Production of dissipative vortices by solid boundaries in 2D flows: comparison between Prandtl, Navier-Stokes and Euler solutions

Marie Farge*†[1], Romain Nguyen Van Yen[2], and Kai Schneider[3]

[1]Laboratoire de Météorologie Dynamique (LMD) – Ecole Normale supérieure de Paris – France
[2]Freie Universität Berlin – Germany
[3]Laboratoire de Mécanique, Modélisation et Procédés Propres (M2P2) – Aix Marseille Université – France

Abstract

Turbulent boundary layers are ubiquitous in geophysical fluid flows and we will study the Reynolds number dependence of the drag due to the interaction between topography and atmospheric flows, or between basins and oceanographic flows. For this we will revisit the problem posed by Euler in 1748, that lead d'Alembert to formulate his paradox and address the following problem: does energy dissipate when a boundary layer detaches from a solid boundary in the vanishing viscosity limit? To trigger detachment we consider a vortex dipole impinging onto a wall and we compare the numerical solutions of the Euler, Prandtl, and Navier-Stokes equations. We observe the formation of a boundary layer whose thickness scales as predicted by Prandtl's 1904 theory. But after a certain time Prandtl's solution becomes singular, while the Navier-Stokes solution collapses down to a much finer thickness. We then observe that the boundary layers rolls up into vortices which detach from the wall and dissipate a finite amount of energy, even in the vanishing viscosity limit, in accordance with Kato's 1984 theorem.

Reference

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Phys. Rev.Lett., 106(8), 1-4

*Speaker
†Corresponding author: marie.farge@ens.fr