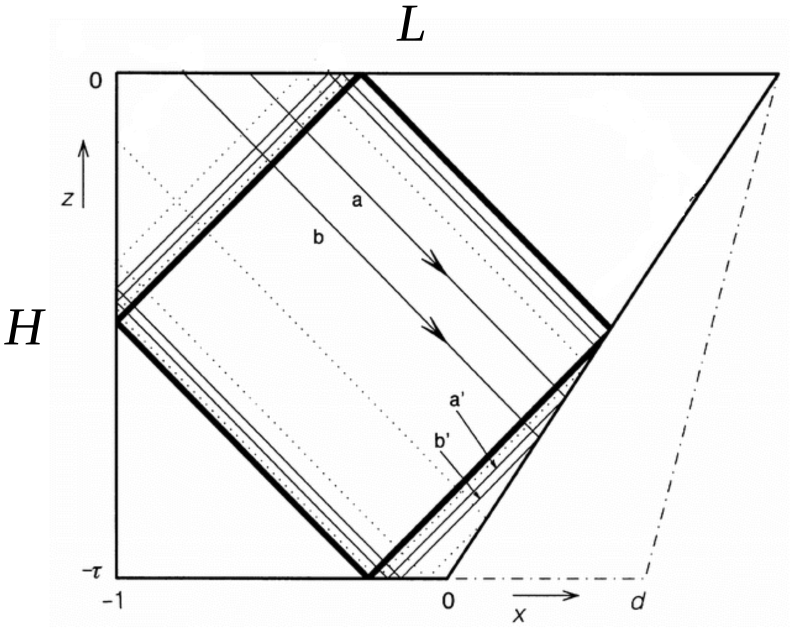
Forcing frequency

$$\Omega = \frac{\omega}{N} = \pm \sin\theta$$

Dispersion relation

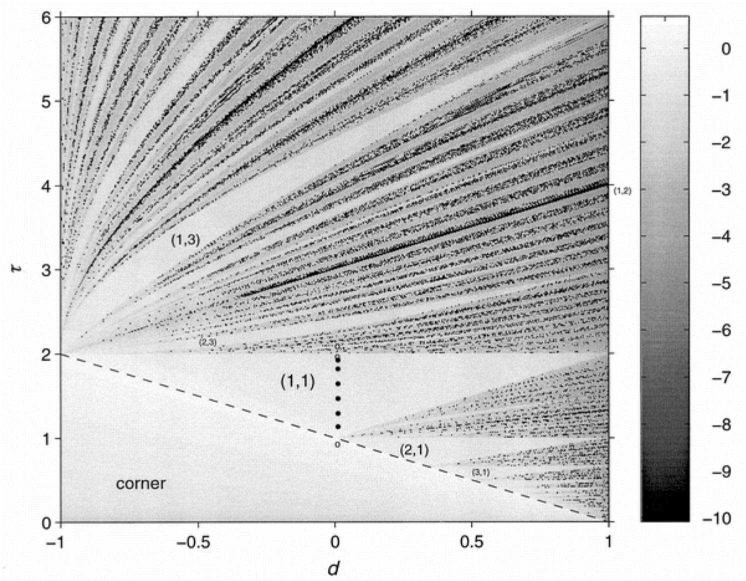


Formation of attractors in a basin with a slope



$$\tau = \left(\frac{1-\Omega^2}{\Omega^2}\right)^{1/2} \frac{H}{L}$$ control parameter of forcing

d control parameter of slope (from -1 to +1)



(d, τ) - diagram of regimes

Grey scale shows the value of Lyapunov exponents

Maas & Lam (JFM 1995)
Maas, Benielli, Sommeria & Lam (Nature1997)

Attractor in viscous stratified fluid
Linear dynamics

$$\tau = \left(\frac{1-\Omega^2}{\Omega^2}\right)^{1/2}\frac{H}{L}$$

d

ideal fluid

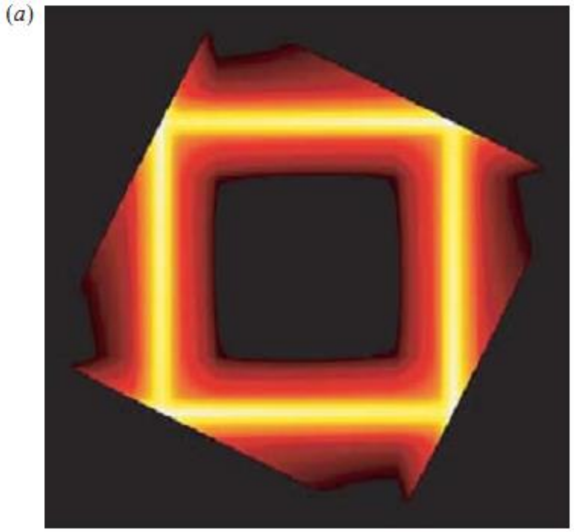
$$\frac{H^2N}{\nu}$$

Stokes number

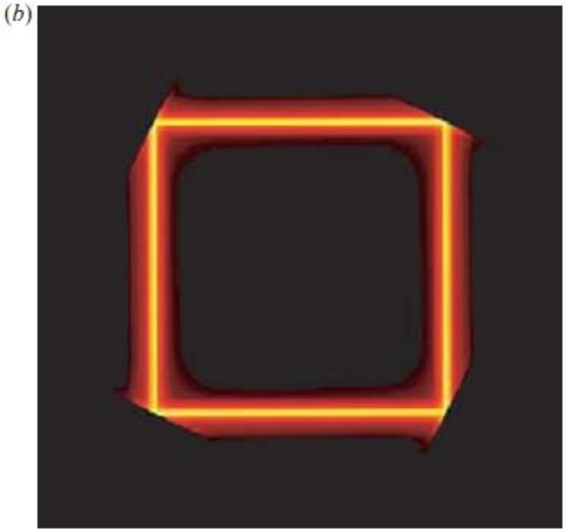
viscous fluid

Key mechanism:

Geometric focusing *versus* viscous broadening = equilibrium width of attractor beams



lower Stokes number



higher Stokes number

Ogilvie (JFM 2005)
Hazewinkel, van Breevoort, Dalziel & Maas (JFM 2008)
Grisouard, Staquet & Pairaud (JFM 2008)

Attractor in viscous fluid with mixing
Nonlinear dynamics

$$\tau = \left(\frac{1-\Omega^2}{\Omega^2}\right)^{1/2}\frac{H}{L}$$

$$d$$

ideal fluid

$$\frac{H^2N}{\nu}$$ Stokes number

$$\frac{a}{H}$$ forcing amplitude

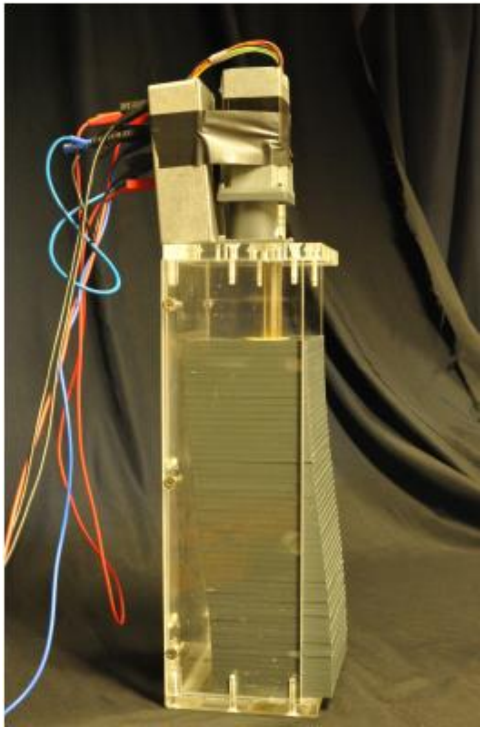
$$Sc = \frac{\nu}{\kappa}$$ Schmidt number = 700

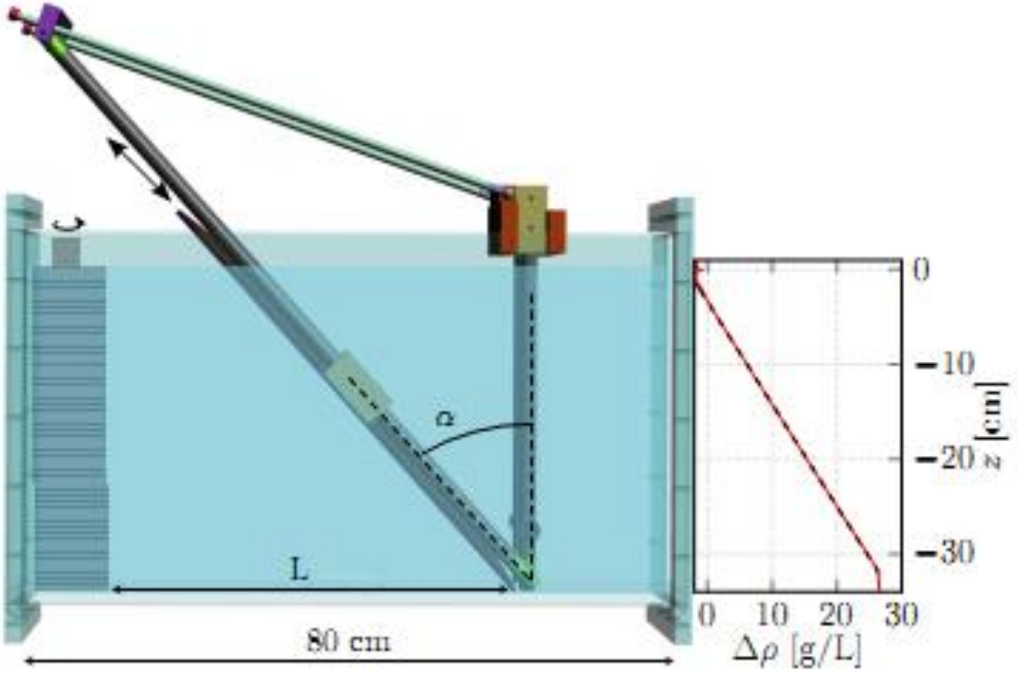
parameters
of the problem

Goal: energy cascade in wave attractors

Experimental setup

Wave generator



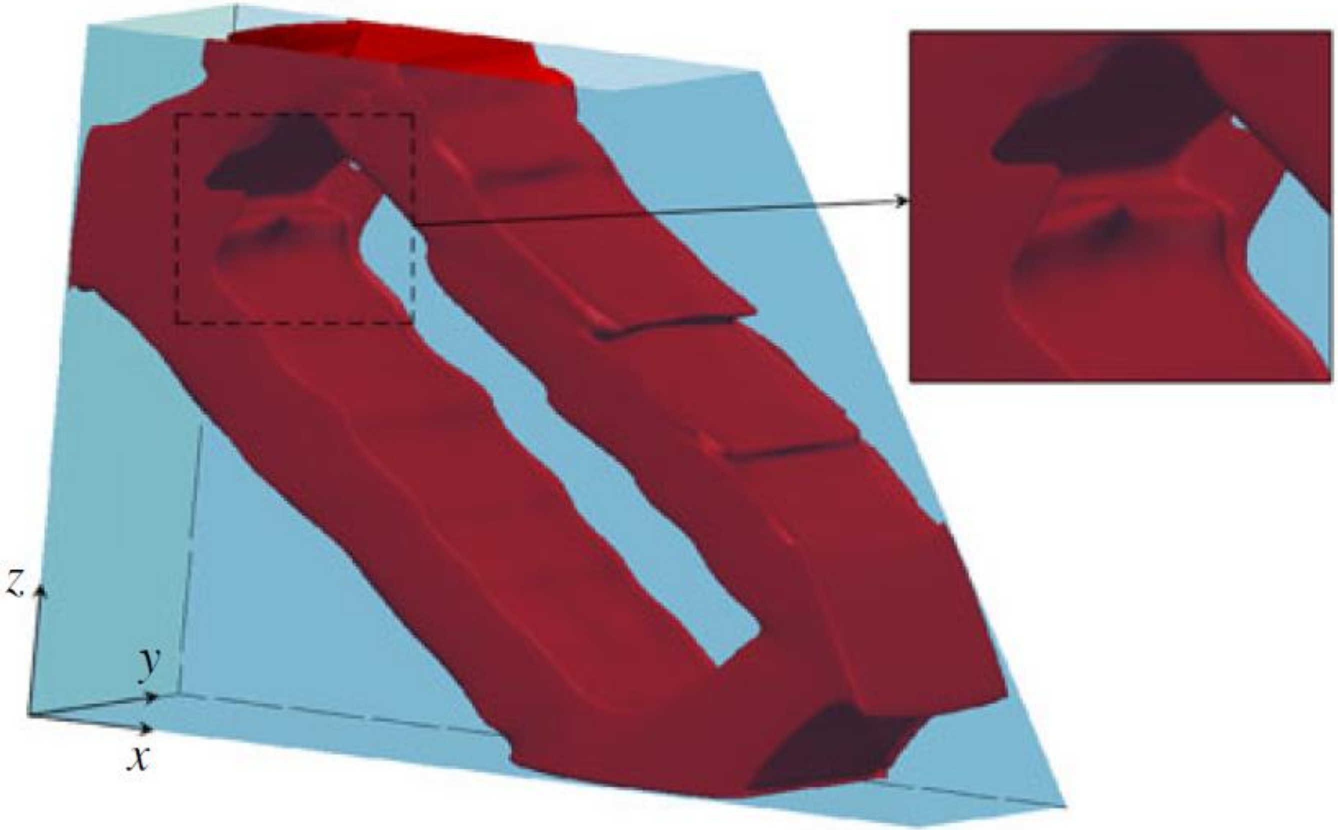


Generator profile:

$$\eta(z,t) = a\cos(\pi z/H)\cos(\omega_0 t)$$

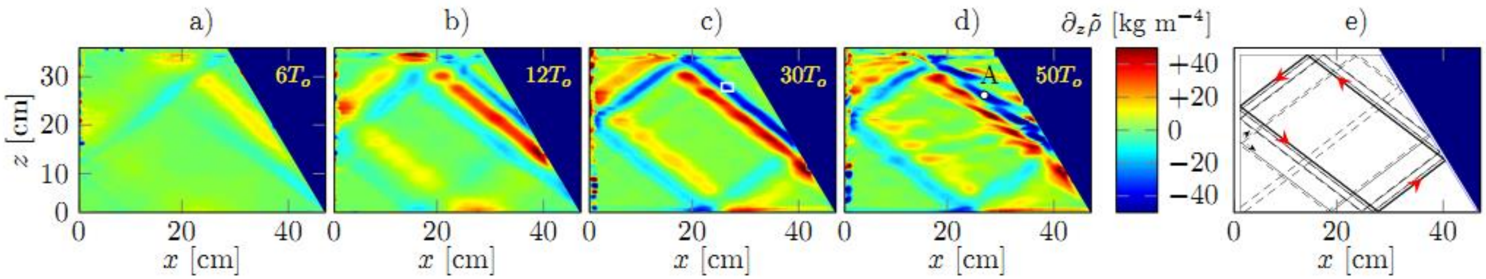
Measuring techniques: Synthetic Schlieren and PIV

Numerical calculations



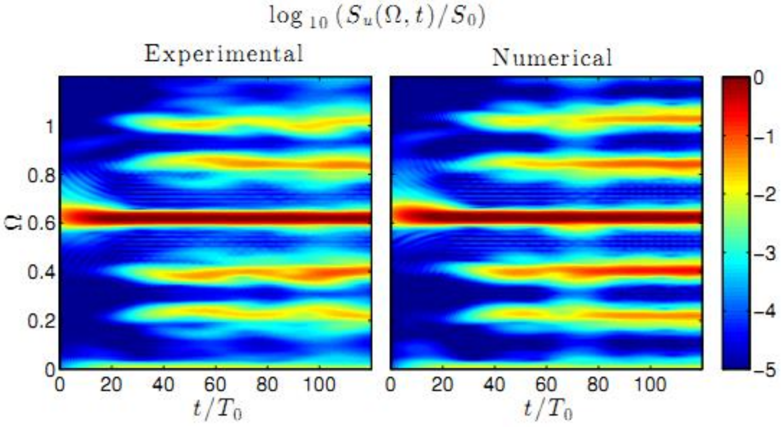
Model: Navier-Stokes in Boussinesq approximation + continuity + salt transport
Method: spectral elements 2D and 3D, code Nek5000 (Fischer & Ronquist 1994)
BC: no-slip at rigid walls, stress-free at free surface

Development of triadic resonance



Scolan, Ermanyuk & Dauxois (PRL 2013)

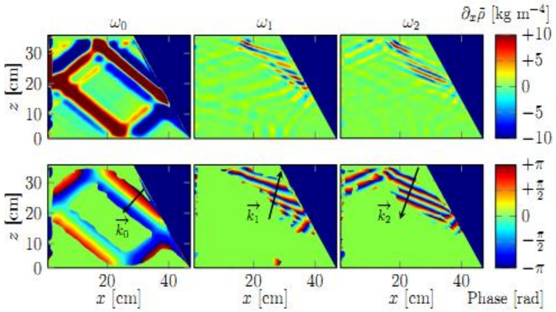
$$\omega_0 = \omega_1 + \omega_2$$



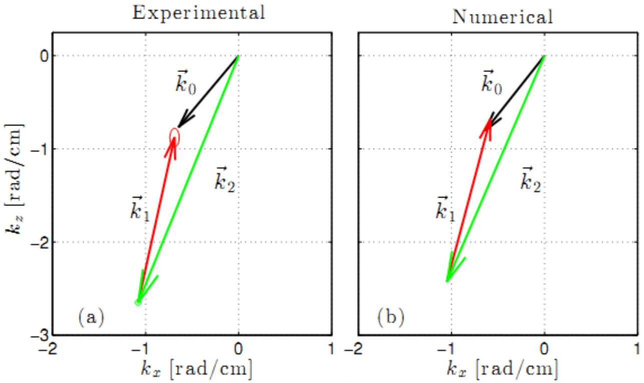
Time-frequency diagram

$$S_r(\omega,t) = \left\langle \left| \int_{-\infty}^{+\infty} v_r(x,z,\tau)\, e^{i\omega\tau} h(t-\tau)\, d\tau \right|^2 \right\rangle_{xz}$$

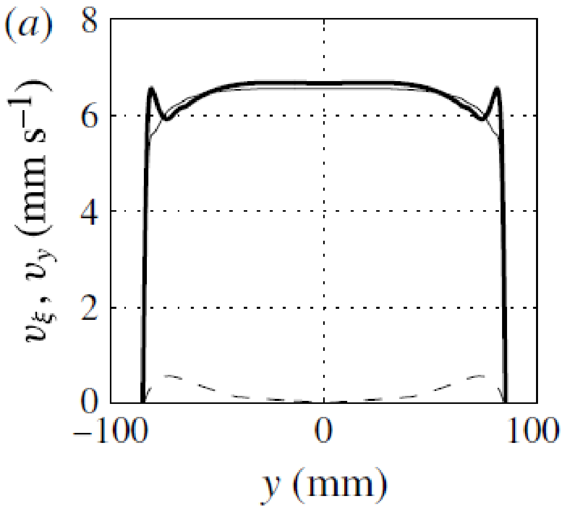
$$\mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2$$



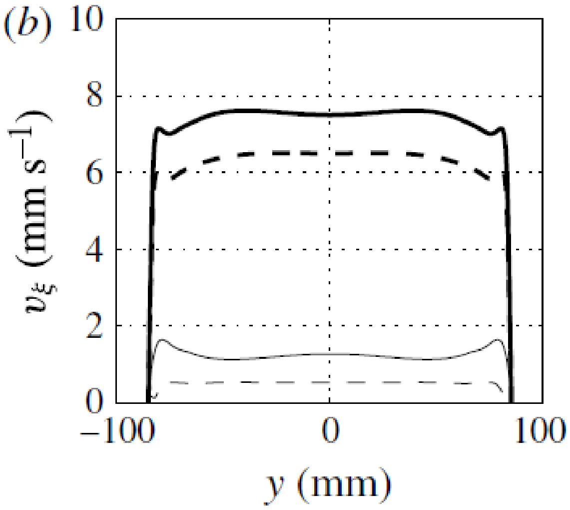
Real part and phase of the Hilbert transform for the primary and two secondary waves



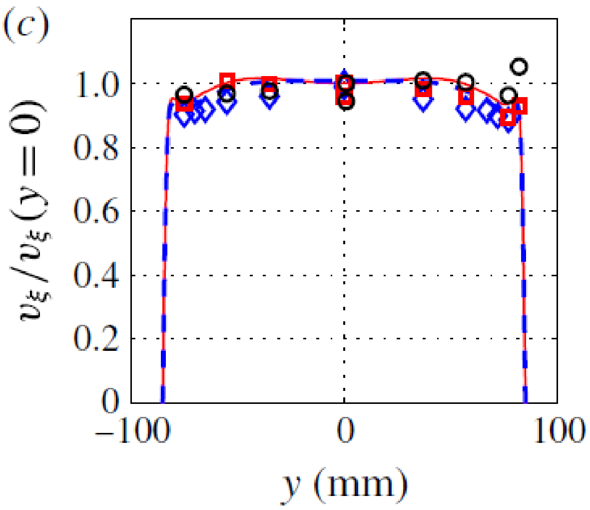
Secondary currents in wave atractors in 3D



Velocity magnitude in
+ and – directions along the beam.
Dash line – cross-tank direction

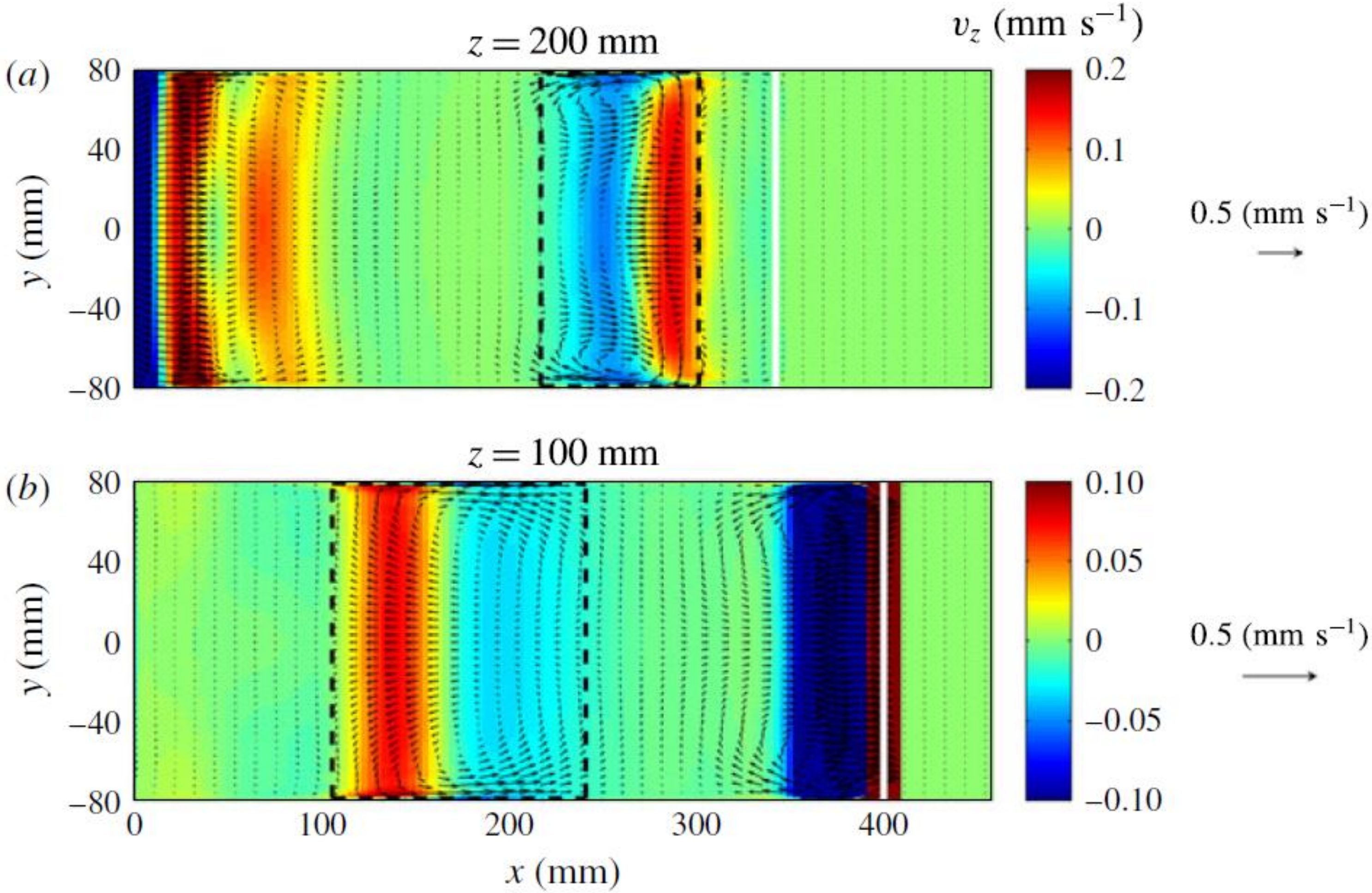


Velocity amplitude filtered
at ω_0 (thick dash line)
for the same case as in (a).
Velocity amplitudes filtered at
ω_0, ω_1 and ω_2
(thick solid and thin dash &
solid lines)



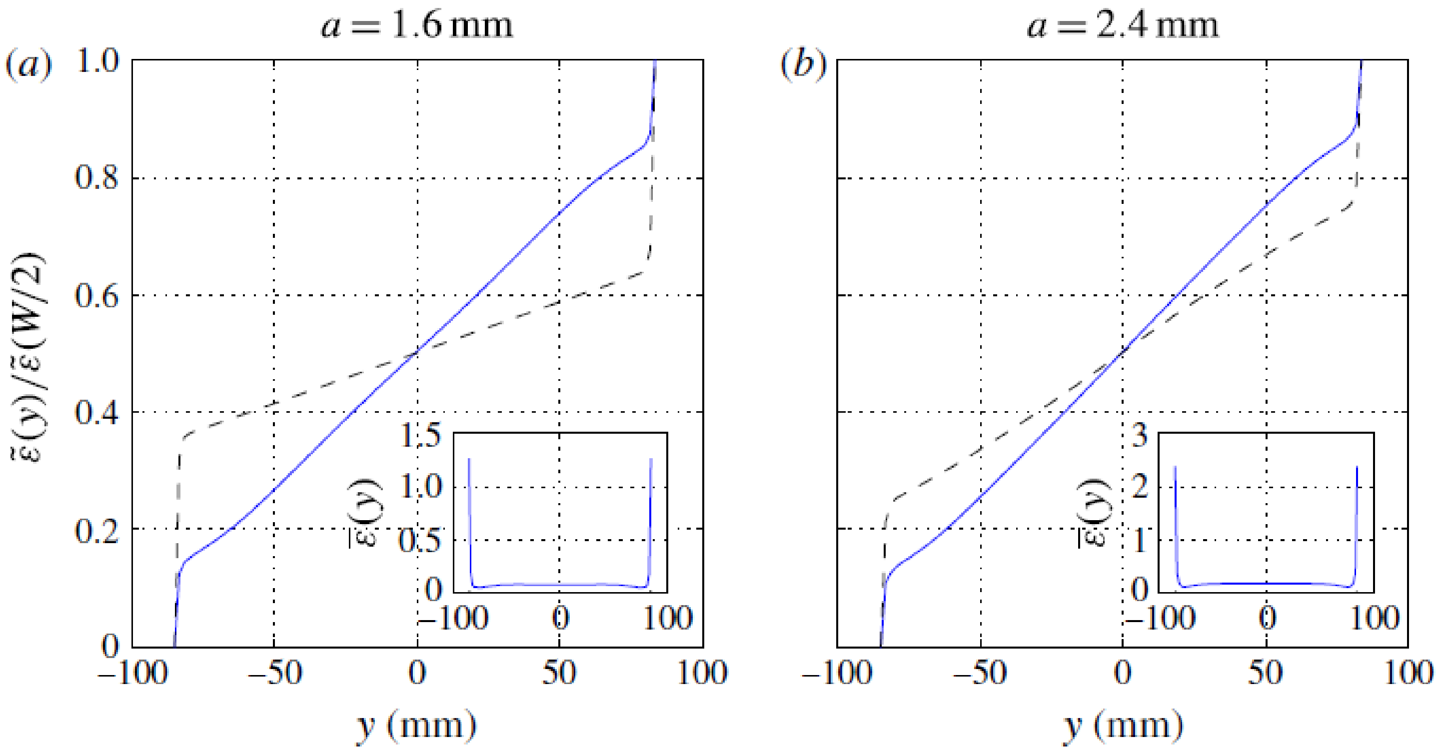
Velocity amplitude filtered
at ω_0 and normalized by the value at
midplane $y = 0$.
Red and blue lines – numerics
for $a = 1.6$ and $a = 2.2$ mm.
Symbols – experiments:
diamonds, squares and circles
correspond to $a = 1.5$, 3 and 5 mm

Secondary currents in wave atractors in 3D



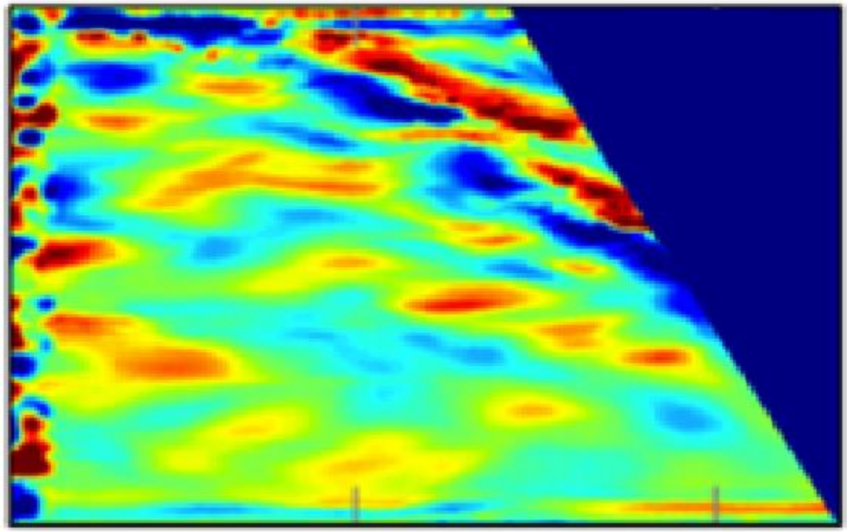
Currents filtered at zero frequency for two horizontal cross-sections of the tank

Dissipation across the tank

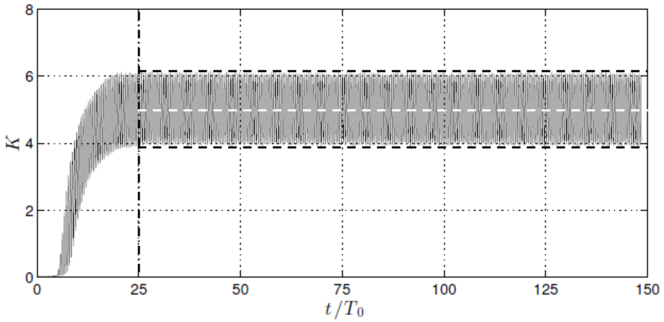


Blue solid line – dissipation in the most energetic beam of the attractor,
dash line – dissipation in the middle of the tank

Well-developed instability in a wave attractor.
Is this wave turbulence? What is beyond?

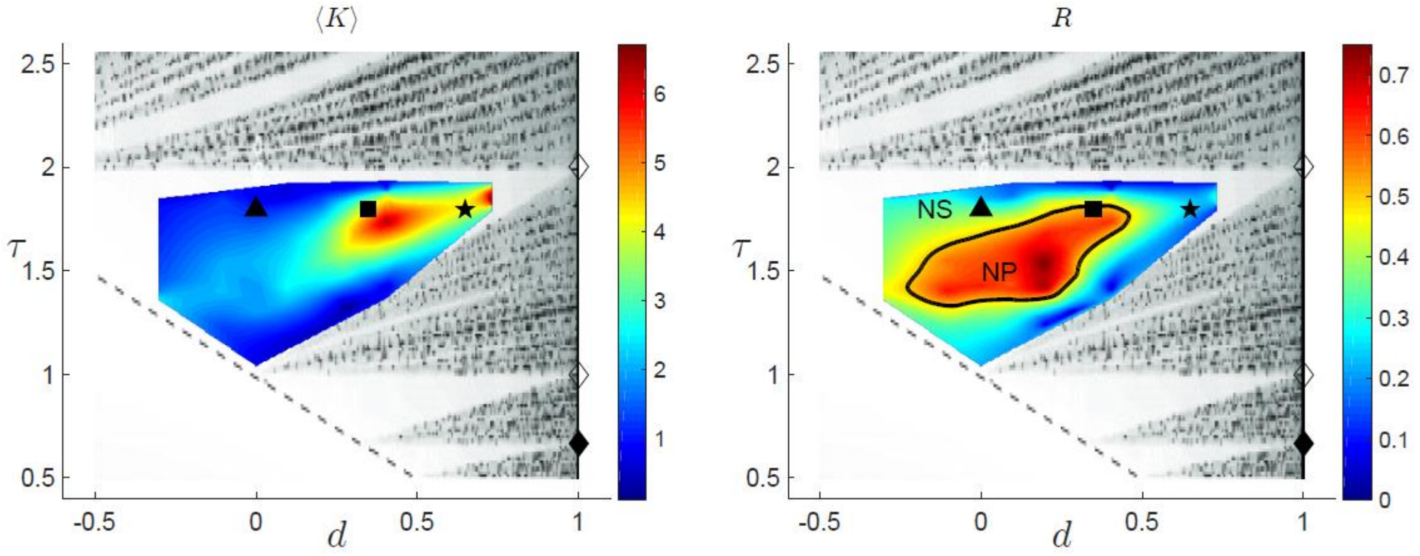


Choice of the operating point at the Arnold tongue



Total kinetic energy vs. time in linear case

$$K = \frac{\int_S \mathrm{d}x\mathrm{d}z\,\frac{1}{2}(v_x^2 + v_z^2)}{\frac{1}{2}(a\omega)^2 S}$$



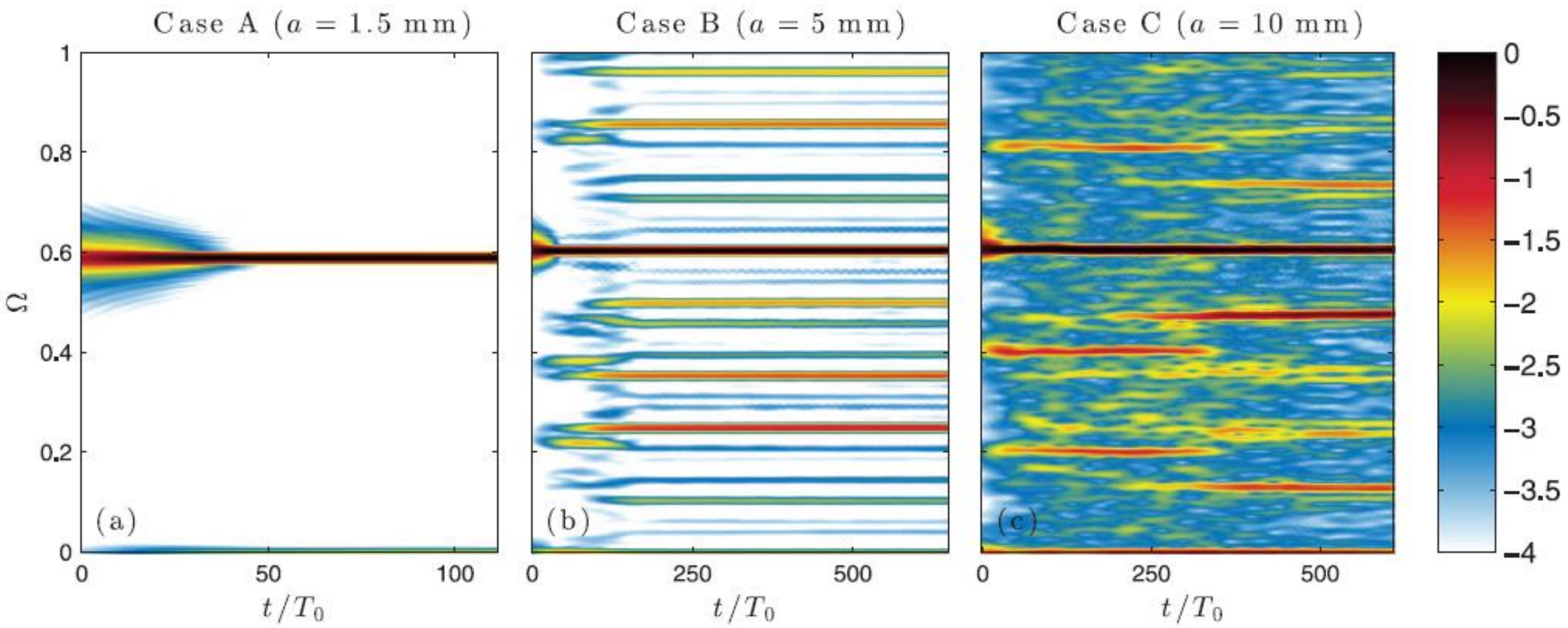
Susceptibility <*K*>= $\dfrac{\left\langle \int_S \mathrm{d}x\mathrm{d}z\,\frac{1}{2}(v_x^2 + v_z^2) \right\rangle}{\frac{1}{2}(a\omega)^2 S}$

$$R = K_{min}/K_{max}$$

R=0 standing waves

R=1 propagating waves (thin beams)

Cascade of triadic interactions in well-focused attractor (■)



Time-frequency diagrams

$$S_u(\Omega, t) = \left\langle \left| \int_{-\infty}^{+\infty} u(x, z, \tau) e^{i\Omega\tau} h(t-\tau)\, \mathrm{d}\tau \right|^2 \right\rangle_{xz}$$

	Type	Ω_0	H cm	L cm	α °	a mm	$t_{\max}$ T_0
A	Exp.	0.59	30.0	45.0	27.3	1.5	149
B	Exp.	0.61	30.3	44.4	25.4	5	693
C	Exp.	0.60	30.1	44.2	24.8	10	651

Wave turbulence analysis

Method: Yarom & Sharon (Nature Physics 2014)

2D PIV velocity field: $u(x,z,t)$ and $w(x,z,t)$

3D Fourier transform: $\hat{u}(k_x,k_z,\omega)$ and $\hat{w}(k_x,k_z,\omega)$

Energy spectrum:

$$E(k_x,k_z,\omega) = \frac{1}{2}\frac{1}{ST}\left(|\hat{u}(k_x,k_z,\omega)|^2 + |\hat{w}(k_x,k_z,\omega)|^2\right)$$

Dispersion relation: $\Omega = \pm\sin\theta$

Interpolation: $E(k_x,k_z,\omega) \longrightarrow E(k,\theta,\omega)$

Integration: $E(\theta,\omega) = \int_{k_{min}}^{k_{max}} E(k,\theta,\omega)kdk$

Wave turbulence and/or mixing events?
Splitting the scales…

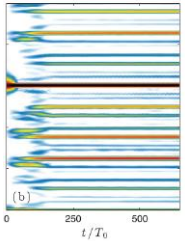
$k \in [0.22, 1]\ rad/cm$

$\lambda \in [28.5, 6.3]\ cm$

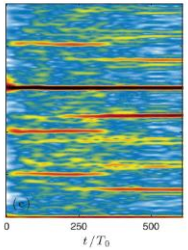
$k \in [1, 1.86]\ rad/cm$

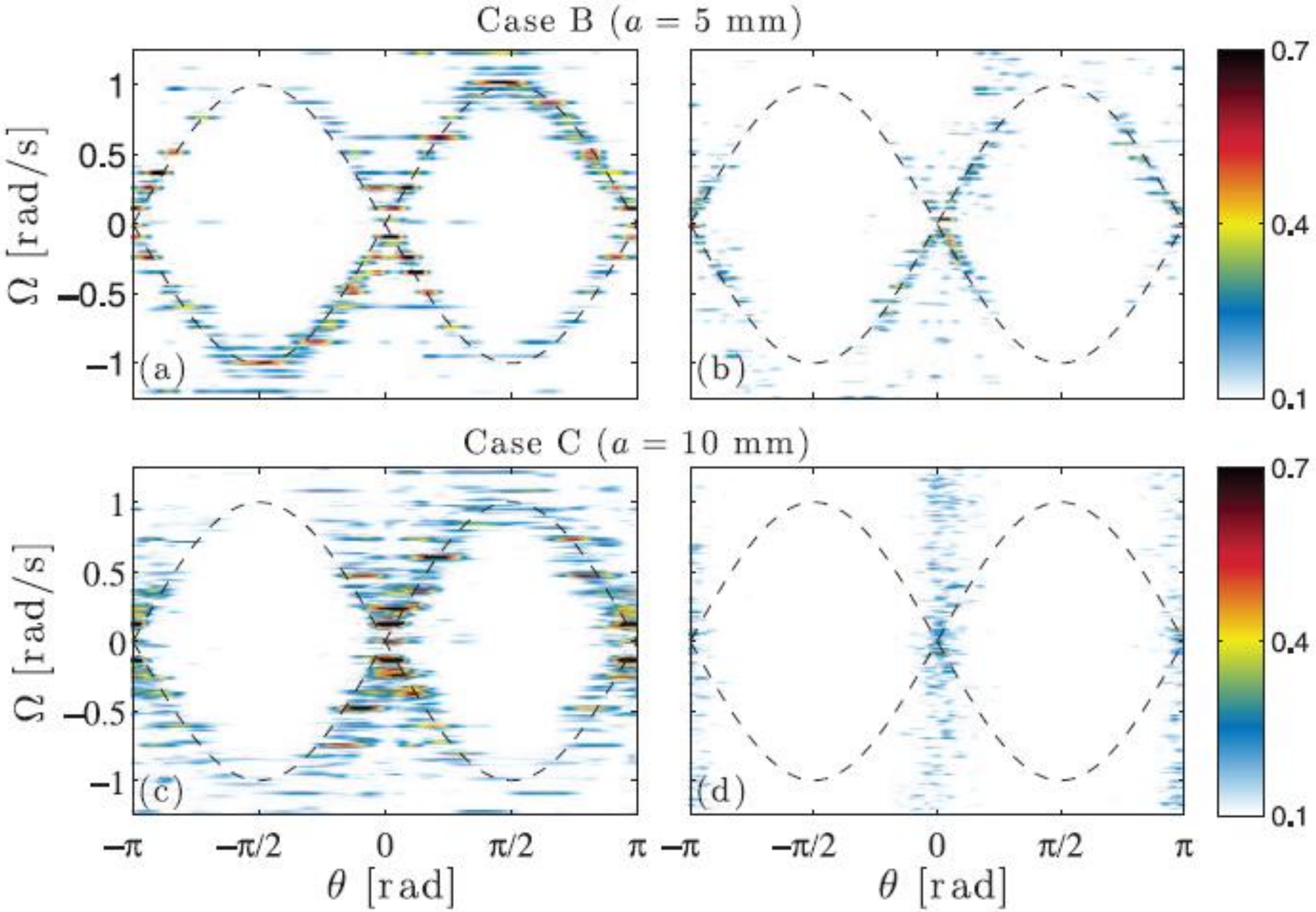
$\lambda \in [6.3, 3.4]\ cm$

Cascade B



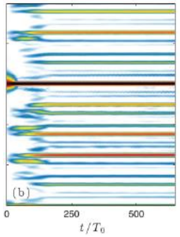
Cascade C

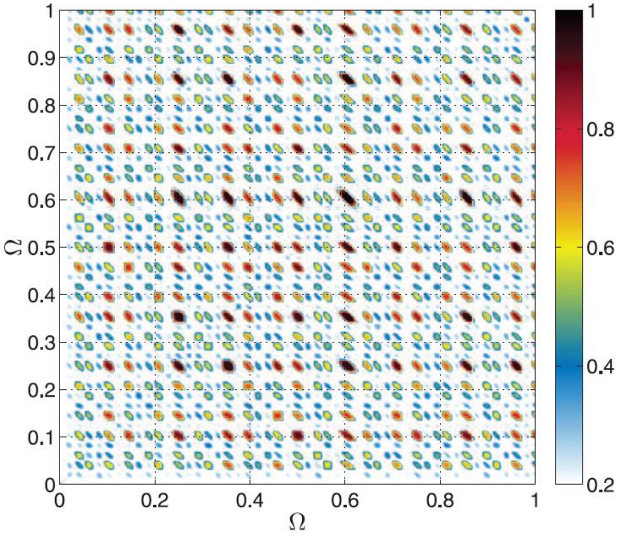




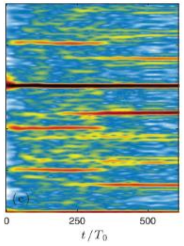
Triadic cascade portrayed by bicoherence

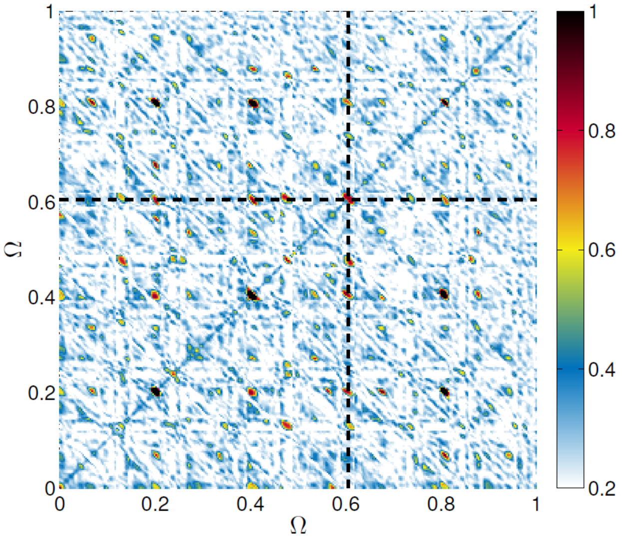
Cascade B





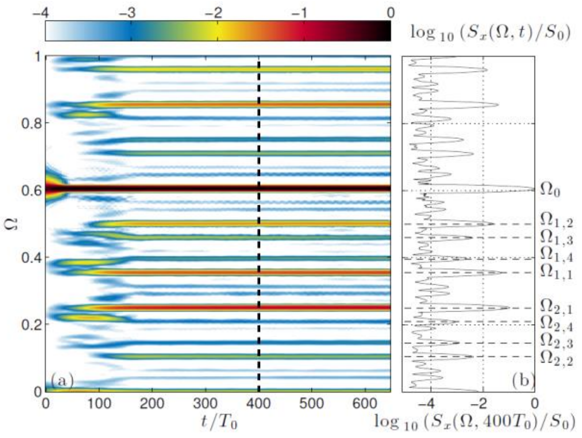
Cascade C

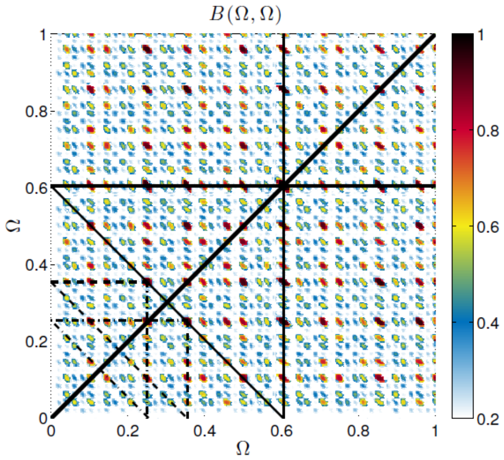




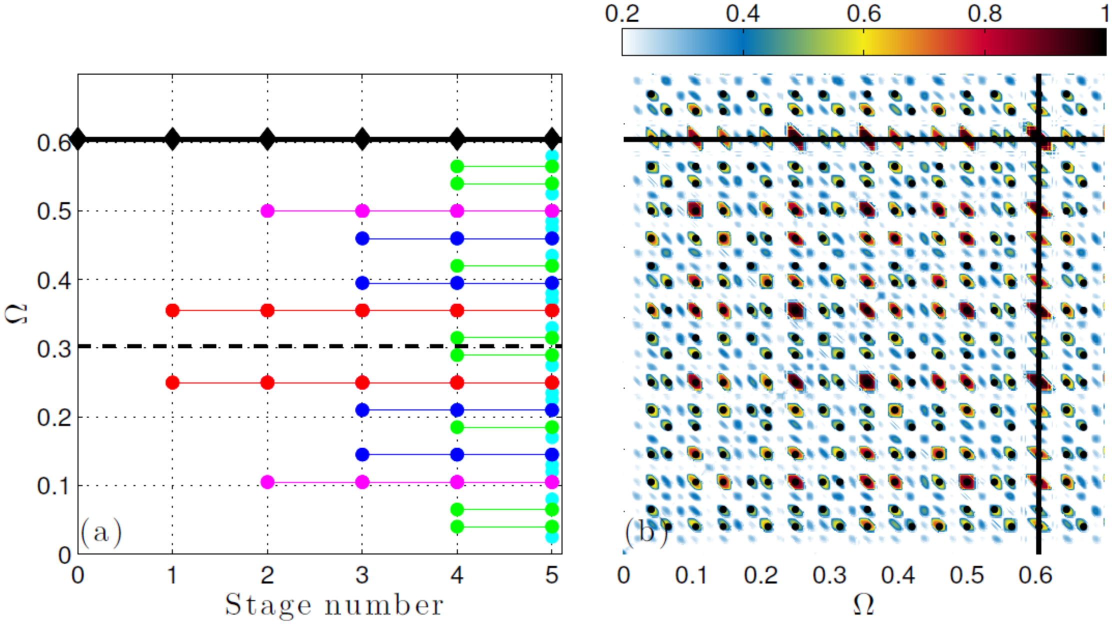
Bispectrum: $M(\Omega_i, \Omega_j) = F(\Omega_i) F(\Omega_j) F^*(\Omega_i + \Omega_j)$

What defines the choice of discrete frequencies in cascade B?
“Differential rule…” versus “true and quasi resonances”





Differential rule:



What defines the choice of discrete frequencies in cascade B?
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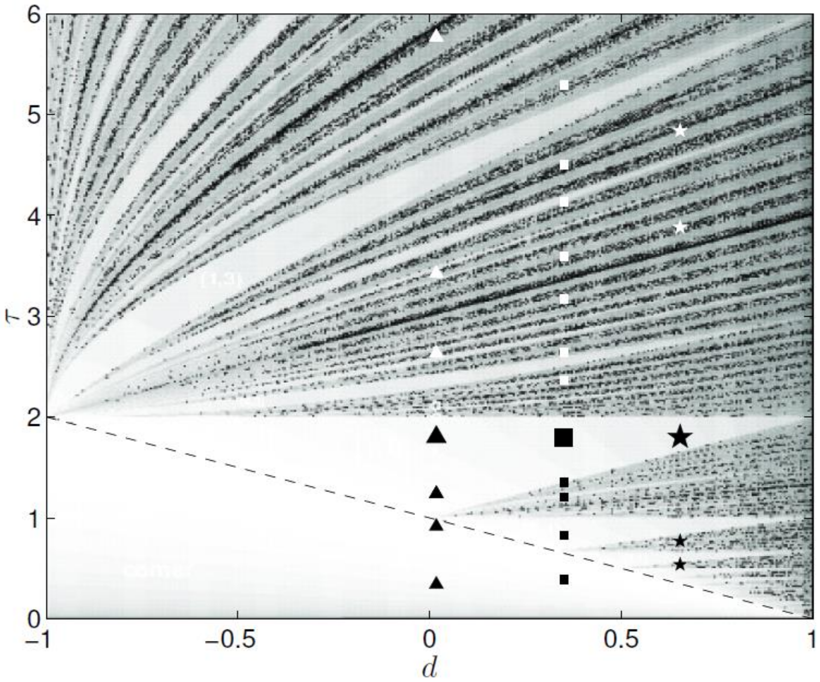
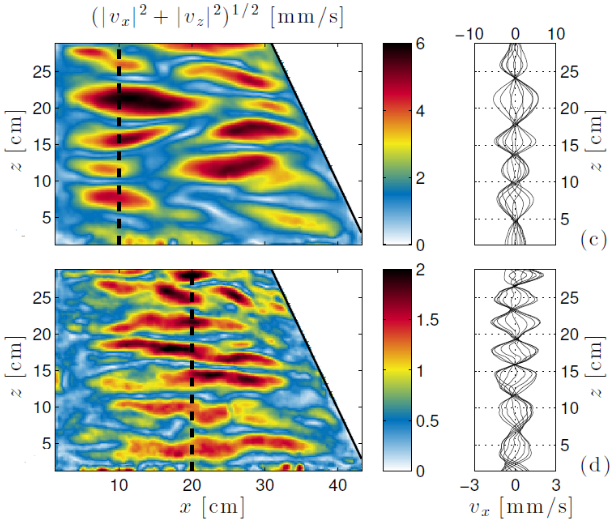
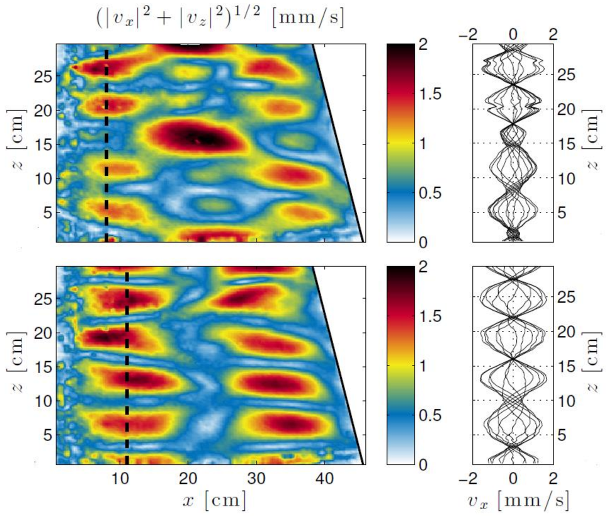


(■)



(★)



Mixing

Horizontal vorticity:

$$\xi = \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}$$

Gradient Richardson number:

$$Ri = \frac{N^2}{(du/dz)^2}$$

Modified Richardson number:

$$Ri_\xi = \frac{N^2}{\xi^2}$$

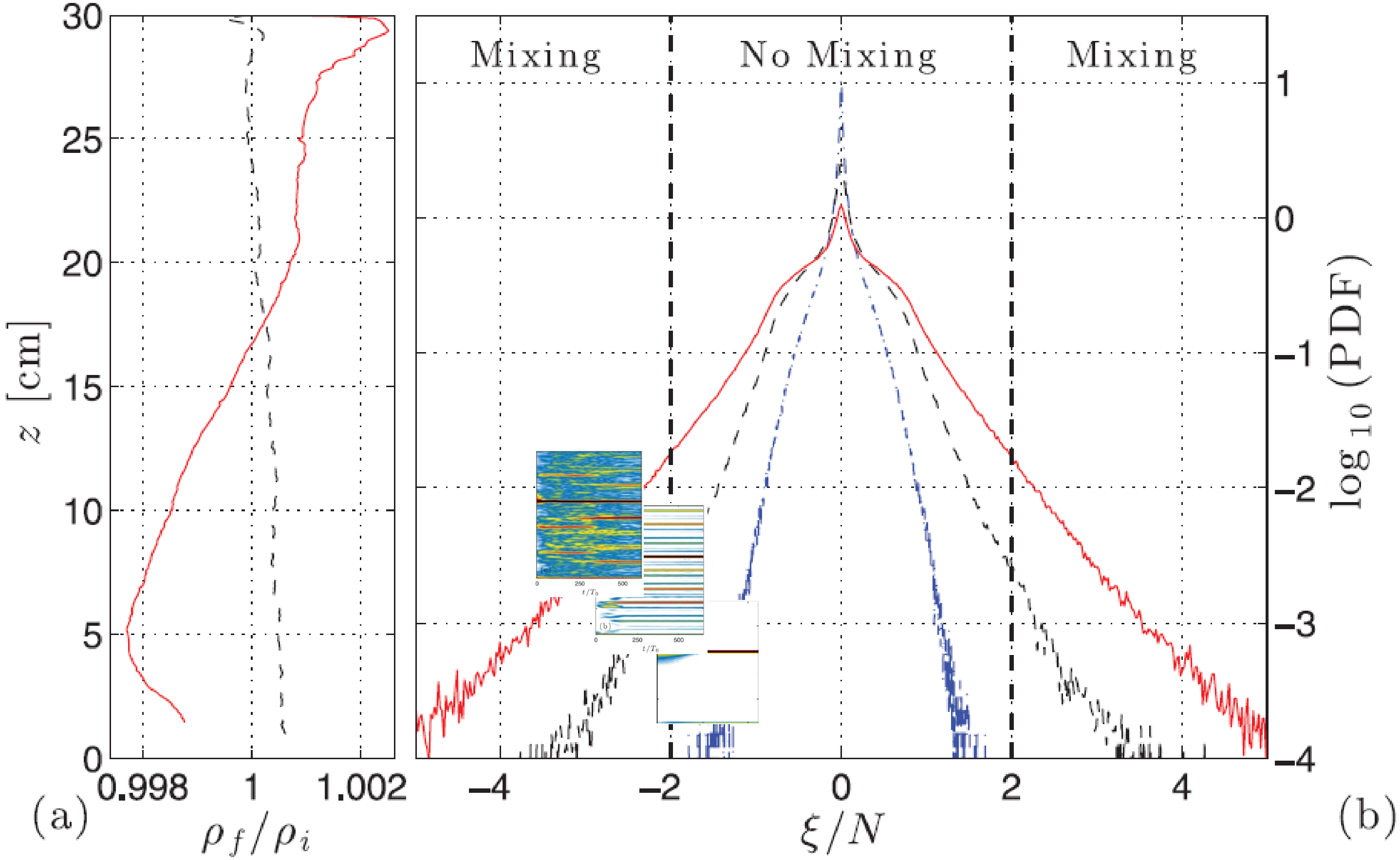
Extension of the Miles-Howard condition

$$Ri > \frac{1}{4}$$

Mixing at

$$Ri_\xi < \frac{1}{4} \longrightarrow \left| \frac{\xi}{N} \right| > 2$$

Statistics of mixing events



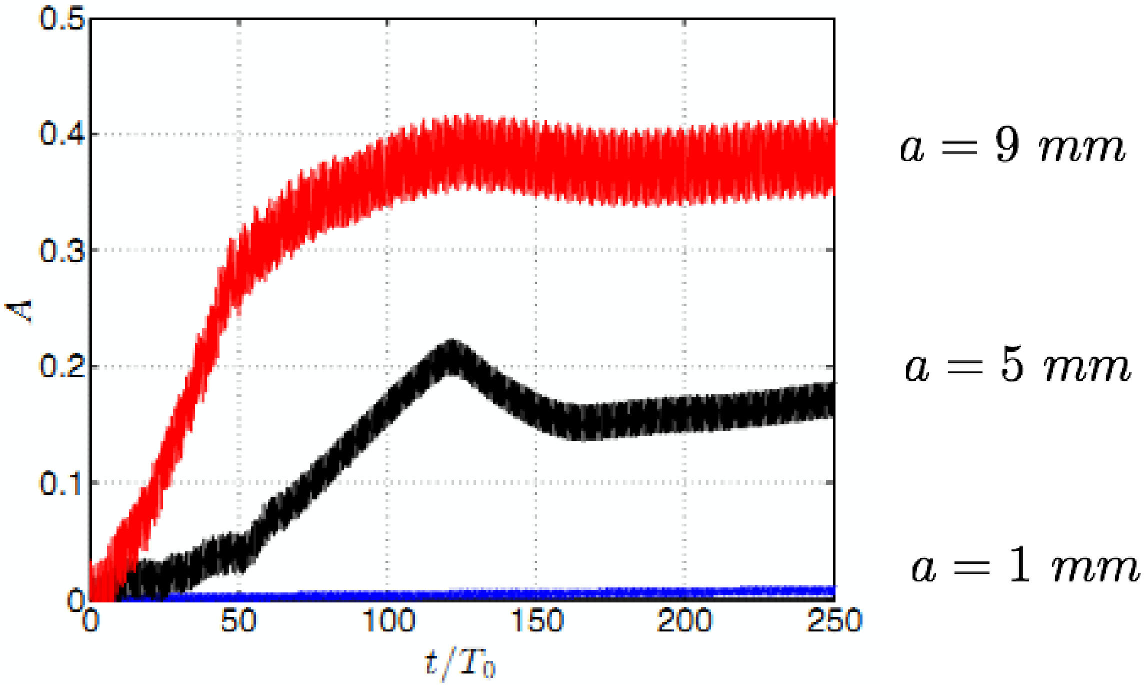
whole-field horizontal vorticity PDF

Mixing

Change of potential energy: $A(t) = \dfrac{(E_p(t) - E_p(0))}{(E_p^* - E_p(0))}$

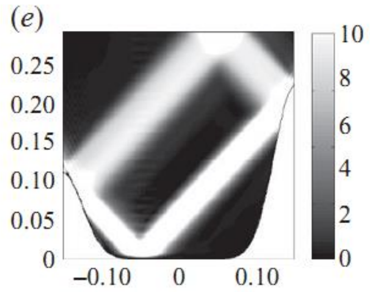
Cascade C: $A \approx 25\%$

2D numerics

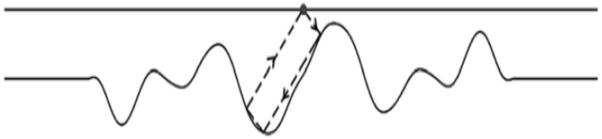


Conclusions

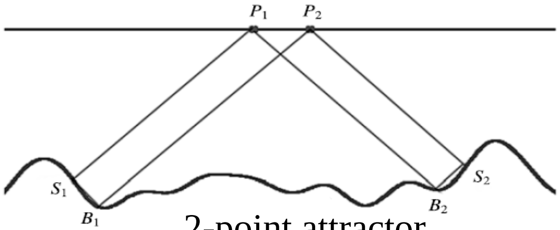
Attractors between ridges



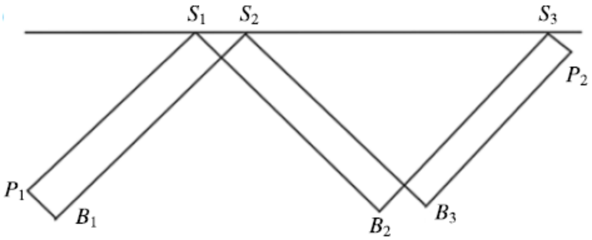
Echeverri *et al.* (JFM 2011)



1-point attractor

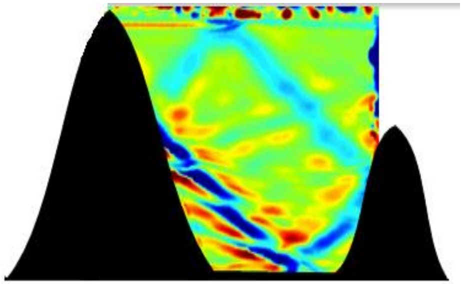


2-point attractor



3-point attractor

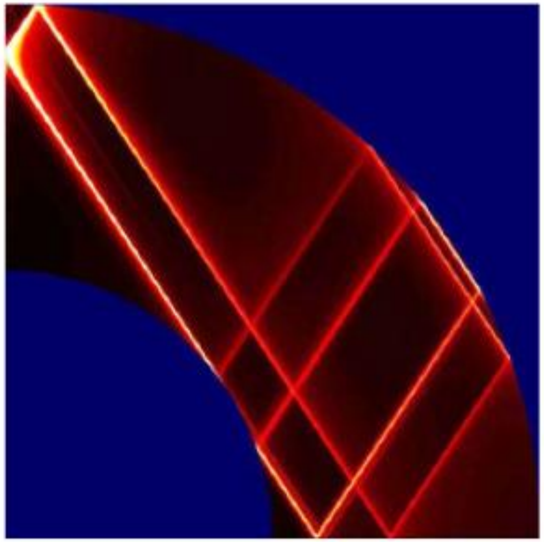
Guo & Holmes-Cefron (JFM 2016)



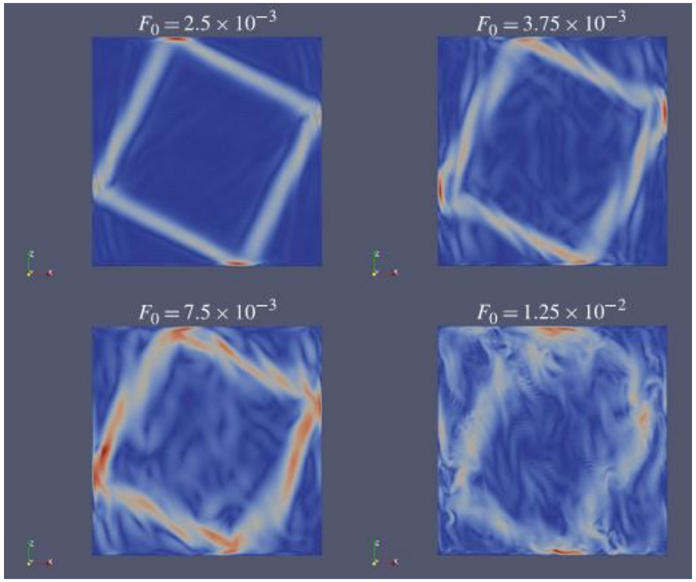
Wave attractor as a source of wave turbulence and mixing

Conclusions

rotating spherical shells



Ogilvie (JFM 2009)
Rieutord, Valdettaro,
Georgeot, Bariteau… (JFM… 2001-2013)



Jouve & Ogilvie (JFM 2014)

Wave attractor as a source of wave turbulence and mixing

Publications

Scolan H., Ermanyuk E.V., Dauxois T. (2013)
Nonlinear fate of internal wave attractors *PRL* **110**, 234501

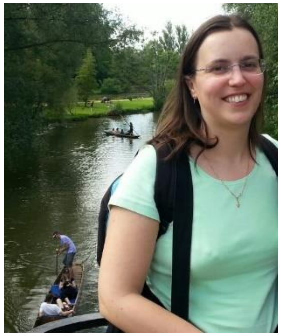
Brouzet C., Ermanyuk E.V., Joubaud S., Sibgatullin I.N., Dauxois T. (2016)
Energy cascade in internal-wave attractors *EPL* **113**, 44001

Brouzet C., Sibgatullin I.N., Scolan H., Ermanyuk E.V., Dauxois T. (2016)
Internal wave attractors examined using laboratory experiments
and 3D numerical simulations *JFM* **793**, 109-131

Thierry
Dauxois

Christophe
Brouzet

Sylvain
Joubaud



Helene
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