Parametrically forced axisymmetric gravity waves and jetting in a circular cylinder

Shyama Prasad Das

and

D. Krishnaraja

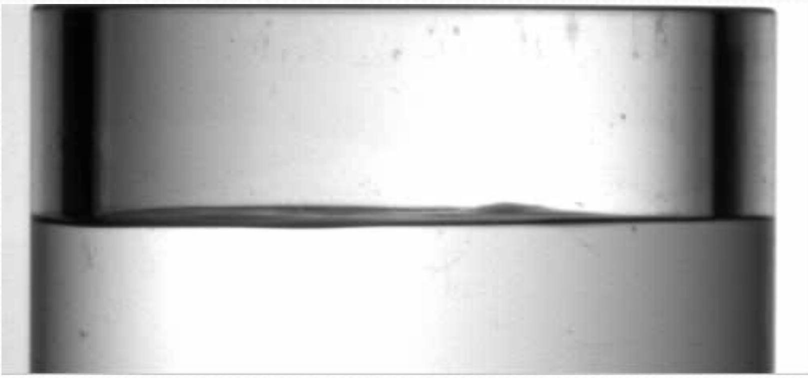
Department of Mechanical Engineering

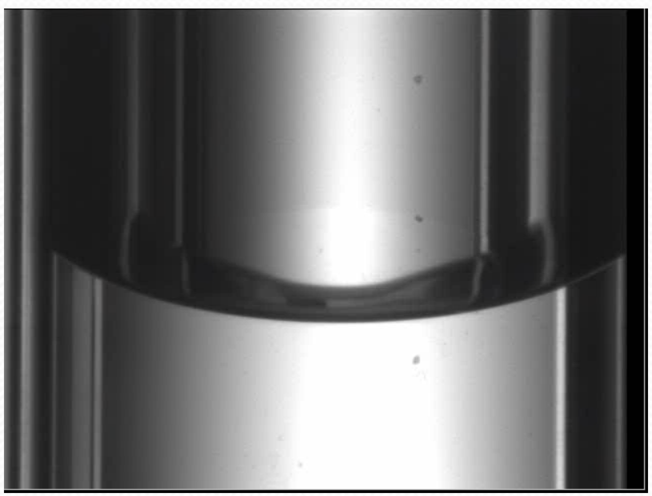
Indian Institute of Technology Madras

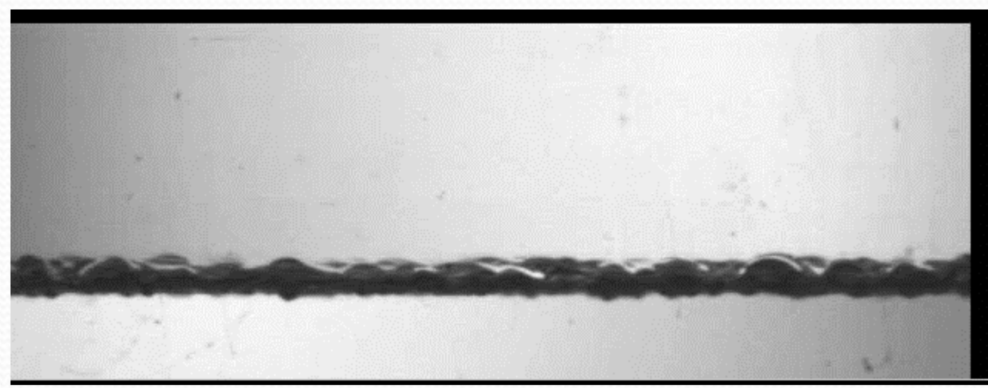
Chennai 600036, India

Emil Hopfinger Colloquium 2016, 11–13 May 2016, Grenoble, France

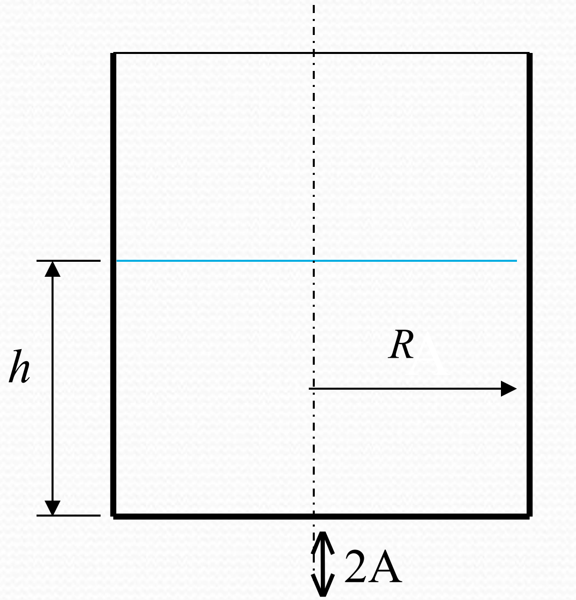
Waves in a Cylindrical Container



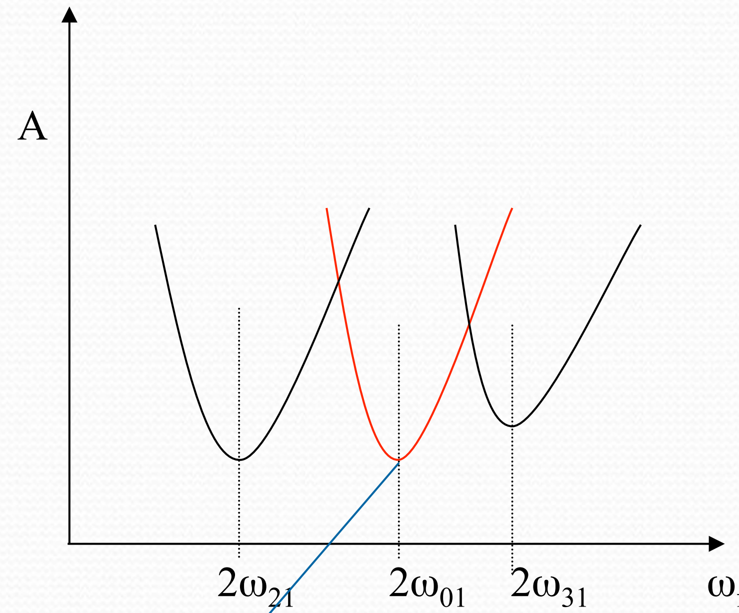




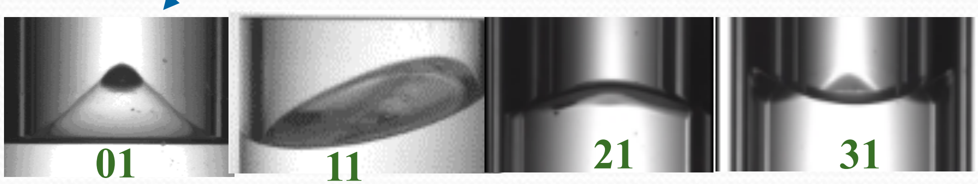
Introduction



$$a(t) = A\omega_f^2 \sin(\omega_f t)$$



Faraday instability thresholds



Liquids:

FC-72→ σ/ρ = 6.5 cm^3/s^2, ν = 0.0040 cm^2/s

h/R=1.2 , R=5cm

Das and Hopfinger (JFM,2008)

Theoretical Concepts

- Dispersion relation:

$$\omega_{mn}^2 = g k_{mn}\left(1 + \frac{\sigma k_{mn}^2}{g\rho}\right)\tanh(k_{mn} h)$$

- Wave numbers obtained from Boundary condition:

$$\left.\partial\phi/\partial r\right|_{r=R} = J_m'(k_{mn}R) = 0 \qquad \phi \text{ -- Velocity Potential}$$

$k_{01}R=3.832$, $k_{02}R=7.016$ (axisymmetric modes)

$k_{11}R = 1.841$, $k_{21}R = 3.054$, $k_{31}R = 4.201$ (asymmetric modes)

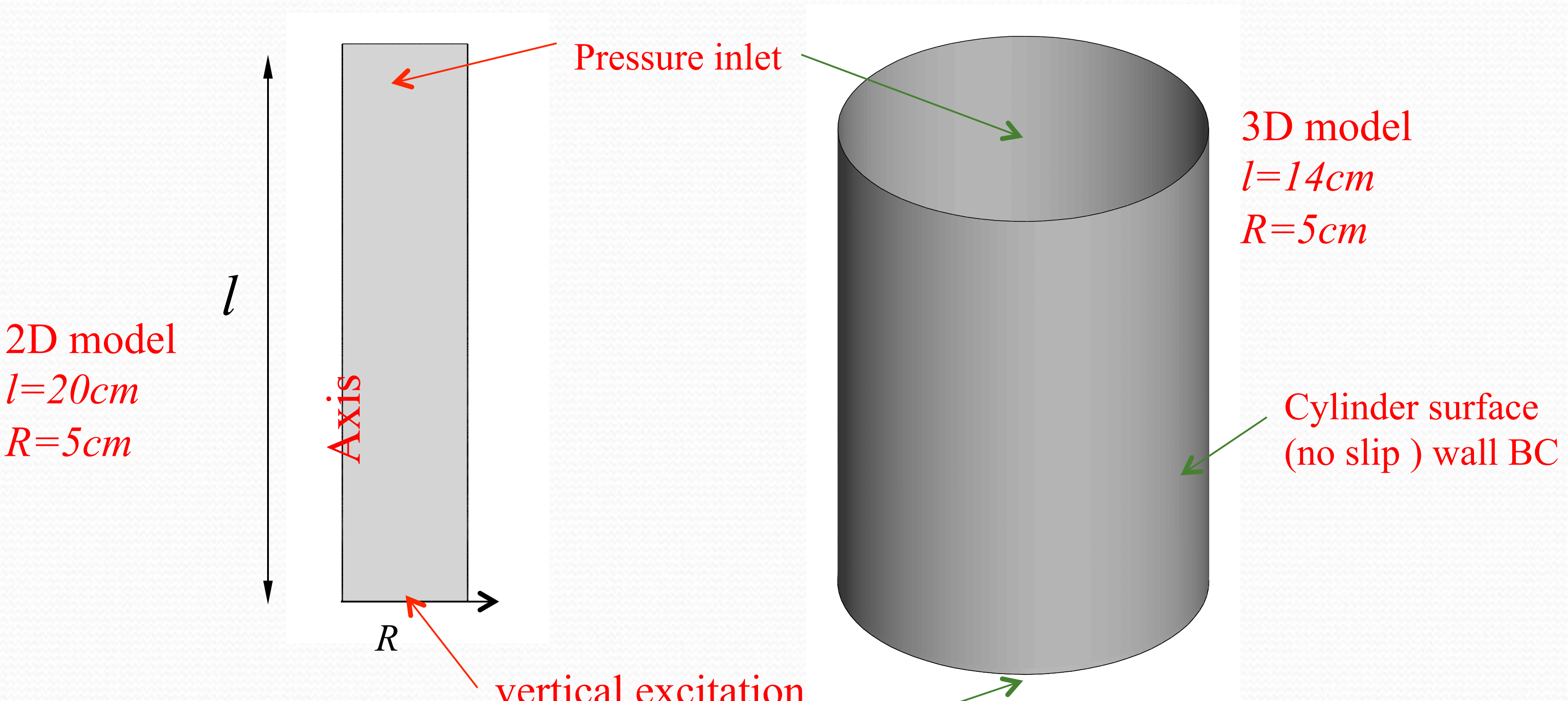
Natural frequency of mode (0,1):

$$\omega_{01} = \left[\frac{3.832\, g}{R}\left(1 + \frac{(3.832)^2}{Bo}\right)\tanh(3.832\, h/R)\right]^{1/2} \qquad Bo = \rho g R^2/\sigma$$

$$\omega_0 \equiv \omega_{01} = 27.472\, rad/s$$

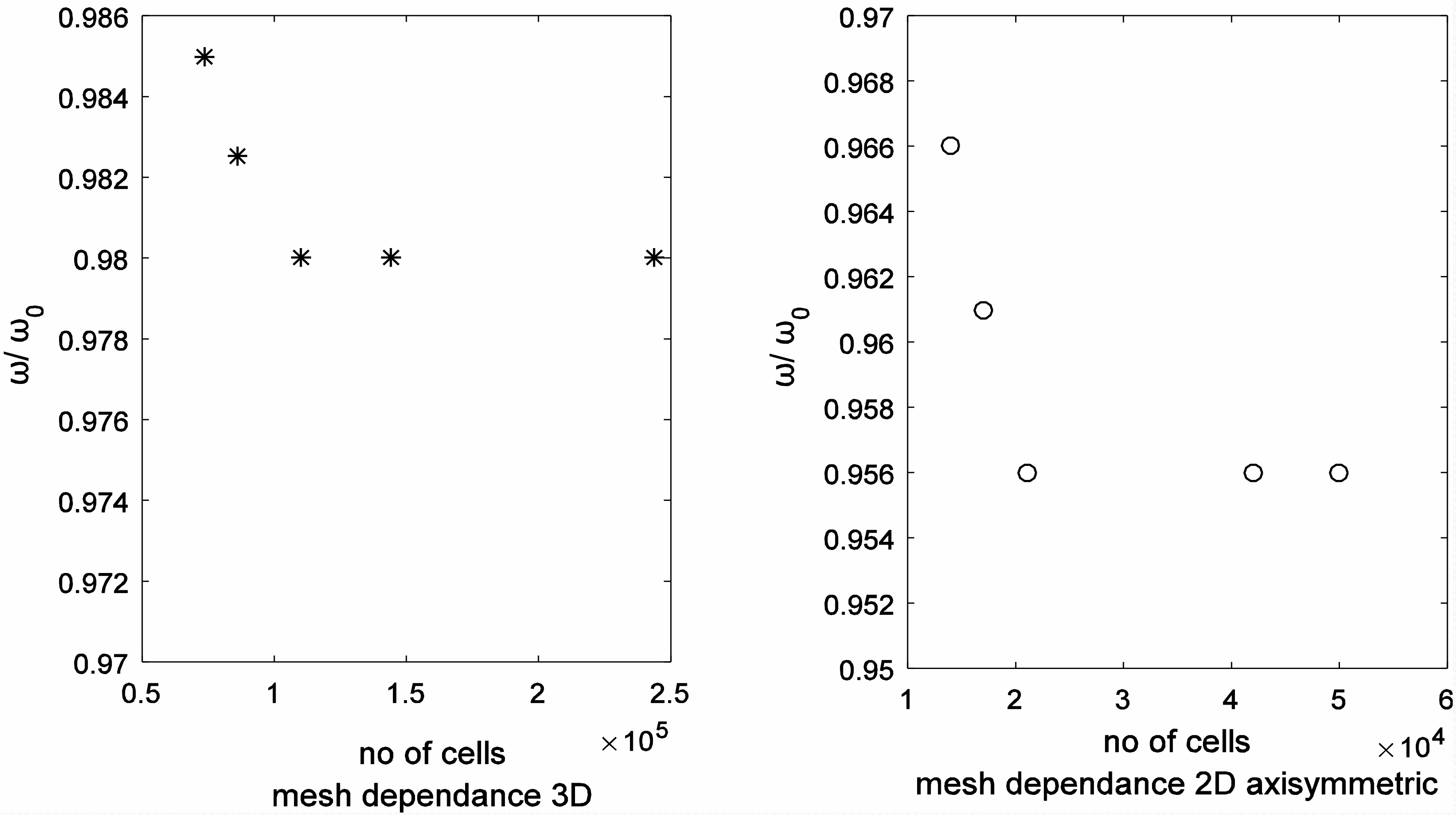
Problem description

- CFD solver : ANSYS FLUENT 15.0
- Model : VOF Multiphase



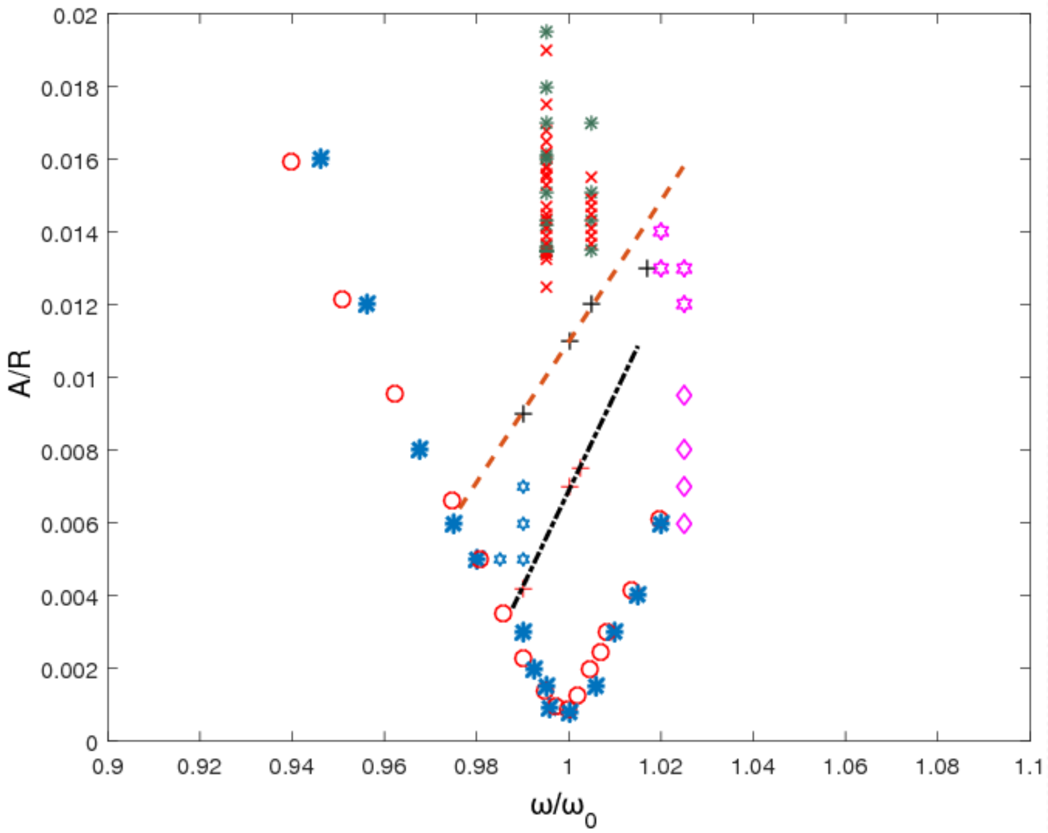
$$a(t) = A\omega_f^2 \sin(\omega_f t)$$

Grid Independence



Phase Diagram

FC-72, R = 5 cm



- Instability threshold
- Wave breaking threshold
- Unstable wave motion
- Period tripling
- Period quadrupling
- Bifurcation into other modes
- Coexistence of two modes

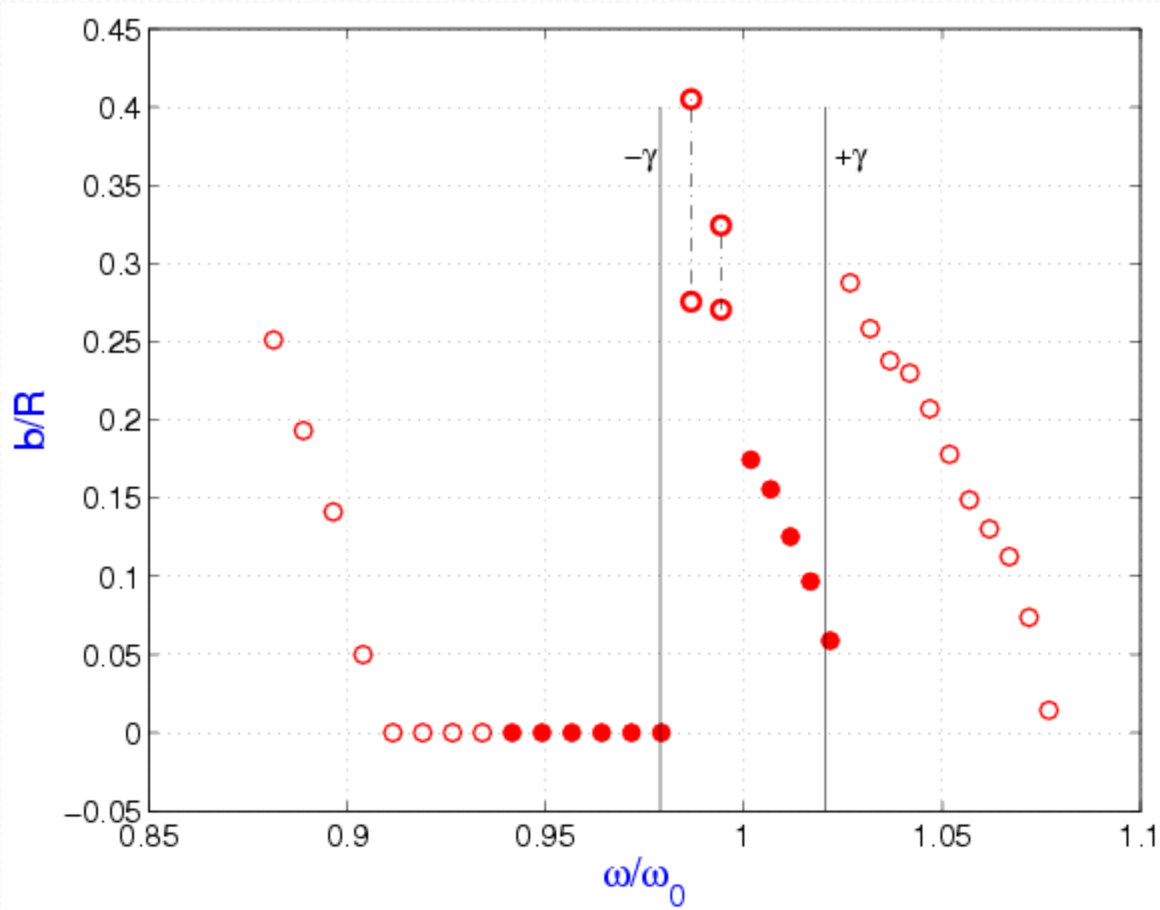
* Instability threshold forcing amplitude
\+ Wave breaking threshold
O Instability threshold (Das and Hopfinger, JF 2008)

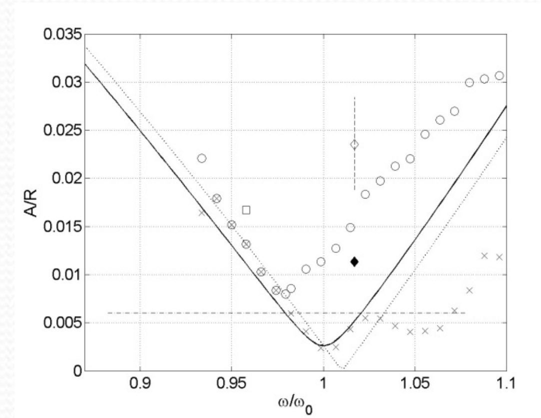
Mode (2,1) ←

→ Mode (3,1)

$$\omega_0 \equiv \omega_{01} = 27.4719 rad / s$$

Wave Amplitude Response





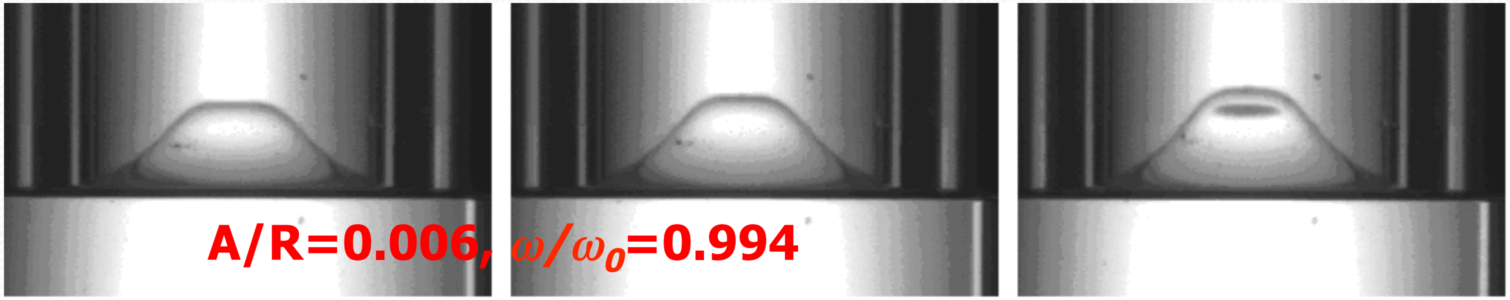
Wave amplitude response, A/R = 0.006, R = 2.5 cm

Das and Hopfinger (JFM,2008)

Wave amplitude Modulation

Experiments in a small container (2.5cm radius)

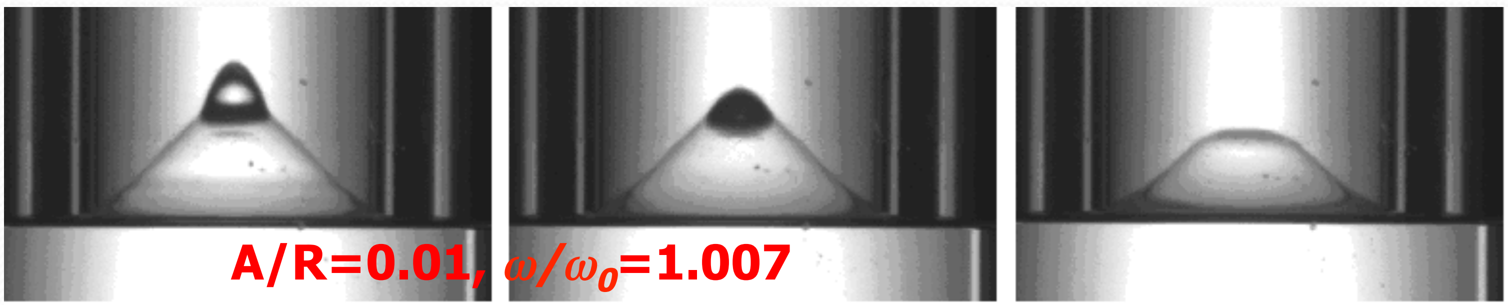
1



A/R=0.006, ω/ω_0=0.994

t = 3T, 23T, ... t = 8T, 28T, ... t = 13T, 33T, ...

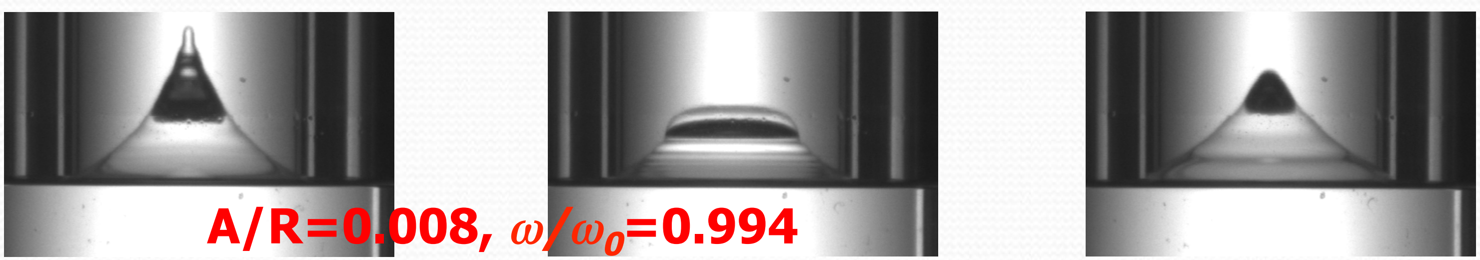
2



A/R=0.01, ω/ω_0=1.007

t = 1T, 15T, ... t = 4T, 18T, ... t = 8T, 22T, ...

3

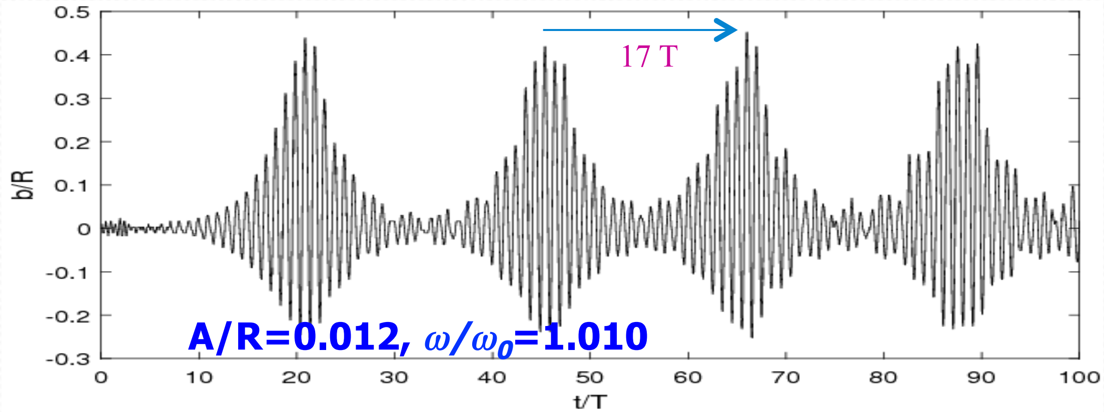


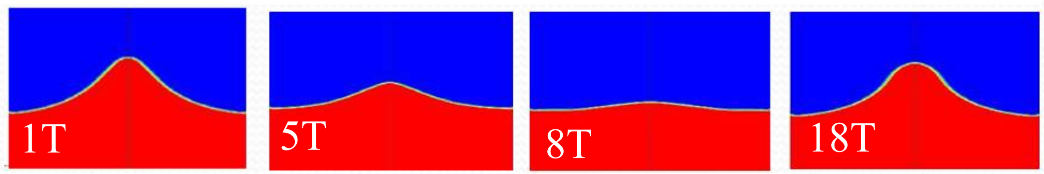
A/R=0.008, ω/ω_0=0.994

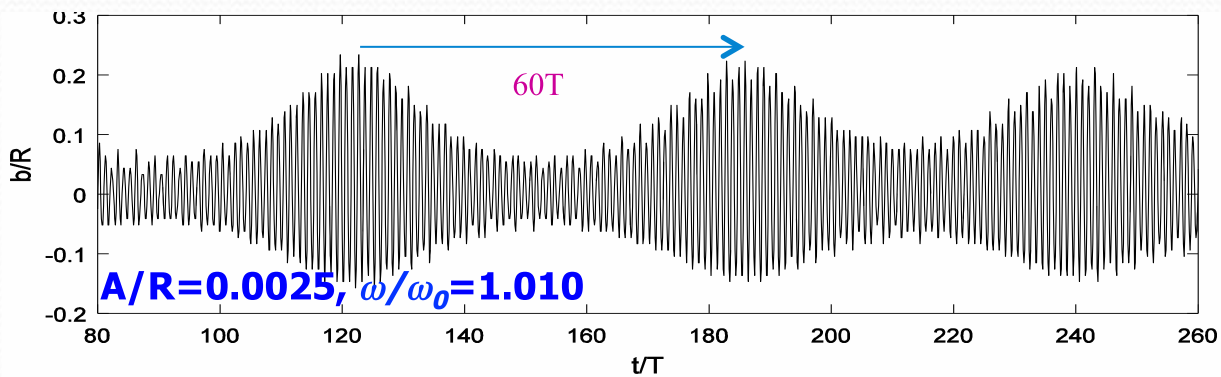
t=4T, 7T, ... t=5T, 8T, ... t=6T, 9T, ...

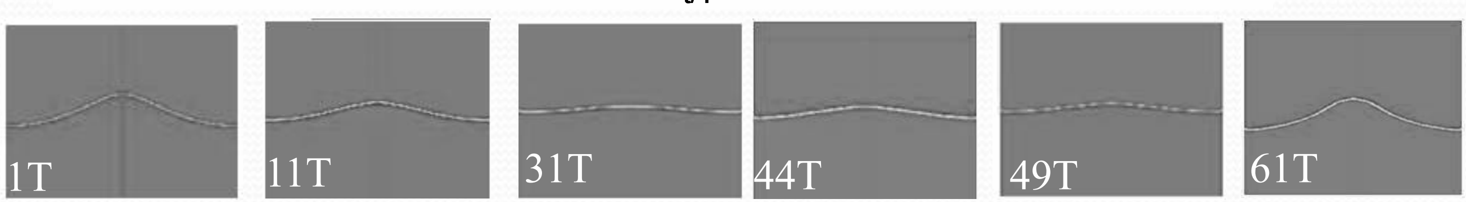
Wave Amplitude Modulation

Slow time scale modulations

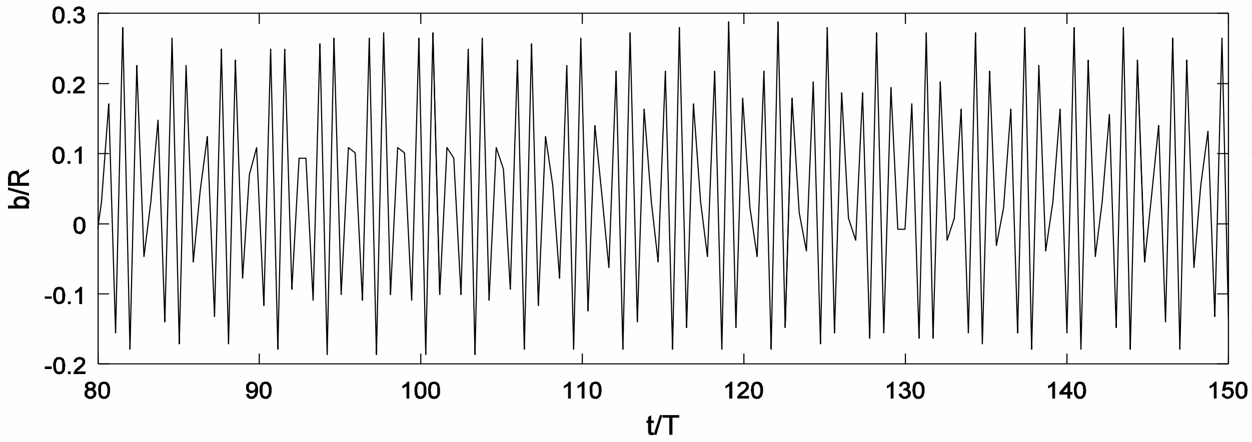


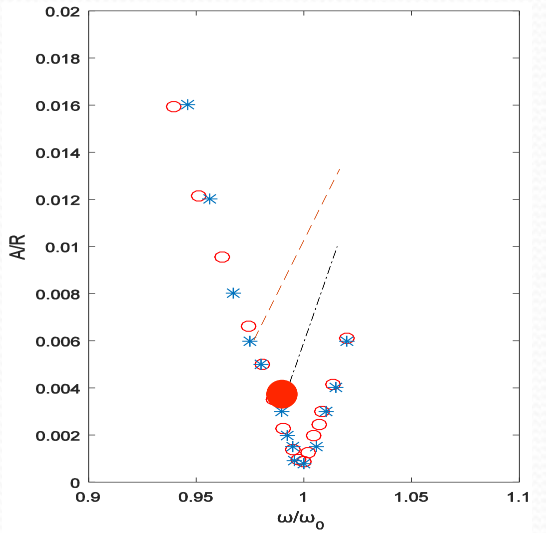




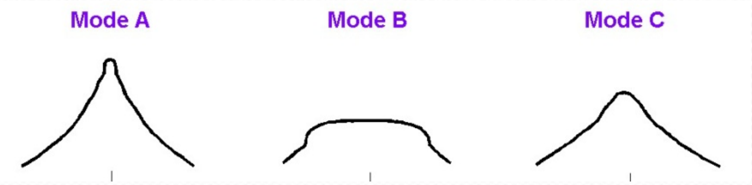


Wave Amplitude Modulation

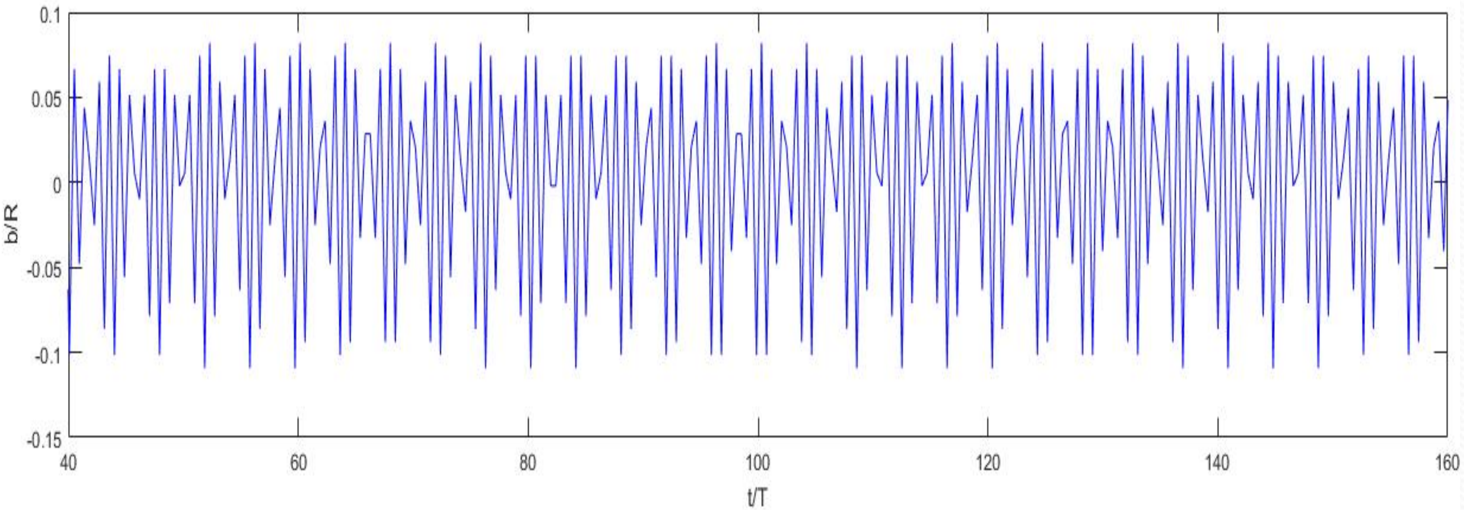


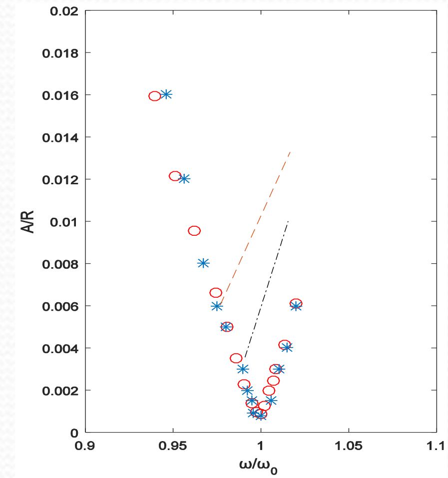


Period tripling $A/R=0.005$, $\omega/\omega_0=0.9825$;



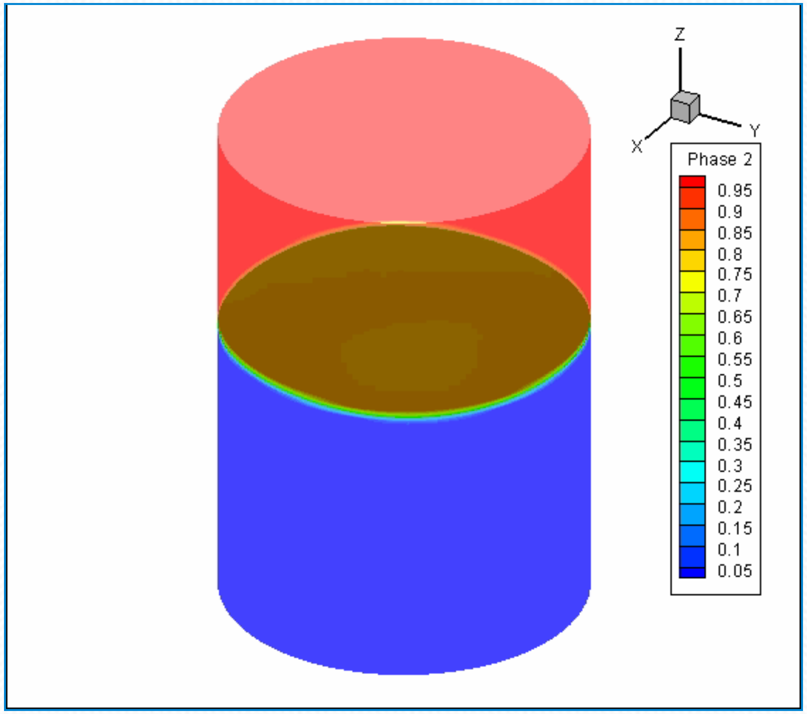
Wave Amplitude Modulation

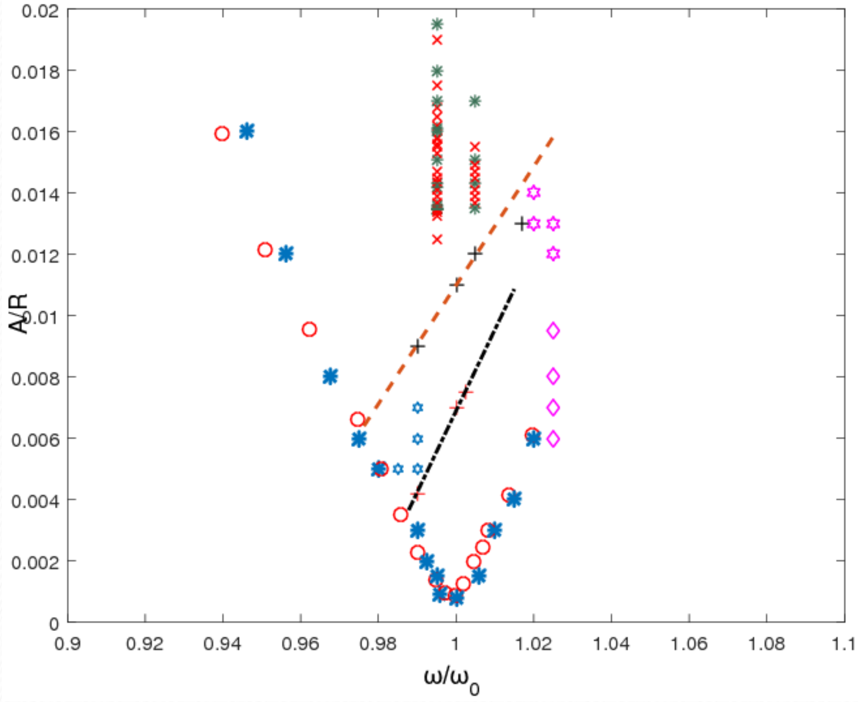




Period quadrupling A/R=0.01, ω/ω_0=1.015;

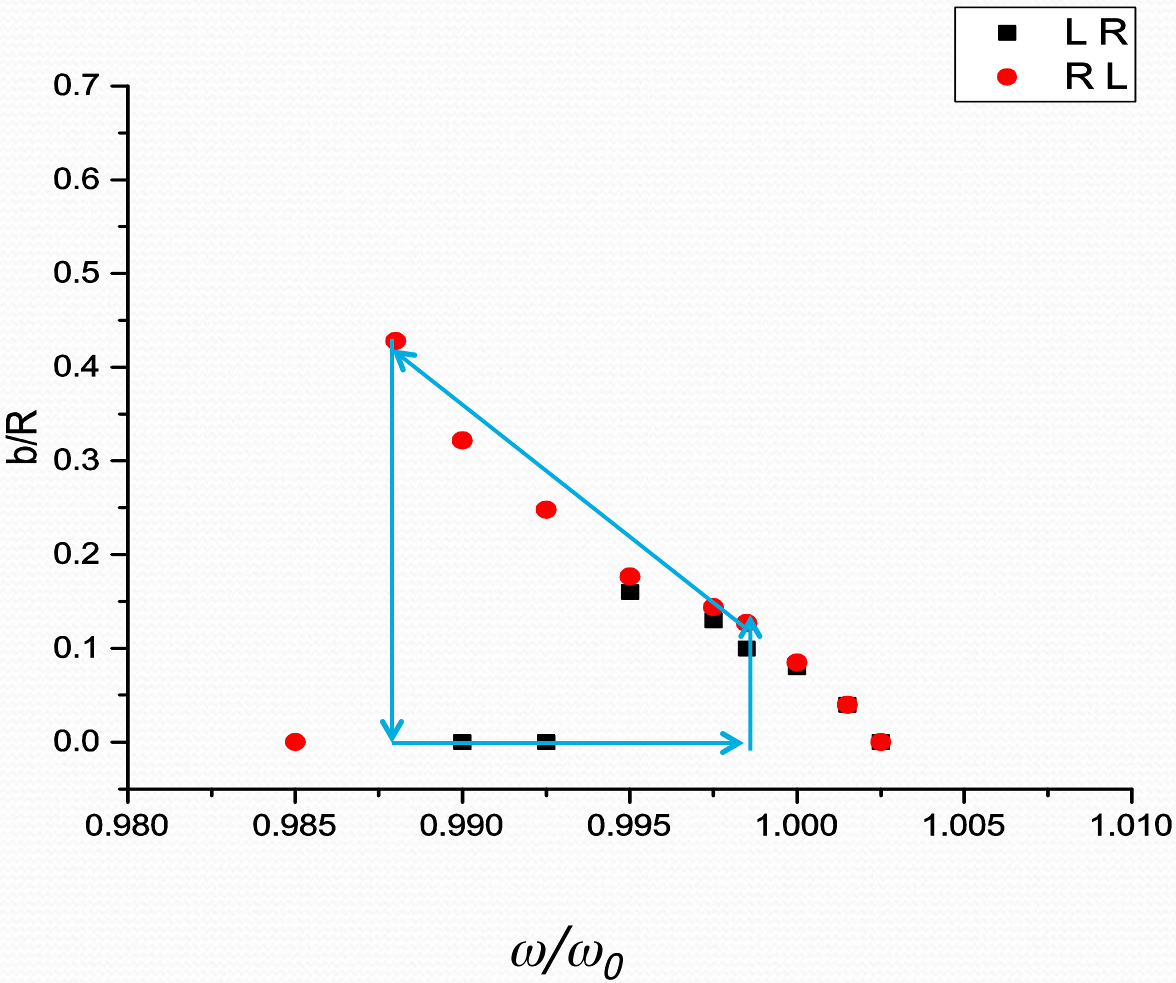
Coexistence

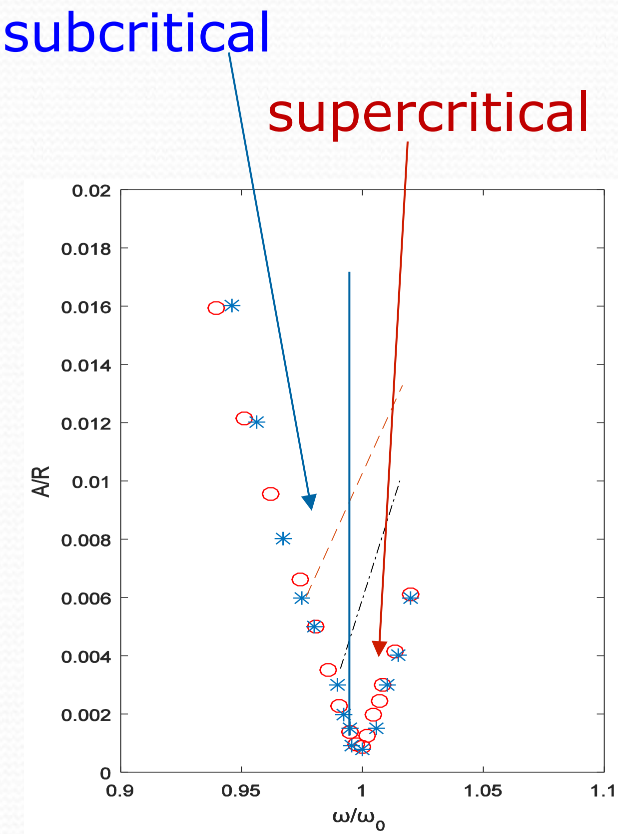




Coexistence of mode 01 and 31 A/R=**0.012**, ω/ω_0=1.020

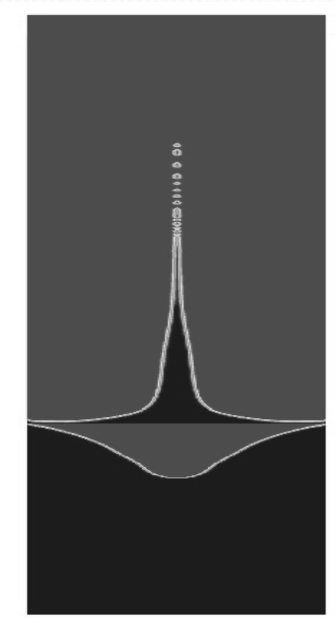
Stability behaviour



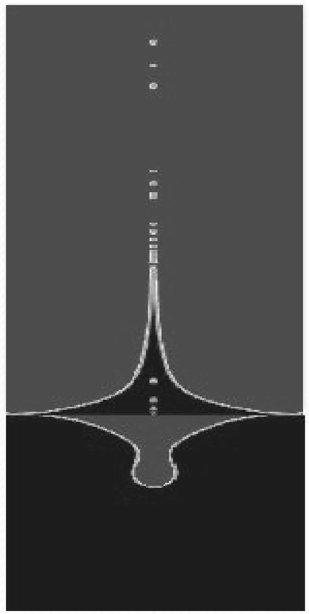


Wave amplitude response, $A/R = 0.0015$

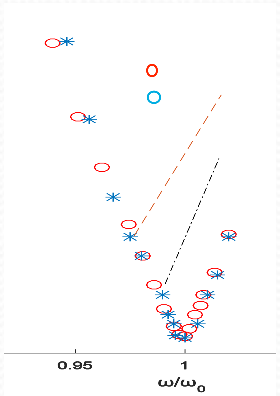
Jetting

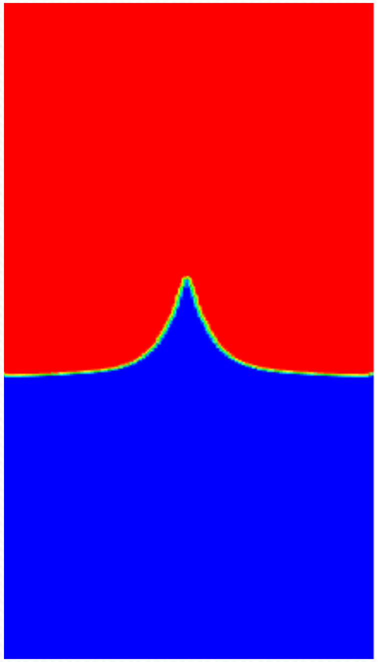


A/R=0.0139, ω/ω_0=0.995



A/R=0.0135, ω/ω_0=0.995

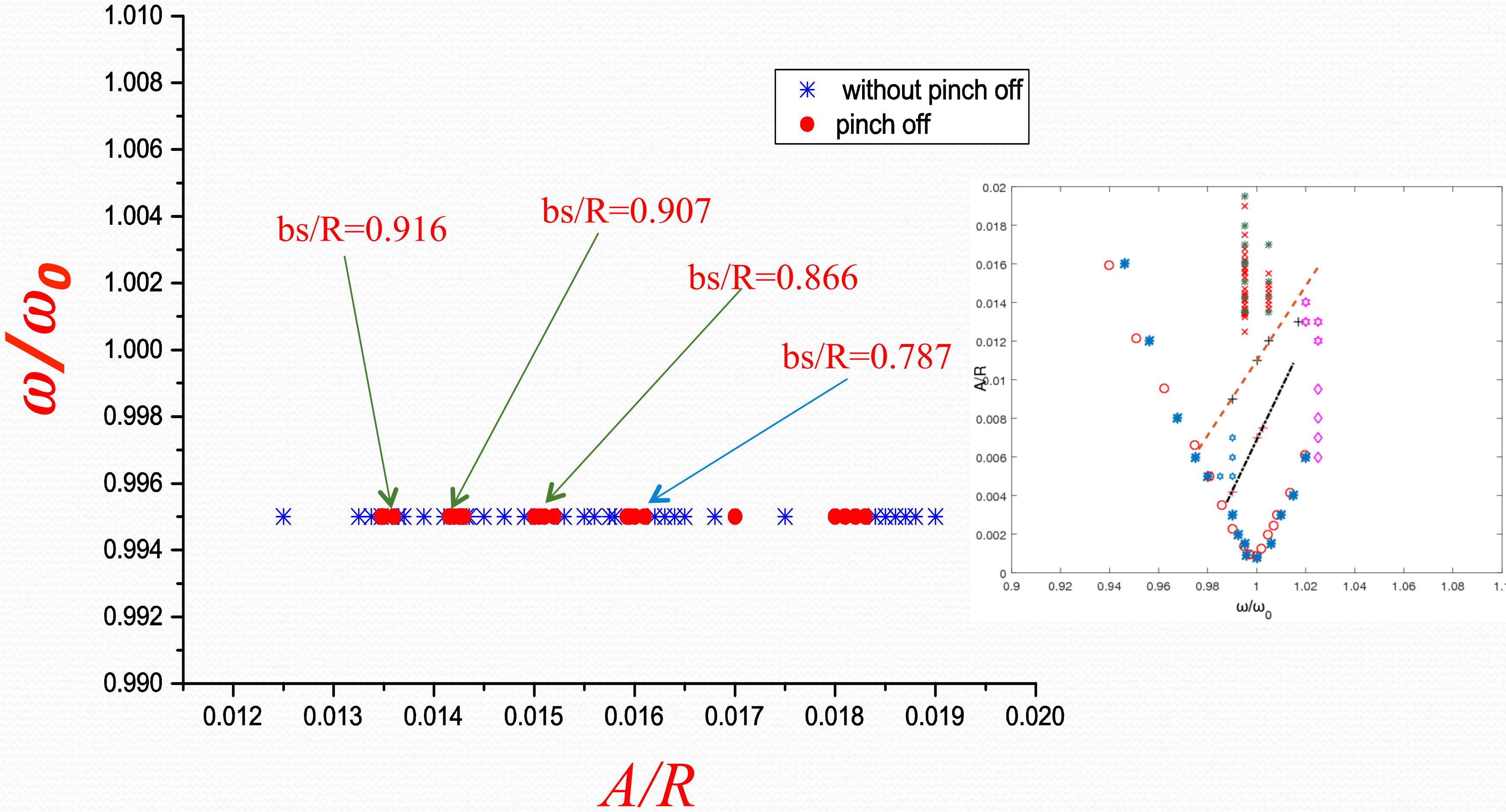




A composite of two photographs
- wave depression (cavity) below
- jet formation above

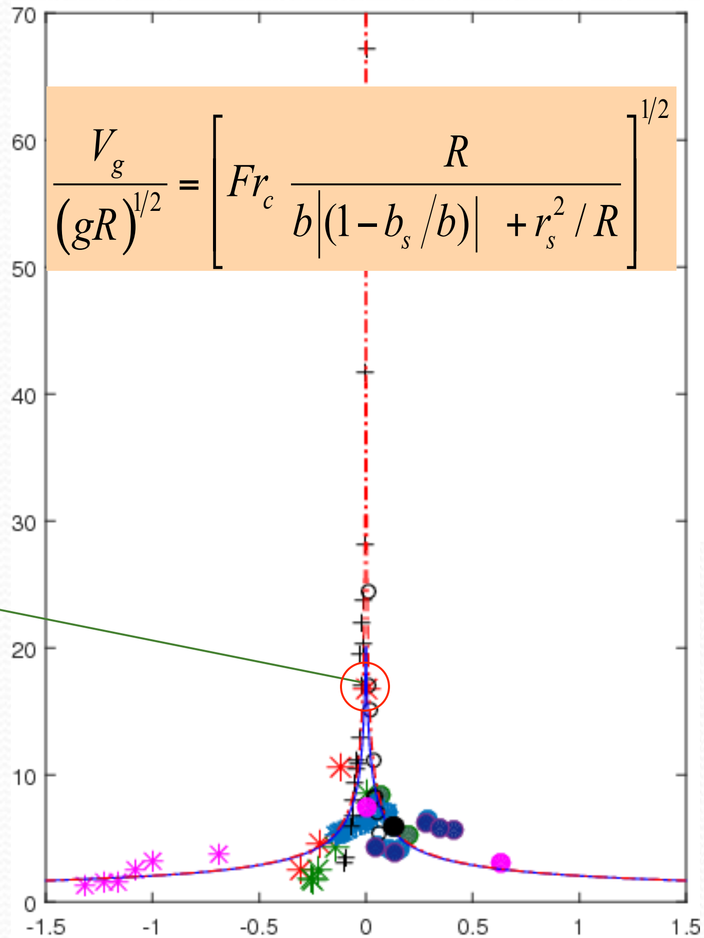
Jet velocity depends on:
- ❑ Surface tension, viscosity, velocity of fluid particle
- ❑ Size of cavity (depth, radius)

Bands of pinch-off



Scaling of jet velocity

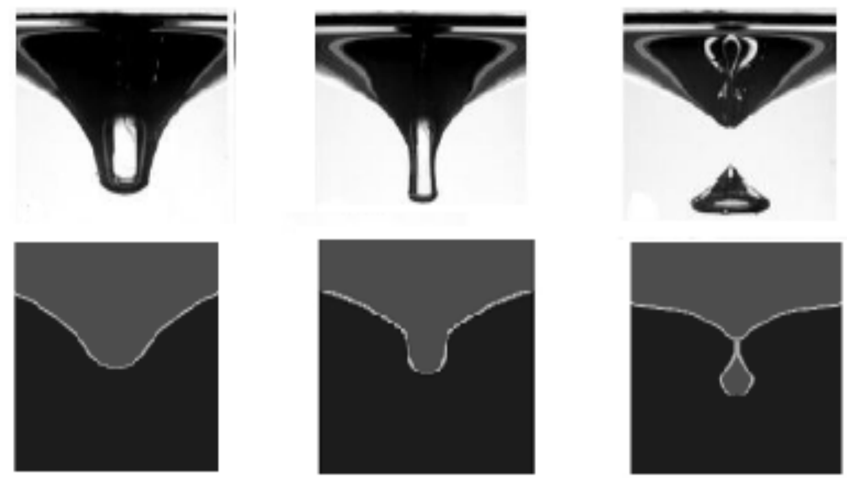
$$\frac{V_g}{(gR)^{1/2}} = \left[Fr_c \frac{R}{b\left|(1-b_s/b)\right| + r_s^2/R} \right]^{1/2}$$



A/R=0.013475,
ω/ω$_0$=0.995

Cavity shape

Glycerin water ν = 1.94 cm²/s , σ/ρ = 65 cm³/s² Zeff (nature 2000)



FC-72 ν = 0.0040 cm²/s σ/ρ = 6.5 cm³/s²

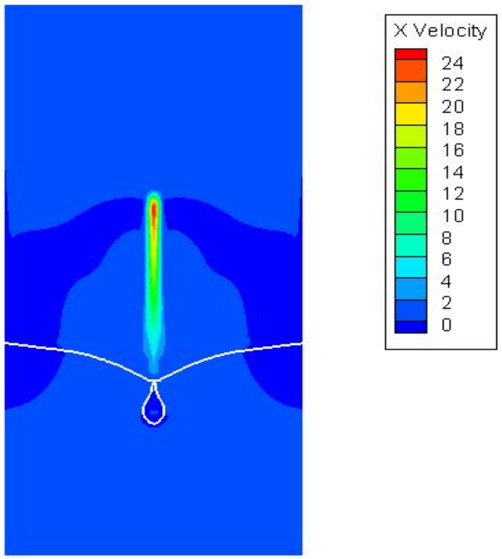
b/bs=0.916

b/bs=0.907

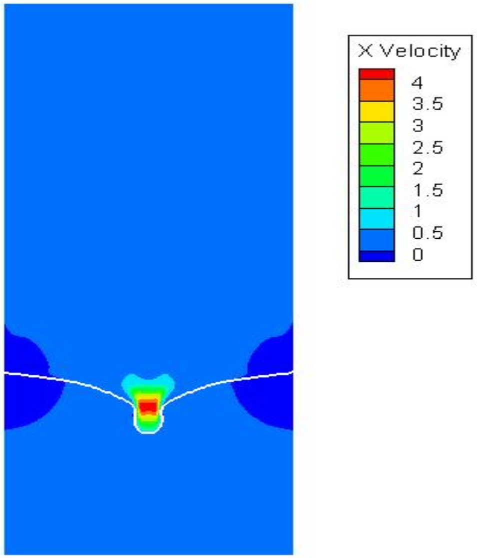
b/bs=0.866

b/bs=0.884, Das and Hopfinger (JFM,2008)

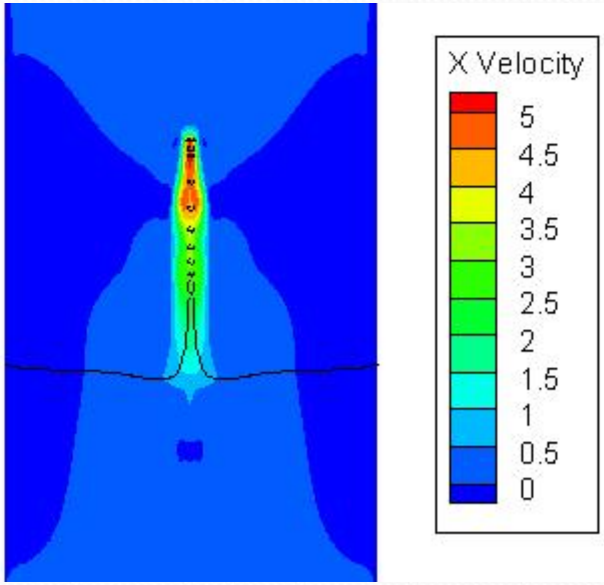
Velocity contour of jetting and cavity



Pinch-off cavity



No pinch off



Jetting

Conclusion

- Stability threshold is in good agreement with the experiments (Das & Hopfinger, 2008) and theory (Hendersen & Miles, JFM, 1984) theory
- Wave amplitude modulations in a slow time scale or period tripling or quadrupling in the stable wave regime
- Parametric instability of axisymmetric mode is of supercritical nature at and above natural frequency and subcritical below natural frequency
- Jet velocity (cavity collapse) scales on wave particle velocity
- Surface tension and viscosity affect the cavity size and its aspect ratio, hence the jet velocity

Thank you