On the near wall dissipation

Sedat TARDU

Frédéric BAUER

LEGI, B.P. 53 X 38041 Grenoble Cédex

France

Introduction

- Wall bounded turbulent flow (fully developed turbulent channel flow)
- Dissipation 12 terms:

$$\varepsilon_K^* = \nu \overline{\frac{\partial u_k}{\partial x_i}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right)}$$

- Very hard experimentally ➔ Local isotropy ➔ Yes or not? If yes where?
- Reynolds number dependence (hard both in DNS and experiments)
- *Fine structure* of the dissipation

*Dissipation statistics conditioned by fixed « amplitudes » of the velocity components (*level-crossing statistics*)

Sreenivasan & co-workers ➔ Contribution to the dissipation is maximum when the velocity goes locally through zero (stagnation points u=v=w=0, production by definition is zero, dissipation is maximum) ??

Intermittency and local inertia .. ?

DNS

*Large computational domains as in (Hoyas, Jiménez 2006)

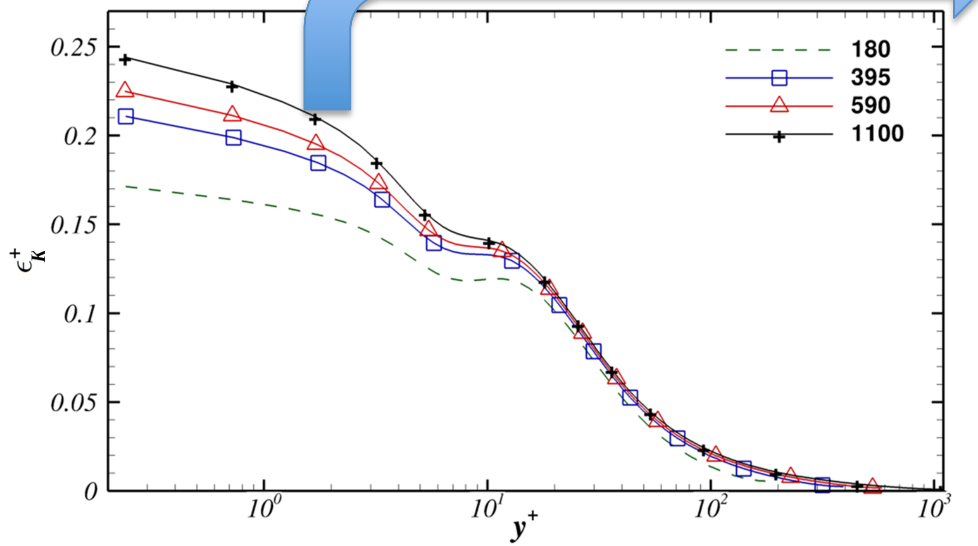
** NS with ***Dispersion Relation Preserving spatial schemes***, See Bauer, Tardu and Doche, Comp. Fluids 2014. Similar to compact schemes but 20% more rapid.

ALL THE QUANTITIES ARE SCALED BY THE INNER VARIABLES, viscosity and wall shear velocity, HEREAFTER

Re_τ	Re_τ actual	Resolution ($N_x x N_y x N_z$)	Δx^+	Δy^+	Δz^+	l_x/h	l_z/h
180	178	771x129x387	8.80	0.49 (0.31η) 5.59 (1.52 η)	5.84	12π	4 π
395	396	1691x283x849	8.81	0.48 (0.33η) 5.57 (1.26η)	5.85	12π	4 π
590	588	1651x423x1113	8.98	0.48 (0.34η) 5.56 (1.15η)	5.00	8 π	3 π
1100	1104	3079x789x2075	8.98	0.48(0.34η) 5.55 (0.98η)	5.00	8 π	3 π

Simulations parameters in the streamwise, wall normal and spanwise directions (x, y, z). Both smallest (first line) and largest (second line) grid spacing are given for wall normal direction. The number in parenthesis is the wall-normal grid spacing scaled by Kolmogorov length η.

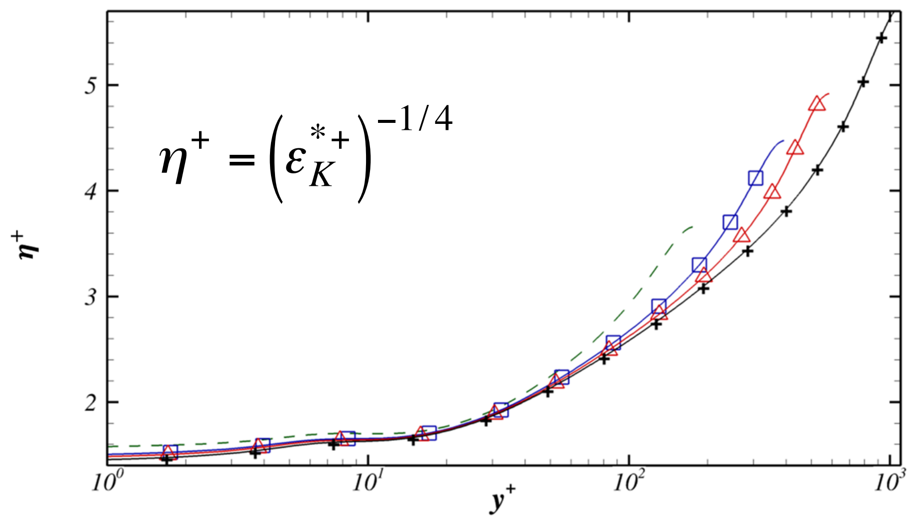
Mean Dissipation and Kolmogorov scale

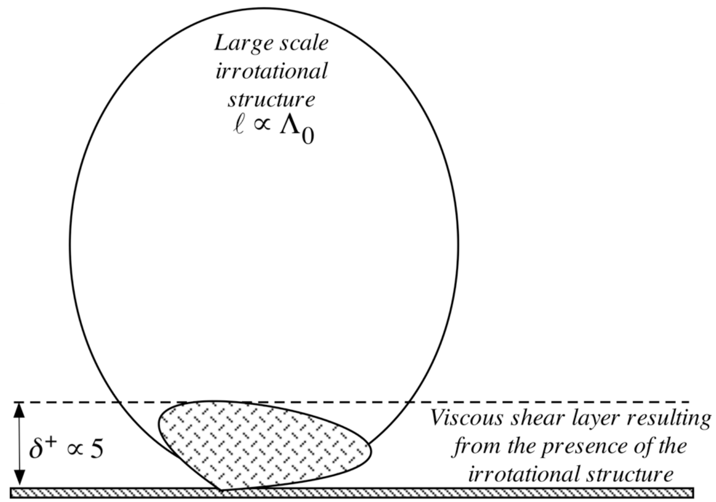


Re number dependence constrained to the low buffer and viscous sublayers: Large-scale, irrotational passive structures force the flow in the inner layer. The passive structures remain irrotational but penetrate to the wall and induce wall parcels of spanwise vorticity.

➔*Effect on the dissipation near the wall since the former is equal to the enstrophy at the wall.*

(Hoyas&Jiménez, 2008).



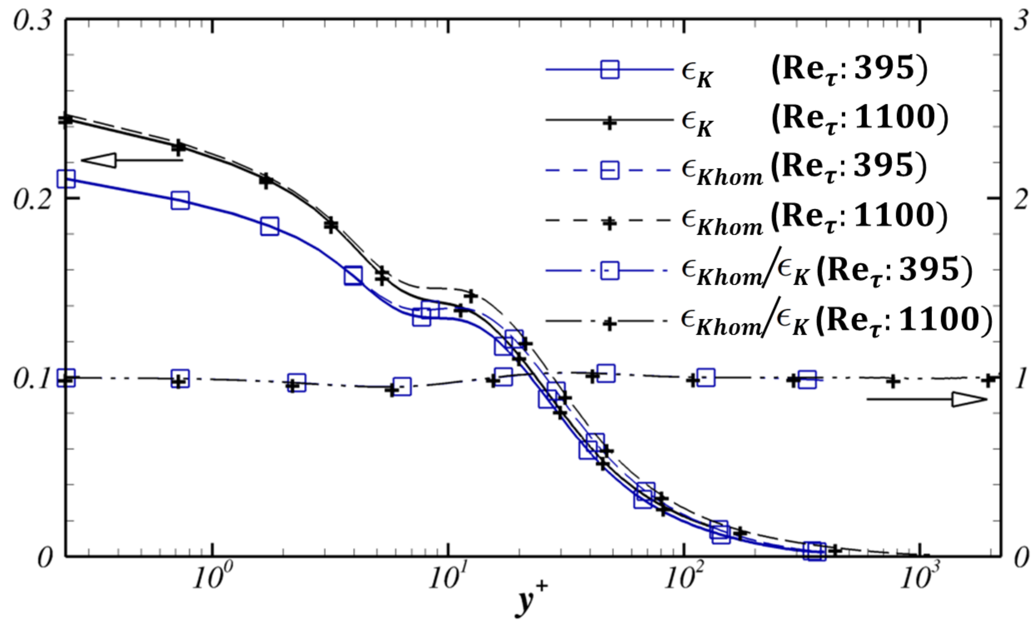


Approximations I

- 12 terms in $\varepsilon_K^* = \nu \overline{\frac{\partial u_k}{\partial x_i}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right)}$

- *HOMOGENEOUS,* $\varepsilon_{K\,hom}^* = \nu \overline{\frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_i}}$ *invariance with respect to translations and rotations* $\overline{\frac{\partial u_k}{\partial x_i}\frac{\partial u_i}{\partial x_k}} = 0$

➔ *good approximation*



Approximations II

Axisymmetric (Georges et al., 1991) –with respect to the streamwise direction. Two forms (u,v,w in the streamwise x, wall normal y and spanwise z directions):

$$\varepsilon^*_{K\,ax1} = \nu\left\{\frac{5}{3}\overline{\left(\frac{\partial u}{\partial x}\right)^2} + 2\overline{\left(\frac{\partial u}{\partial z}\right)^2} + 2\overline{\left(\frac{\partial v}{\partial x}\right)^2} + \frac{8}{3}\overline{\left(\frac{\partial v}{\partial z}\right)^2}\right\}$$

$$\varepsilon^*_{K\,ax2} = \nu\left\{-\overline{\left(\frac{\partial u}{\partial x}\right)^2} + 2\overline{\left(\frac{\partial u}{\partial y}\right)^2} + 2\overline{\left(\frac{\partial v}{\partial x_1}\right)^2} + 8\overline{\left(\frac{\partial v}{\partial y}\right)^2}\right\}$$

Isotropic:

$$\varepsilon^*_{K\,iso} = \left(\overline{\omega_i\omega_i}\right)_{iso} = 15\nu\overline{\left(\frac{\partial u}{\partial x}\right)^2}$$

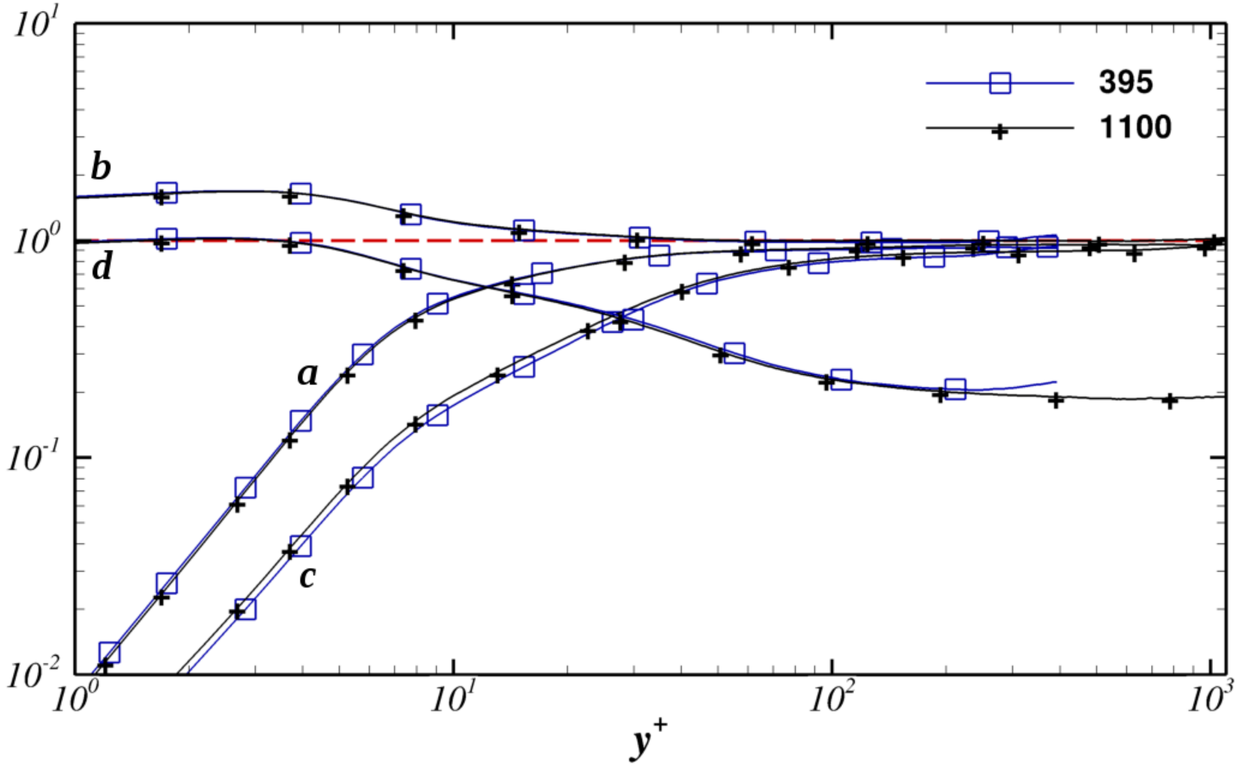
* *Near wall approximation (Antonia et al, 1991)*

$$\varepsilon^*_{K\,w} \approx \frac{5}{4}\nu\overline{\left(\frac{\partial u}{\partial y}\right)^2}$$

$$\varepsilon^{*+}_{K\,w_{y=0}} = D^{+*}_{K0} = \left(\overline{\omega_i\omega_i}\right)^+_0 = \left(\overline{\omega_z\omega_z}\right)^+_0 + \left(\overline{\omega_x\omega_x}\right)^+_0$$

$$= \frac{5}{4}\overline{\tau'\tau'}^+ \text{ (Streamwise Wall shear stress intensity !!!)}$$

Approximations II



(a) $\varepsilon^*_{K\,ax1}/\varepsilon^*_K$ (axisy. 1)

(b) $\varepsilon^*_{K\,ax2}/\varepsilon^*_K$ (axisy. 2)

(c) $\varepsilon^*_{K\,Iso}/\varepsilon^*_K$ (Isotropic)

(d) $\varepsilon^*_{K\,w}/\varepsilon^*_K$ (Near wall)

Conclusions approximations

*The axisymmetric form

$$\varepsilon^*_{K\,ax2} = \nu\left\{-\overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2} + 2\overline{\left(\frac{\partial u_1}{\partial x_2}\right)^2} + 2\overline{\left(\frac{\partial u_2}{\partial x_1}\right)^2} + 8\overline{\left(\frac{\partial u_2}{\partial x_2}\right)^2}\right\}$$

is acceptable through the whole layer.

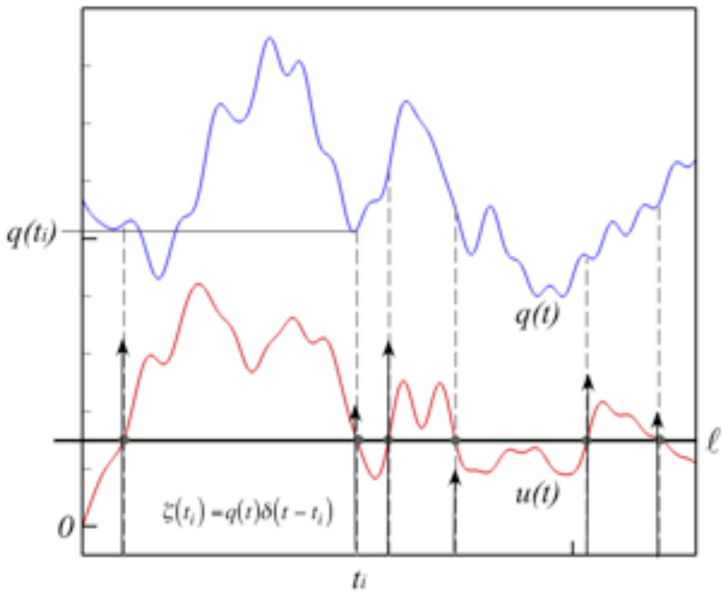
- One approaches (without exactly reaching) the local isotropy only in the meso-layer:

$$y^+ > 100$$

- In the viscous layer the spanwise component of enstrophy dominates.

CONDITIONAL (Palm) STATISTICS

- Determine the ensemble averages of the dissipation conditioned by level crossings of the velocity components. Palm statistics (DETAILS in *Tardu & Bauer, JoT 2015, Tardu PoF, 2016, Tardu&Bauer 2016*).



$$\hat{E}\left(q \middle| u_i = \ell\right) = \overline{q}_{\ell_{u_i}} = \frac{E\left\{q \times \sqrt{\left(\frac{\partial u_i}{\partial x}\right)^2 + \left(\frac{\partial u_i}{\partial z}\right)^2} \;\middle|\; u_i = \ell_{u_i}\sigma_{u_i}\right\}}{E\left\{\sqrt{\left(\frac{\partial u_i}{\partial x}\right)^2 + \left(\frac{\partial u_i}{\partial z}\right)^2} \;\middle|\; u_i = \ell_{u_i}\sigma_{u_i}\right\}}$$

Fine structure of the dissipation
Dissipation and zero-crossings

- Main argument advanced so-far (*Sreenivasan & coworkers*):

Liepmann scale (based on the zero-crossing frequency) is approximately equal to the Taylor scale (*which is correct*) ➔

$$f_0 \propto \sqrt{\overline{\left(\frac{\partial u}{\partial x}\right)^2}} \propto \bar{\varepsilon}_{iso}$$

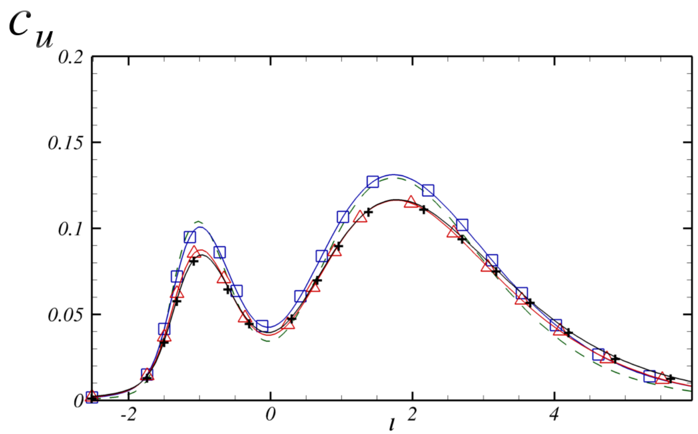
At zero-crossings where the local production is exactly zero, it is expected that the dissipation dominates.

Not sure: *Do not ***confuse*** mean dissipation and contribution of zero-crossings

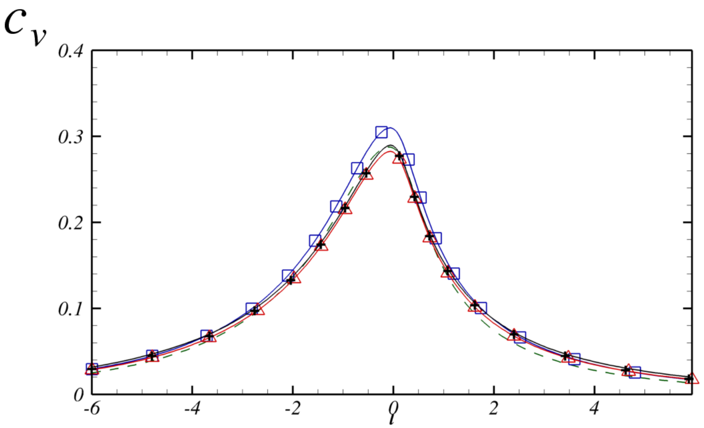
* Even if this argument reveals to be correct, the contribution can be dominant ***only*** for isotropic part (that is negligible in a large part of the wall layer)!

Spanwise crossings of the *wall normal velocity* dominates the dissipation next to the wall and **NOT** the streamwise velocity

- Contribution of the level-crossings to the dissipation in the viscous sublayer $y^+ = 1$



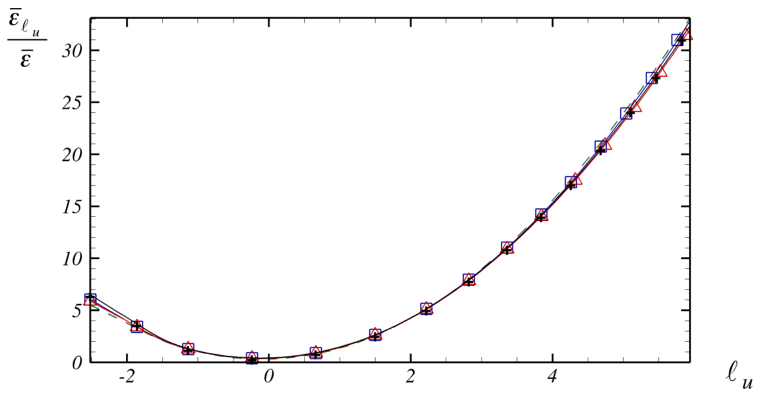
Contribution of u: note that the maxima are **not** at l=0 !



Contribution of the wall normal velocity Maximum at l=0 at 3 times larger than u

*That the mean dissipation is largest at the "stagnation points – u=v=w=0 that can**not coexist locally (can be shown theoretically*)*** *is* ***<u>incorrect</u>.***

- Mean dissipation (unbiased correct) at u and v crossings $y^+ = 1$

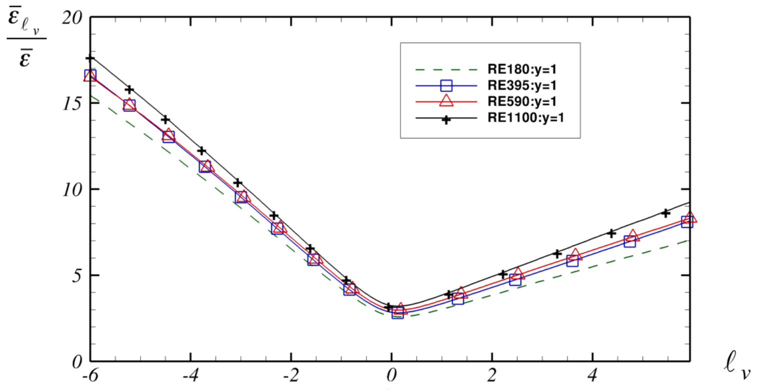


Streamwise velocity - level crossings

* $\bar{\varepsilon}_{\ell_u} \approx 0$ at $u = 0$!!!

* Large values at intermittent large - rare - ℓ_u

* Scale remarkably well with the local $\bar{\varepsilon}$

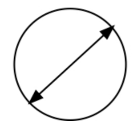


Wall normal velocity - level crossings

$\bar{\varepsilon}_{\ell_v} \neq 0$ but again large dissipation at large - rare - ℓ_v

LOCAL DISSIPATION DEPENDS ON u, v and w in the whole layer.

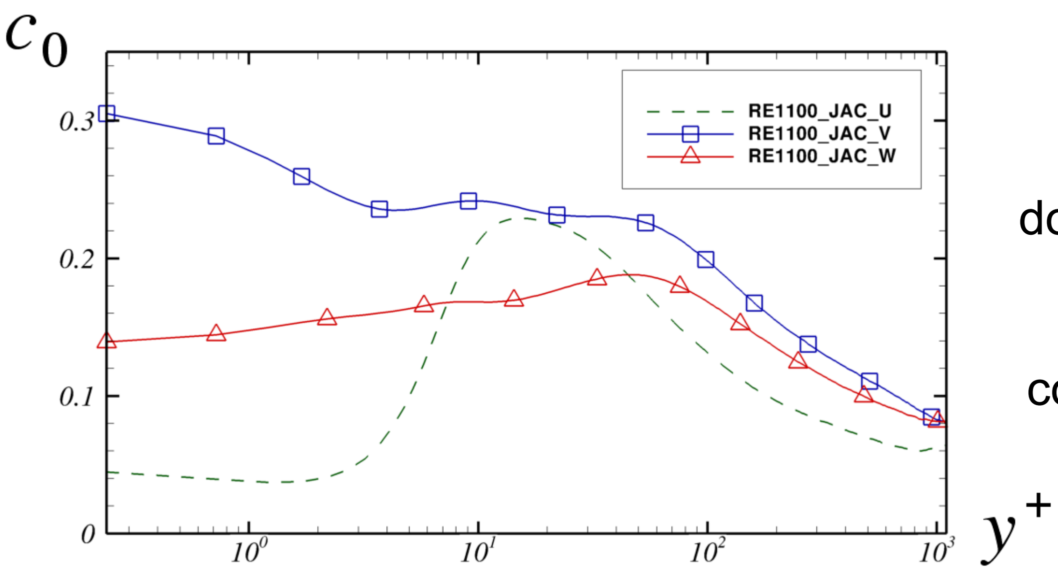
Possible within a structure of Kolmogorov scale η



* If $u_i = u_j = 0$ then $\frac{\partial}{\partial x_k}\left(u_i u_j\right) = 0$

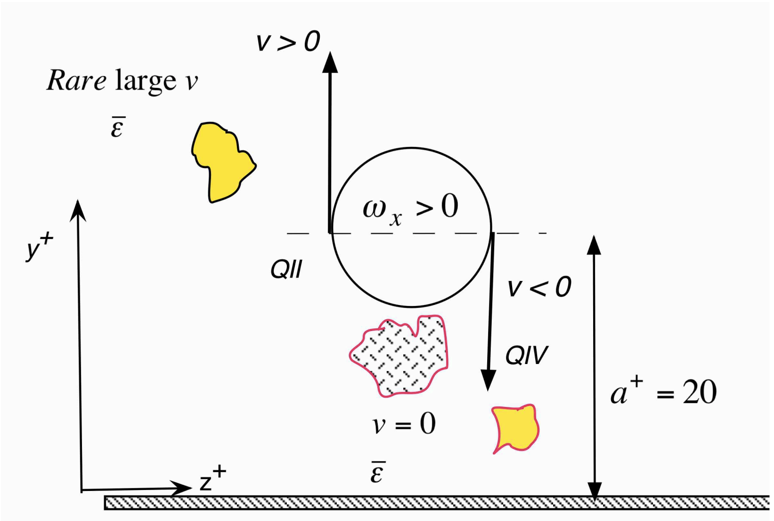
==> Tangential crossings of the shear stress signal $u_i u_j$ ==> Asymptotically zero (Ylvisaker 1962)

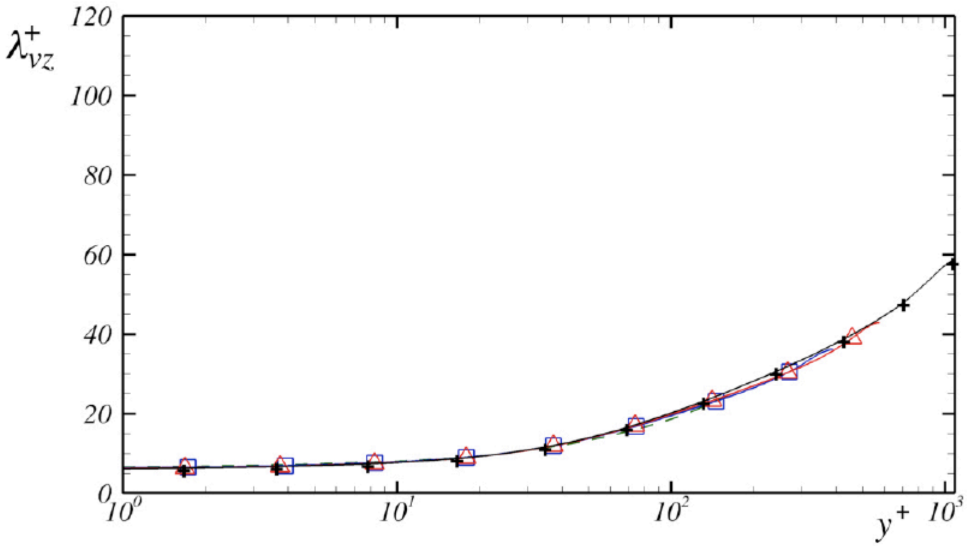
Contribution of the zero-crossings to the dissipation in the whole layer → CHANGER CETTE FIGURE



Wall-normal velocity crossings dominate the dissipation in the low buffer layer.
Beyond the buffer layer the contributions become weaker and of equal magnitude.

Comments





The contribution is directly related to the frequency of the level - crossings

$$c_\varepsilon \propto \frac{\bar{\varepsilon}_{\ell_{u_i} b}}{\bar{\varepsilon}} \frac{f_{\ell_{u_i}}}{f_s}$$

The frequency of level crossings follows well the Gaussian relationship :

$$f^+_{\ell_{u_{iG}}} = \frac{1}{\pi \lambda^+_{u_i}} e^{-\frac{\ell^2_{u_i}}{2}} = \frac{\sqrt{\overline{\left(\partial u_i / \partial x_j\right)^2}}}{\pi \sigma_{u_i}} e^{-\frac{\ell^2_{u_i}}{2}}$$

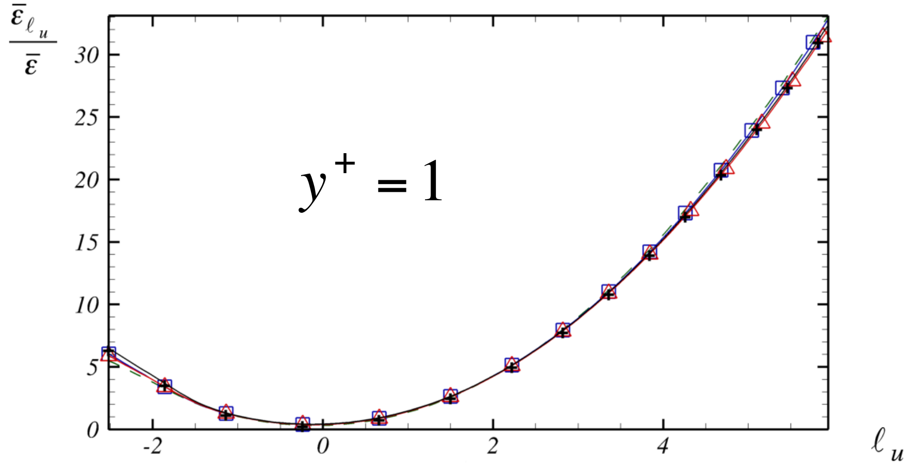
where $\lambda^+_{u_i}$ is the *Taylor scale*. The SMALLEST

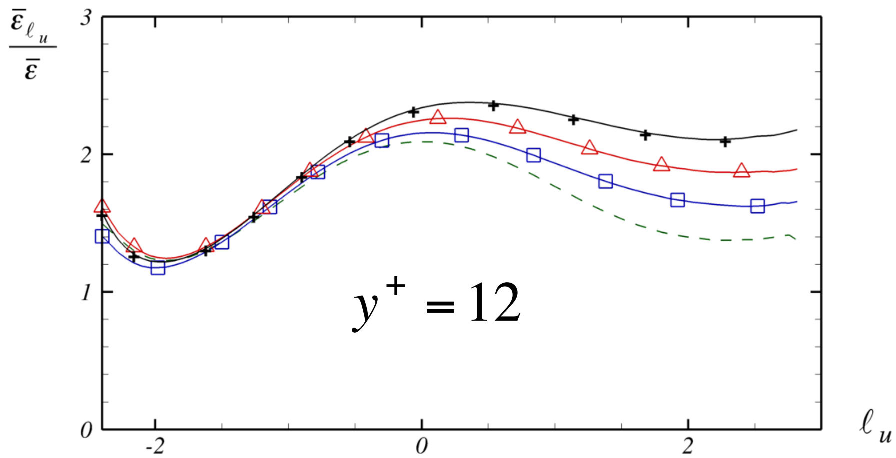
Taylor scale near the wall is that related to $\frac{\partial v}{\partial z}$ (λ^+_{vz}) :

==> *Contribution* of zero - crossings of v is largest

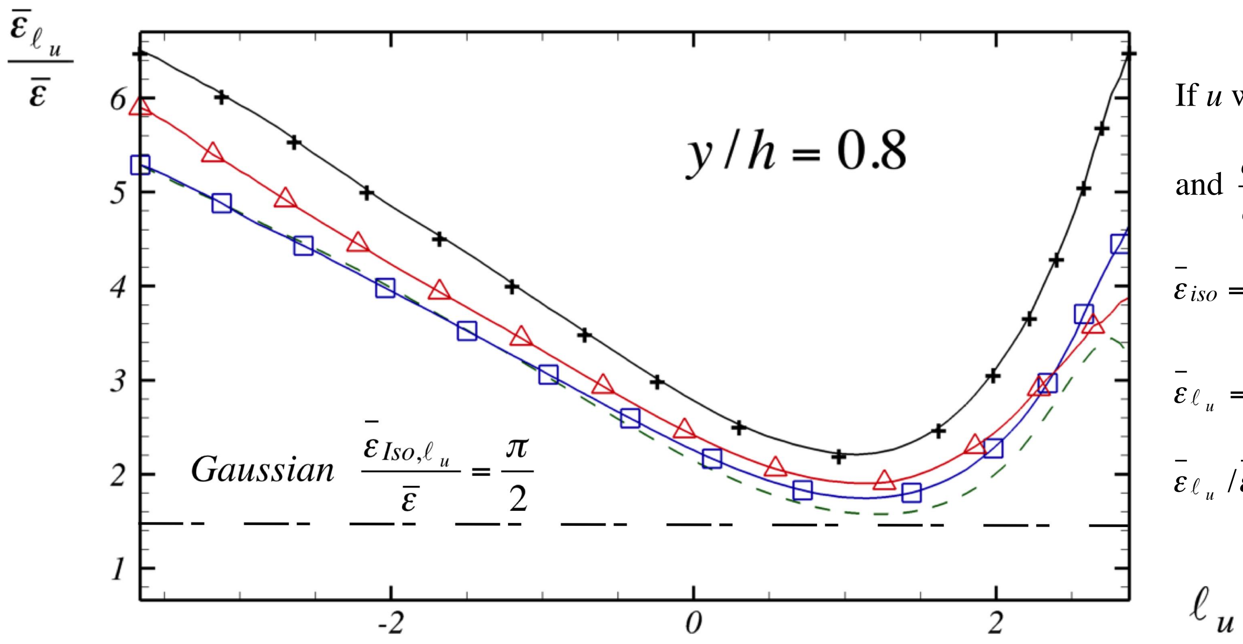
==> Carrefull for interpretation!!

The interdependence of the local dissipation and the velocity fluctuations becomes rapidly blurred above the viscous sublayer





Isotropic zone



If u was Gaussian then u and $\frac{\partial u}{\partial x}$ would be independent and $\frac{\partial u}{\partial x}$ would also be Gaussian.

$$\bar{\varepsilon}_{iso} = 15\nu \overline{\frac{\partial u}{\partial x}^2}$$

$$\bar{\varepsilon}_{\ell_u} = 15\nu \frac{\pi}{2} \overline{\frac{\partial u}{\partial x}^2} \quad \text{(streamwise level crossings)}$$

$$\bar{\varepsilon}_{\ell_u} / \bar{\varepsilon}_{iso} = \frac{\pi}{2} \quad \text{(independent of the threshold } \ell_u\text{)}$$

Intermittency (long tails of the pdf's) even within the meso - layer (particularly at $\ell_u < 0$)

Discussion-I

$$E\left(\varepsilon_K \middle| u_i = \ell_{u_i}\sigma_{u_i}\right)/\bar{\varepsilon}_K = f\left(\ell_{u_i}\right)$$

Dissipation - seen as a random variable - is statistically dependent on u_i

Random variable $\varepsilon_K = \varepsilon_{KHom} = \nu \frac{\partial u_k}{\partial x_j}\frac{\partial u_k}{\partial x_j}$

==> Conditional joint probability density is such that - dependence- :

$$p\left(\frac{\partial u_k}{\partial x_j}, u_i\right) \neq p\left(\frac{\partial u_k}{\partial x_j}\right)p\left(u_i\right)$$

That also implies - Correlated - :

$$E\left(\varepsilon_K u_i\right) \neq E\left(\varepsilon_K\right)E\left(u_i\right) \neq 0$$

Therefore :

$$E\left(\varepsilon_K u_i\right) = E\left(u_i \frac{\partial u_k}{\partial x_j}\frac{\partial u_k}{\partial x_j}\right) = E\left(\frac{\partial u_k}{\partial x_j}\frac{\partial u_i u_k}{\partial x_j} - \frac{1}{2}\frac{\partial u_k^2}{\partial x_j}\frac{\partial u_i}{\partial x_j}\right) \neq 0$$

or :

$$E\left(\frac{\partial u_k}{\partial x_j}\frac{\partial u_i u_k}{\partial x_j}\right) \neq \frac{1}{2}E\left(\frac{\partial u_k^2}{\partial x_j}\frac{\partial u_i}{\partial x_j}\right)$$

AND there is NO REASON that should not be the case

Discussion 2

Note the appearence of the anisotropy tensor (seen as a random variable :

$$b_{ik} = \frac{u_i u_k}{2K} - \frac{\delta_{ik}}{3}$$

==> The interdependence of the local dissipation is blurred towards the outer layer wherein the dissipation is well approximated by local isotropy.

BUT $E\left(\varepsilon_{KIsot} \middle| u_i = \ell_{u_i}\sigma_{u_i}\right)/\bar{\varepsilon}_K = f\left(\ell_{u_i}\right)$ because u and $\dfrac{\partial u}{\partial x}$ are STILL dependent.

(Gaussianity is NEVER reached otherwise recall that one should have :

$$\frac{\bar{\varepsilon}_{iso,\ell_{\mathrm{u}}}}{\bar{\varepsilon}} = \frac{\pi}{2}$$

which is NOT the case.

Main Conclusions

- Local isotropy approximation can only be in the meso-layer.
- Level-crossing statistics provide a nice way to show that isotropic turbulence is not Gaussian
- The conditional dissipation reaches values as large as 30 times the local mean at rare intermittent level-crossings in the low buffer layer.
- The interdependence of the local dissipation and the velocity fluctuations becomes rapidly blurred above the viscous sublayer and is a function of both intermittency and anisotropy.