Internal wave in the ocean, local, global stability and transient growth

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Abstract

Internal gravity waves that exist in a continuously stratified fluid are particularly important in the ocean. They transport energy and are thought to generate turbulent mixing which contributes to the deep ocean circulation. Through fully non-linear and linear direct numerical simulations the stability of a gravity wave beam, we show that the stability properties and transient growth intensity strongly depend on the mean flow intensity. Small scale instabilities dominate for small mean flow as for internal tide. For lee waves or more generally strong mean flows, large scales lead the instability but small scales dominate the short time growth. Computing the linear impulse response of a monochromatic internal wave we propose an interpretation based on an extension of absolute and convective theory to 2D periodic base flow and make the connection with the classical triadic instability theory [1].

1 Non-linear global mode

We compute in a two dimension domain the stability properties of an internal gravity wave beam solution of the non-linear Navier-Stokes equation under the Boussinesq approximation and a linear stratification. The wave is generated using a penalization localized in space that forces an internal gravity wave of specific horizontal wavelength and frequency propagating down as in the experiment [2]. In this case equivalent to a tidal wave where the tidal flow oscillates over a topography and generates a wave, experiment results of [2] are recovered. The beam destabilizes (see figure 1 (b)) and we show that the instability mechanism involves a small triadic resonance. To extend the results of [2], we consider the effect of a horizontal mean advection velocity U_∞. The forcing frequency ω' in the wave maker frame is $\omega' = \omega_0 + U_\infty k_{0x}$ where ω_0, the frequency in the fluid frame and $U_\infty k_{0x}$ the doppler effect. In all the computations ω_0 is kept constant in order to have locally in the fluide the same wave. A limit case appears when mean advection velocity of $U_\infty = -U_\phi$ where $U_\phi = \omega_0/k_{0x}$ then ω' decreases to zero: the forcing is stationary. This case is equivalent to a lee wave appearing when a stratified fluid flows over a topography.

We show that the stability property of beams made locally of the same gravity wave strongly depends on mean advection velocity. For small magnitude of advection velocity, small scale instabilities develop as in the tidal case. Then the beam stabilizes at intermediate advection velocity and destabilizes again when we keep increasing the advection. At this second threshold, down to the lee wave case, the instability is of much larger scale than for the tidal case reported by [2]. Increasing the Reynolds number or the beam size makes the stable domain to disappear but the instability scale selection stays the same with small scale instability in the tidal rgime with scale decreasing when increasing the Reynold number, and large scale instability in the lee rgime with scales indpendante on the Reynolds number or beam size.

We show that the instability may be interpreted using the triadic instability [1] and that scale selection corresponds to different branch of triadic resonance. We confirm the presence of a stability region for intermediate value of the mean advection velocity by computing the linear eigenmode as Floquet mode with an Arnoldi-Krylov technique and showing that the leading eigenmode has a negative growthrate.

2 Linear global stability and transient growth

We confirm the presence of a stability region for intermediate value of the mean advection velocity by computing the linear eigenmode as Floquet mode with an Arnoldi-Krylov technique and show that the leading eigenmode has a negative

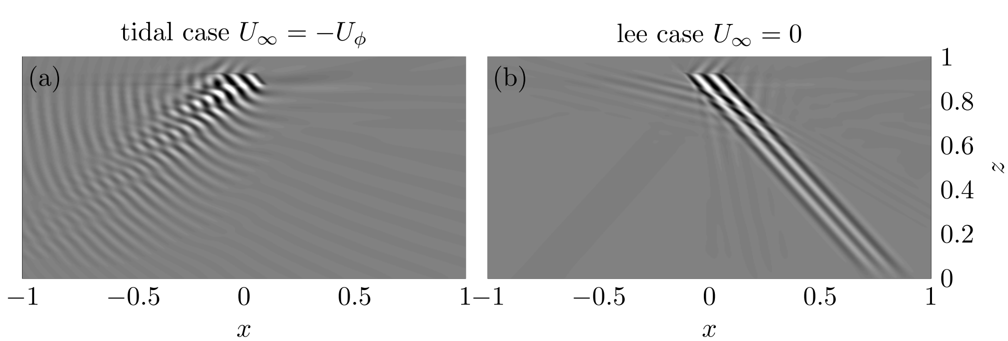

Figure 1: Buoyancy perturbation field of an internal wave beam simulation for the two limit case, the lee (a) and the tidal (b) case.

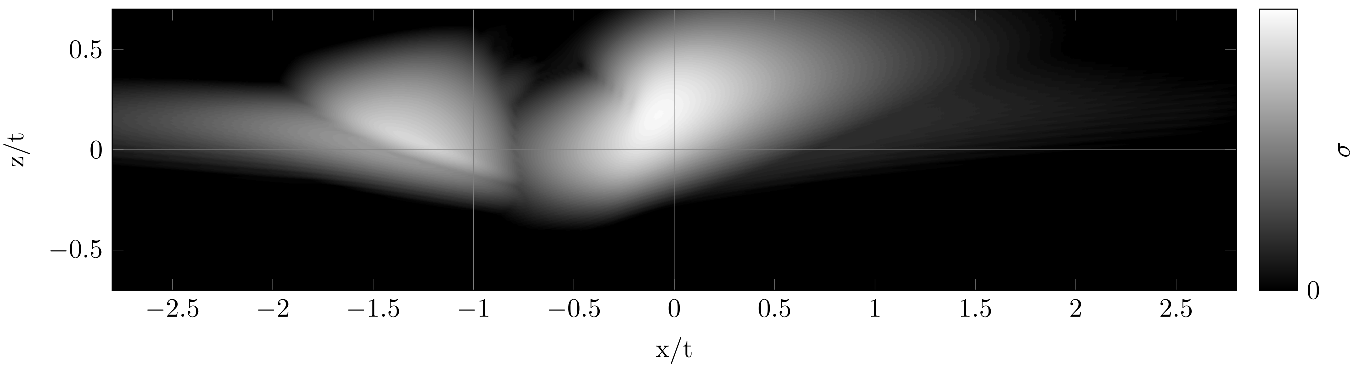

Figure 2: Energy growth rate of the impulse response along rays of different velocities starting from the initial impulse

growthrate. In the lee wave case the flow is unstable and a selective frequency damping method [3] is used to first compute the steady base flow solution. Then we implement a direct-adjoint method to access the optimal perturbation that maximize the total energy growth at different time horizon using a similar Arnoldi-Krylov technique. At short time horizon, the optimal perturbation is small scale and is advected by the flow while at large time, the perturbation switches to a large scale solution and converges to the perturbation characteristic observed through nonlinear simulation (previous section). Analyzing optimal perturbations and optimal responses in fourrier space seems to indicates that small scale short time transient corresponds to the small scale triadic instability advected by the flow whereas the long time large scale instability corresponds to large scale bra,nah of the triadic instability that is able to sustain the flow.

3 Linear local stability

We propose an interpretation of the different instability selection in term of absolute and convective instability. We conjecture that, In the case of the lee wave, the large scale instability is absolute whereas the small scale instability is convective and leads short time transient growth because its local growth rate is larger. When the mean advection flow is varied the properties of small scale and large scale instabilities exchange and in the tidal case the short scale instability is absolute and the large scale convective. This conjecture is confirmed by computing the impulse response around a plane monochromatic internal gravity wave in an extended two dimension periodic domain. The spatiotemporal evolution of a localized in space and time perturbation (a multipolar gaussian vorticity perturbation) evidences the formation of three different wavepackets. From their fourrier space characteristic, we show that each of these wave packets corresponds to a different branch of triadic instability. Two of the wavepacket are small scale and have a small group velocity compared to the fluid while the third wave packet is large scale and has a much larger group velocity compared to the fluid. Using the triadic theory with finite detuning [4] we derive the group velocity at the maximum growthrate of the three different branches of triadic instability and find good agreement with the velocity of the three wave paquet maxima in the impulse response.

Analyzing the impulse response energy not versus space but along rays i.e. at x/t and z/t constant (fig. 2), we compute the absolute growth rate along all possible rays and show that our conjecture seems valid.

In the lee reference frame the large scale mode corresponding to the large scale triadic instability has a positive absolute growth rate and therefore the lee wave is absolutely unstable toward the large scale mode whereas in the tidal reference frame the small scale mode corresponding to the small scale triadic instability has a positive absolute growth rate and therefore the tidal wave is absolutely unstable toward the small scale mode.

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